Monetary and Fiscal Policy According to HANK-IO

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Abstract

This paper studies monetary and fiscal policy transmission in a multi-sector heterogeneous-agent New Keynesian model with an input-output network ("HANK-IO"). We document systematic household-sector linkages in micro data and calibrate our model to match them. To identify when these linkages have implications for policy transmission, we analytically characterize an as-if benchmark that features a strict decoupling between household and sectoral heterogeneity. Away from this benchmark, novel earnings and expenditure heterogeneity channels emerge that govern the propagation of demand and supply shocks. Quantitatively, these channels shape the transmission of stabilization policy to aggregates, as well as its distributional and sectoral consequences.

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1 Introduction

Economists have made substantial progress in documenting and modeling the implications of cross-sectional heterogeneity for policy transmission in disaggregated economies. Recent work on household behavior has emphasized the importance of heterogeneity in spending propensities and the incidence of shocks. On the production side, a long tradition of research has documented sectoral heterogeneity and studied its importance for the propagation of shocks in multi-sector models. This paper takes as its starting point a set of empirical regularities that point to systematic linkages *between* households and sectors at a disaggregated micro level.

Motivated by these stylized facts, this paper proposes a new framework to study and quantify the implications of household-sector linkages for the transmission of monetary and fiscal policy. We develop a multi-sector heterogeneous-agent New Keynesian model with an input-output production network, which we call the "HANK-IO" model.[1] Our framework is inspired by both the burgeoning heterogeneous-agent New Keynesian (HANK) literature (Kaplan et al., 2018; Auclert et al., 2023) and the long tradition of multi-sector business cycle models (Long and Plosser, 1983). We analytically characterize an as-if benchmark that features a strict decoupling between household and sectoral heterogeneity. Away from this benchmark, novel earnings and expenditure heterogeneity channels may shape the propagation of demand and supply shocks. After disciplining our model empirically to match the household-sector linkages we document in micro data, we show that these channels contribute quantitatively to the transmission of policy to aggregates, as well as the distributional and sectoral consequences of policy.

Our baseline economy departs from a canonical HANK model (Auclert et al., 2023) and enriches its production side by introducing multiple sectors and input-output linkages (Baqaee and Farhi, 2020). Households face idiosyncratic uncertainty that leads them to make different consumption, savings, and portfolio decisions (ex-post heterogeneity). We also allow for permanent differences in household characteristics or types (ex-ante heterogeneity). In particular, household types may differ in their preferences over consumption goods and their labor endowments. Production in our economy takes place across a rich network of sectors, and we allow for sectoral heterogeneity across factor and input shares, input-output linkages, competitiveness, and price rigidity. Our key point of departure from the canonical HANK framework is that we allow for systematic links between household types and production sectors in terms of both earnings and expenditure patterns. We discipline these household-sector linkages empirically using micro data.

As-if benchmark. We begin our analysis with an instructive *as-if* benchmark to illustrate conceptually under what conditions household-sector linkages may play a role for policy transmission. Our as-if benchmark assumes that all households share the same homothetic preferences over consumption goods and there is a single labor factor. Under these two assumptions, there is no

[1] We choose this name in memory of the late Emmanuel Farhi whose work with David Baqaee on heterogeneous-agent economies with input-output production networks ("HA-IO") has inspired this paper (Baqaee and Farhi, 2018).

role for earnings or expenditure heterogeneity as households are symmetrically exposed to price inflation and changes in labor demand. Our first result is that the HANK-IO model features a strict decoupling between household and sectoral heterogeneity under these two assumptions. We show that the macroeconomic dynamics of our as-if benchmark admit an intertemporal IS-LM representation as a system of two forward-looking equations: a dynamic IS curve that determines output as a function of the time path of real interest rates, and a dynamic LM curve that pins down real interest rates as a function of future aggregate demand. The dynamic IS curve is shaped by household but not by sectoral heterogeneity: Given a path of real interest rates, it takes the same form as the IS equation of a canonical one-sector HANK model. Conversely, the dynamic LM curve is shaped by sectoral but not by household heterogeneity: It maps a given path of aggregate demand to the same path of real interest rates as a canonical representative-household multi-sector model. The as-if benchmark therefore highlights that household and sectoral heterogeneity are decoupled in this sense in the absence of earnings and expenditure heterogeneity.

Monetary and fiscal policy transmission in the as-if benchmark is characterized by an Intertemporal Keynesian Cross. When the monetary authority adopts a rule that neutralizes real interest rate effects, fiscal policy transmission is governed by the same Keynesian multiplier as in the one-sector HANK model of Auclert et al. (2023). Intertemporal marginal propensities to consume (iMPCs) remain a sufficient statistic for the output effects of government spending. This result obtains in spite of substantial sectoral heterogeneity that is masked by the dynamic LM equation. For a given path of real interest rates, the aggregate fiscal multiplier is therefore unaffected by sectoral heterogeneity and, in fact, identical to that in any HANK economy that admits the same IS curve representation.

Likewise, for a given change in the path of real interest rates, monetary policy transmission is solely governed by household heterogeneity. The sufficient statistics for policy are again identical to those in a one-sector HANK economy. Sectoral heterogeneity does, however, affect the transmission from nominal to real rates, which is governed by the LM curve. Our result shows that once the monetary authority has implemented a desired path of real rates, the production network structure of the economy no longer matters for policy transmission—the path of real rates is a summary statistic for the effect of monetary policy on aggregate activity. These results are manifestations of our decoupling result under the IS-LM representation: Demand shocks interact with the IS curve and are shaped by household but not by sectoral heterogeneity.

Finally, we unpack the LM curve and derive an aggregation result that traces the macroeconomic effects of sectoral technology shocks in our as-if benchmark. We decompose the impact on real GDP into a pure technology effect—accounting for increased productivity of resources at a given allocation—and changes in allocative efficiency—summarizing the effects on output from a reallocation of resources across firms and workers. Remarkably, our aggregation result for the as-if benchmark is identical to that of Baqaee and Farhi (2020). This is despite our economy featuring rich and dynamic household heterogeneity, whereas theirs is a static representative-household setting.

For given changes in markups and factor shares, the aggregate consequences of microeconomic technology shocks are not directly shaped by household heterogeneity. In other words, changes in sectoral markups and the labor income share are sufficient statistics for the implications of household heterogeneity.

The role of household-sector linkages. Away from the as-if benchmark, household-sector linkages shape the transmission of policy and shocks through novel earnings and expenditure heterogeneity channels. Our next analytical result characterizes an Intertemporal Keynesian Cross for monetary and fiscal policy away from the as-if benchmark. When households supply different labor factors, an *earnings heterogeneity channel* emerges. It is captured by a cross-sectional covariance across household types between iMPCs and changes in households' earnings shares. Earnings heterogeneity amplifies the aggregate effects of policy when it leads to a redistribution of income shares to factors (and in states of the world) with large spending propensities.

When households consume different consumption baskets, an *expenditure heterogeneity channel* emerges that comprises two distinct effects. A change in the price path of a household's basket elicits income and intertemporal substitution effects. If relative prices increase for households (and in states of the world) with large spending propensities, then the resulting drop in their effective purchasing power leads to a fall in aggregate consumer spending. This effect is captured by a covariance across household types between iMPCs and changes in relative bundle prices. Moreover, when households experience different rates of inflation in their respective bundle prices, then their effective real rates of return on savings differ as well. The expenditure heterogeneity channel of policy then also comprises a covariance across households between their relative rates of inflation and intertemporal interest rate response elasticities. Intuitively, if relative real savings rates increase for those household types that have large intertemporal elasticities of substitution, then the aggregate effect of policy is dampened.

Finally, we derive an aggregation result for sectoral technology shocks. Unlike in the as-if benchmark, gains from allocative efficiency are now shaped by household-sector linkages. Expenditure heterogeneity has implications for allocative efficiency because changes in purchasing power affect household labor supply and consequently firms' cost structure. This new force is governed by a covariance across household types between factor shares and changes in the relative price of households' consumption baskets: If consumer prices increase especially for those households whose labor factors have large cost-based Domar weights, then output falls and allocative efficiency deteriorates. Intuitively, when the price of a household's consumption bundle increases, a given nominal wage has less purchasing power and the real wage falls. If prices increase for households whose factors play a dominant role in firms' cost structures (large cost-based Domar weights), then output falls. Crucially, the relevant covariance is with respect to *cost-based* factor shares because markups drive a wedge between prices and marginal costs. Earnings heterogeneity also shapes the transmission of technology shocks to allocative efficiency.

When the overall income share falls, households receive less labor income, which elicits a positive labor supply response and increases output. Heterogeneity in factor share responses can amplify or dampen the resulting change in allocative efficiency. When factors with the largest cost-based Domar weights experience a relative decrease in their income share, then resources are reallocated towards the more monopolized and distorted sectors. The strength of this effect is governed by households' labor supply elasticities.

Our results point to an important conceptual distinction between demand and supply shock propagation in HANK-IO. What matters for the aggregation of sectoral technology shocks—in particular their transmission through changes in allocative efficiency—are covariances with respect to cost-based factor shares. Gains from allocative efficiency result from a reallocation of resources to relatively more distorted and monopolized sectors. The cost-based factor share precisely captures those markups that drive a wedge between prices and marginal costs, and it is the appropriate measure to capture whether resources find more productive uses. The transmission of monetary and fiscal policy (demand) shocks, on the other hand, is determined by covariances with respect to iMPCs. Intuitively, demand propagation is governed by spending propensities in response to changes in income and prices.

Empirical and quantitative analysis. Drawing in parts on existing and novel results, we document empirical regularities that speak to systematic linkages between households and sectors at the micro level. On the expenditure side, higher-income households spend relatively more on sectors that (*i*) have higher labor but (*ii*) lower capital shares, and (*iii*) are more central in the investment network. Middle-income households spend relatively more on sectors that (*iv*) have high intermediates shares, (*v*) are more flexible, (*vi*) are more network-central, (*vii*) have lower markups, and (*viii*) have a higher government spending share. On the earnings side, we show that higher-income households earn relatively more from sectors that (*i*) have higher intermediates and (*ii*) capital shares but (*iii*) lower labor shares, (*iv*) are less price rigid, (*v*) are more network-central, (*vi*) have lower markups, (*vii*) a higher government spending share, and (*viii*) a higher investment share. While these empirical regularities point to a potential role for the interaction of household and sectoral heterogeneity in the propagation of shocks, we currently lack a framework to assess their quantitative importance.

To match these empirical regularities on household-sector linkages, we develop a quantitative model that enriches our analytical framework along two dimensions, introducing capital as an additional production factor and nonhomothetic CES consumption preferences. Allowing households to trade a second, illiquid asset is important to match the intertemporal marginal propensities to consume that govern the new earnings and expenditure heterogeneity channels of policy. Explicitly modeling capital formation also allows us to capture the concentrated investement network documented by Vom Lehn and Winberry (2022). Finally, estimating a nonhomothetic CES demand system allows us to match empirical expenditure patterns across the income distribution (Comin et al., 2021). We calibrate our model to match the income and wealth distribution of households, key moments of sectoral heterogeneity across a 22-sector production network, and the household-sector linkages we document empirically.

Our quantitative analysis yields four main results. First, sectoral heterogeneity *amplifies* monetary and fiscal policy transmission relative to a benchmark HANK model with a single sector. As in prior work on multi-sector RANK economies (Pasten et al., 2017; Rubbo, 2023), this amplification is primarily driven by heterogeneous price rigidities across sectors and the input-output network. Second and in contrast to this first result, household-sector linkages *dampen* the aggregate effects of monetary and fiscal policy (by 31% and 29%, respectively). Our quantitative HANK-IO framework thus yields a new perspective on the implications of sectoral heterogeneity: While sectoral heterogeneity in price rigidities and input-output linkages amplify policy transmission, accounting for household-sector linkages in models with both household and sectoral heterogeneity dampens it. Third, the aggregate on-impact multiplier of targeted fiscal policy is 6% larger than that using empirically observed expenditure weights. Finally, household-sector linkages have distributional implications and imply substantial dispersion in the incidence of policy transmission.

Related literature. Our paper builds on much previous work that has documented and studied the implications of household and sectoral heterogeneity.

Sectoral heterogeneity. We contribute to a long tradition of research on the transmission and propagation of shocks in multi-sector business cycle models. Starting with Long and Plosser (1983), much work has employed structural real business cycle models to assess quantitatively whether sectoral technology shocks can account for observed business cycles patterns.[2] An important strand of this literature characterizes the aggregation properties of these models.[3] It has long been appreciated that Hulten's theorem applies to first order in frictionless competitive economies, and the aggregation of sectoral shocks is governed by sales shares (Domar weights). This literature has recently been reinvigorated with renewed focus on misallocation and potential gains in allocative efficiency in inefficient economies with distortions.[4] In an important contribution, Baqaee and Farhi (2020) derive an aggregation result for inefficient multi-sector economies. See Baqaee and Rubbo (2022) for a recent review.

Another strand of the multi-sector business cycle literature introduces nominal rigidities in the New Keynesian tradition.[5] Many of these papers investigate the implications of sectoral heterogeneity and input-output linkages for monetary policy and inflation dynamics. Baqaee et al.

[2] Among many others, see Bak et al. (1993), Horvath (2000), Foerster et al. (2011), Atalay (2017). While these papers focus on sectoral input-output linkages, recent work by Vom Lehn and Winberry (2022) studies the role of the investment network in propagating sectoral shocks.

[3] See for example Hulten (1978), Horvath (1998), Dupor (1999), Gabaix (2011), Acemoglu et al. (2012), Carvalho and Gabaix (2013), Acemoglu et al. (2017). Also see Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) for recent surveys.

[4] See Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Midrigan and Xu (2014), Baqaee and Farhi (2019), Liu (2019), Bigio and La'O (2020), Dávila and Schaab (2023), and many more.

[5] See among others Aoki (2001), Bouakez et al. (2009), Pasten et al. (2017, 2020), and Baqaee et al. (2021).

(2021) show that monetary policy can have supply-side effects and operate through a misallocation channel when policy redirects resources towards more monopolized sectors. Rubbo (2023) studies a general multi-sector representative-agent New Keynesian model and shows that sectoral and aggregate Phillips curve slopes decrease in intermediate input shares. She shows that a divine coincidence price index provides a better fit in Phillips curve regressions than consumer prices.

Relative to existing work, we develop a dynamic structural model featuring rich heterogeneity across both households and sectors. We emphasize the role of systematic household-sector linkages observed in micro data that give rise to novel transmission channels. Analytically, we derive an Intertemporal Keynesian Cross to characterize monetary and fiscal policy in the presence of household-sector linkages, and we extend the aggregation result of Baqaee and Farhi (2020) to a multi-sector heterogeneous-agent New Keynesian model. Quantitatively, we assess the importance of earnings and expenditure heterogeneity for policy transmission.

Household-sector linkages. Our paper contributes to research studying the interaction of household and sectoral heterogeneity. Indeed, we take as our starting point and motivation the large body of empirical work that studies earnings and expenditure heterogeneity across households.[6] A strand of this literature explicitly documents systematic household-sector linkages through earnings and expenditure patterns.[7] Relative to this empirical literature, we document several new empirical regularities on household-sector linkages. Our main contribution is to develop a structural model that matches these facts and to establish the quantitative importance of household-sector linkages for policy transmission.

Our paper is naturally related to a small but growing body of research that studies disaggregated economies featuring both household and sectoral heterogeneity, starting with Baqaee and Farhi (2018).[8] The contribution of our paper is to develop a dynamic structural framework that is apt for quantitative analysis. We leverage the sequence-space representation of our model to obtain analytical results even though our economy features rich and dynamically evolving heterogeneity.

HANK. The burgeoning HANK literature studies the implications of household heterogeneity for business cycles and policy transmission.[9] Our contribution is to unbundle the production side and introduce sectoral heterogeneity to this literature. We extend the Intertemporal Keynesian Cross (Auclert et al., 2023) to an environment with rich sectoral heterogeneity and characterize the

[6] See among many others Kaplan and Schulhofer-Wohl (2017), Jaravel (2019), Cravino et al. (2020), Jaravel (2021), Comin et al. (2021), and Andersen et al. (2022).

[7] Cravino et al. (2020) and Clayton et al. (2018) show that higher-income households spend in and earn from relatively price-rigid sectors. Hubmer (2023) documents that high-income households spend in sectors with higher labor shares. Jaravel (2019) shows that high-income households spend in sectors with higher markups.

[8] Most of these papers study the propagation of shocks in static environments (Flynn et al., 2022; Guerrieri et al., 2022; Andersen et al., 2022). Clayton et al. (2018) assess the importance of heterogeneity in price rigidity across sectors, also emphasizing the importance of earnings and expenditure heterogeneity. Yang (2022) computes optimal monetary policy in a HANK model where inflation has redistributive effects through households' consumption baskets, nominal wealth positions, and earnings elasticities to business cycles. Bellifemine et al. (2022) characterize an Intertemporal Keynesian Cross in a HANK economy with regional heterogeneity.

[9] Important contributions include, among many others, McKay and Reis (2016), McKay et al. (2016), Kaplan et al. (2018), Auclert (2019), and Auclert et al. (2023).

implications of household-sector linkages. Monetary and fiscal policy transmission is governed by earnings and expenditure heterogeneity channels, as well as a supply-side misallocation channel.

2 A Baseline HANK-IO Model

In this section, we develop a new multi-sector heterogeneous-agent New Keynesian framework with input-output linkages, which we call the "HANK-IO" model. Time is discrete and indexed by $t \in \{0, 1, \ldots\}$. We abstract from aggregate uncertainty and focus on one-time, unanticipated shocks.[10] Our model features heterogeneous households and multiple sectors. The economy is populated by a continuum of households of different types, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$. Each has mass μ_i with $\sum_i \mu_i = 1$. There are N production sectors, indexed by $j \in \mathcal{J} = \{1, \ldots, N\}$, each producing a distinct good. Households purchase consumption goods from and supply labor across these sectors.

Households differ in their permanent characteristics (ex-ante heterogeneity across types), and they face idiosyncratic uncertainty that leads them to make different consumption, savings, and portfolio decisions (ex-post heterogeneity within type). Our description of household behavior for a given type i is deliberately close to Auclert et al. (2023). We depart from the canonical HANK model by unbundling the aggregate production function and allowing for systematic links between household types i and different production sectors j—in terms of both earnings and expenditure patterns. We later discipline these *household-sector linkages* empirically using micro data in Section 4.

2.1 Households

The preferences of a household of type i are ordered according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \left[u_i\left(\left\{ c_{ij,t} \right\}_j \right) - v_i\left(N_{i,t} \right) \right],$$

where $c_{ij,t}$ denotes the household's consumption of good j and $N_{i,t}$ denotes hours of work, determined by union demand as specified below. Households of type i are endowed with and supply a distinct labor factor, which we refer to as factor i. Preferences are separable across time and between consumption and work. They can vary across household types but not across households within a type. Consumption preferences are encoded in a homothetic aggregator. Abusing notation slightly, consumption utility can be written as $u_i(c_{i,t})$, where $c_{i,t} = \mathcal{D}_i(c_{i1,t}, \ldots, c_{iN,t})$ is an aggregator index of real household consumption.

[10] Following Auclert et al. (2021a), we consider perfect foresight impulse responses to shocks around a steady state without aggregate risk. Linearizing with respect to these shocks uncovers the same impulse responses as in the model with aggregate risk by certainty equivalence.

Households can trade a single asset, a bond, and face a budget constraint of the form

$$\sum_j p_{j,t} c_{ij,t} + P_t b_{i,t} = (1 + i_t) P_{t-1} b_{i,t-1} + E_{i,t},$$

where $b_{i,t}$ denotes the household's bond position at time t, P_t is the price at which bonds trade, $p_{j,t}$ is the price of good j, and i_t is the nominal net rate of return on bonds between periods $t-1$ and t. The household's post-tax non-financial income is $E_{i,t} = \tau_t \big(z_{i,t} W_{i,t} N_{i,t} + z_{i,t} \zeta_i \Pi_t \big)$. It comprises labor earnings $z_{i,t} W_{i,t} N_{i,t}$, where $W_{i,t}$ is the nominal wage paid to factor i, and corporate dividends $z_{i,t} \zeta_i \Pi_t$, where Π_t denotes aggregate corporate profit and ζ_i is type i's profit share. Non-financial income is taxed at rate τ_t. Households face idiosyncratic income risk captured by the productivity shock $z_{i,t}$, which follows a first-order Markov chain and has mean 1.[11]

We write the household's problem recursively in terms of real wealth, adjusting for purchasing power. Since $\mathcal{D}_i(\cdot)$ is a homothetic aggregator, there exists an ideal price index $P_{i,t}$ for households of type i, with $P_{i,t} c_{i,t} = \sum_j p_{j,t} c_{ij,t}$. Given a bond position b, we define real wealth as $a = \frac{P_t}{P_{i,t}} b$. A household of type i with wealth a and labor productivity z at time t solves the dynamic problem

$$V_{i,t}(a,z) = \max_{\{c_{ij}\}_j, a'} \; u_i\Big(\Big\{ c_{ij,t} \Big\}_j \Big) - v\Big(N_{i,t} \Big) + \beta_i \mathbb{E}_t \Big[V_{i,t+1}(a', z') \Big] \tag{1}$$

$$\text{s.t.} \quad a' = R_{i,t} a + \frac{E_{i,t}(z)}{P_{i,t}} - \sum_j \frac{p_{j,t}}{P_{i,t}} c_{ij,t}$$

$$a' \geq \underline{a}$$

where $V_{i,t}(a,z)$ denotes the household's lifetime value at time t. Adjusting for purchasing power, the household's real rate of return on savings is $R_{i,t} = (1 + i_t) \frac{P_{i,t-1}}{P_{i,t}}$. Finally, households are subject to a borrowing constraint on real wealth.[12]

The budget constraint of problem (1) highlights the sources of earnings and expenditure heterogeneity in this model. Nominal labor income $zW_{i,t}N_{i,t}$ will differ across household types insofar as labor factors are compensated differently and production sectors exhibit different factor intensities. Changes in relative earnings elicit a pure income effect. There are two separate sources of expenditure heterogeneity. First, non-financial income $E_{i,t}$ is deflated using the household's price index $P_{i,t}$. Holding fixed nominal income, an increase in a household's relative bundle price implies a fall in purchasing power. This is also a pure income effect. Second, when households face different rates of inflation $\frac{P_{i,t}}{P_{i,t-1}}$ in their consumption bundles, they perceive different real rates of return on savings $R_{i,t}$, eliciting both income and substitution effects.

[11] Rebating corporate profits in proportion to labor productivity $z_{i,t}$ affords us analytical tractability, as in Auclert et al. (2023). We can allow for a more general dividend ownership structure at the expense of some expositional clarity.

[12] Representing the household problem in terms of wealth a instead of bonds b is without loss. Specifying the borrowing constraint in terms of purchasing power adjusted wealth, however, is a substantive assumption. This ensures that price changes do not change households' effective risk-sharing capacity, which would introduce a distinct source of non-homotheticity.

Aggregation. Households are uniquely identified by their type i, wealth a, and labor productivity z. We denote the joint density at time t by $g_{i,t}(a, z)$, which we also refer to as the cross-sectional distribution. Aggregate household demand for good j is then given by $C_{j,t} = \sum_i \mu_i \iint c_{ij,t}(a, z) g_{i,t}(a, z) \, da \, dz$.

Labor unions. Household labor supply decisions are intermediated by labor unions (Erceg et al., 2000; Auclert et al., 2023). We allow for flexible nominal wage adjustments but maintain the standard assumption of labor rationing: all households of type i work the same hours $N_{i,t}$. Appendix A.1 presents a self-contained treatment of our model's labor market structure and shows that it gives rise to a labor supply schedule for factor i

$$v'(N_{i,t}) = \frac{\epsilon^w - 1}{\epsilon^w} w_{i,t} u_i'(C_{i,t}), \tag{2}$$

where ϵ^w is the elasticity of substitution that governs unions' desired markup of real wages $w_{i,t} = \frac{W_{i,t}}{P_{i,t}}$ over marginal rates of substitution. It is a measure of monopsony in the labor market. Here, $C_{i,t}$ denotes aggregate consumption of all households of type i.

2.2 Production with Input-Output Linkages

Our economy comprises N production sectors. Each sector comprises a retailer and intermediate firms whose dynamic pricing problem gives rise to sectoral Phillips curves, analogous to the standard New Keynesian model.

Retailer. Each sectoral good j is bundled by a retailer using varieties k from a continuum of monopolistically competitive firms according to a CES aggregation technology,

$$y_{j,t} = \left(\int_0^1 y_{jk,t}^{\frac{\epsilon_j - 1}{\epsilon_j}} \, dk \right)^{\frac{\epsilon_j}{\epsilon_j - 1}},$$

with elasticity of substitution ϵ_j. The retailer's demand for inputs from firm k in sector j is given by

$$y_{jk,t} = \left(\frac{p_{jk,t}}{p_{j,t}} \right)^{-\epsilon} y_{j,t}, \tag{3}$$

where $p_{jk,t}$ is the price of firm k, and $p_{j,t}$ is the price of sector j's bundle, which we simply refer to as "good j" going forward.

Dynamic pricing decision. Each firm k in sector j faces a Rotemberg (1982) adjustment cost when changing its price, which is given by $\frac{\chi_j}{2}(\frac{p_{jk,t}}{p_{jk,t-1}} - 1)^2 p_{j,t} y_{j,t}$. Here, χ_j governs the degree of nominal rigidity and may vary across sectors. The firm's problem is to maximize the net present value of profits net of adjustment costs subject to demand (3). It gives rise to the set of linearized sectoral Phillips curves

$$\pi_{j,t} = \beta \pi_{j,t+1} + \frac{\epsilon_j}{\chi_j}\left(mc_{j,t} - \frac{\epsilon_j - 1}{\epsilon_j} \right), \tag{4}$$

that characterize the price dynamics in sector j, with $\pi_{j,t} = \frac{p_{j,t}}{p_{j,t-1}} - 1$. The sectoral Phillips curves (4) express current inflation in terms of (expected) future inflation and current real marginal cost $mc_{j,t}$, which we specify below. In a zero-inflation steady state, markups are $\frac{\epsilon_j}{\epsilon_j - 1}$ and may vary across sectors.

In our derivation of sectoral Phillips curve, we leverage symmetry across all firms within a sector: With Rotemberg adjustment costs, all firms in a sector remain symmetric ex post as long as they are all initialized with the same price $p_{jk,-1} = p_{jk',-1}$. We maintain this assumption throughout the paper. Symmetry within sectors allows us to represent the entire production structure of the economy at the sectoral level. We proceed as if production decisions in sector j were taken by a representative firm, taking as given the evolution of sectoral prices in accordance with (4).

Production structure. Goods in sector j are produced using a constant-returns technology represented by the sectoral production function

$$y_{j,t} = A_{j,t} F_j\left(\left\{ x_{jk,t} \right\}_k, \left\{ N_{ji,t} \right\}_i \right) \tag{5}$$

where $A_{j,t}$ is a Hicks-neutral technology shifter, and $x_{jk,t}$ and $N_{ji,t}$ denote firm j's uses of intermediate inputs from sector k and labor factor i. Sectoral profit is total revenue net of operating expenses, that is

$$\Pi_{j,t} = p_{j,t} y_{j,t} - \sum_k p_{k,t} x_{jk,t} - \sum_i W_{i,t} N_{ji,t} = p_{j,t} y_{j,t} - \boldsymbol{C}_{j,t},$$

where we denote by $\boldsymbol{C}_{j,t}$ the sector's total cost.The real marginal cost of sector j is then given by

$$mc_{j,t} = \frac{\boldsymbol{C}_{j,t}}{y_{j,t}} \tag{6}$$

assuming constant returns. It is a function of technology and prices $(A_{j,t}, \{p_{k,t}\}_k, \{W_{i,t}\}_i)$.

2.3 Monetary and Fiscal Policy

The monetary authority sets the path of nominal interest rates. We consider several policy rules, further specified below, that relate an exogenous monetary policy shock $\{\theta_t\}_{t \geq 0}$ to nominal interest rates.

The fiscal authority sets an exogenous path for sectoral government spending $\{G_{j,t}\}_{t \geq 0}$ and tax revenue $\{T_t\}_{t \geq 0}$. Aggregate government spending is a homothetic aggregator of sectoral expenditures given by $G_t = \mathcal{G}(\{G_{j,t}\}_j)$ associated with a price index $P_{G,t}$. Fiscal policy may be debt-financed, subject to an intertemporal budget constraint that ensures fiscal sustainability. The government issues debt in the bond market and its debt position B_t evolves according to

$$T_t + P_t B_t = (1 - i_t) P_{t-1} B_{t-1} + \sum_j p_{j,t} G_{j,t}. \tag{7}$$

To raise desired revenue T_t, the government sets tax policy τ_t according to

$$T_t = \sum_i \mu_i \iint (1 - \tau_t) \Big(z W_{i,t} N_{i,t} + z \Pi_t \Big) g_{i,t}(a, z)\, da\, dz. \tag{8}$$

2.4 Markets and Equilibrium

Equilibrium requires that the markets for each sectoral good j, for each labor factor i, and for bonds clear. Goods market clearing in sector j requires that total production be equal to total use,

$$y_{j,t} = C_{j,t} + \sum_{k=1}^{N} x_{kj,t} + G_{j,t}, \tag{9}$$

where $C_{j,t} = \sum_i \iint c_{ij,t}(a,z) g_{i,t}(a,z)\, da\, dz$ denotes total consumption and $\sum_{i=1}^{N} x_{ij,t}$ total production use of good j. Labor markets clear when hours worked by households of type i are equal to firms' labor demand for factor i,

$$\mu_i N_{i,t} = \sum_j N_{ji,t}. \tag{10}$$

Finally, the bond market clears when the net bond position of the household sector is equal to the government's outstanding debt, that is

$$\sum_i \iint \frac{P_{i,t}}{P_t} a g_{i,t}(a, z)\, da\, dz = B_t. \tag{11}$$

Definition (Competitive Equilibrium)**.** *Given a symmetric initial price distribution $p_{j,-1}$, initial government debt B_{-1}, and an initial cross-sectional distribution $g_{i,-1}(a,z)$, and taking as exogenously given paths for sectoral technology $\{A_{j,t}\}$ as well as monetary and fiscal policy $\{\theta_t, G_t, T_t\}$ satisfying* (7) *and a monetary policy rule (below), the competitive equilibrium of the HANK-IO model consists of sequences of prices $\{r_t, p_{j,t}, P_{i,t}, W_{i,t}\}$, sectoral allocations $\{y_{j,t}, x_{jk,t}, N_{ji,t}, \Pi_{j,t}\}$, individual allocation rules $\{c_{ij,t}(a,z), a_{i,t}(a,z), N_{i,t}\}$, and cross-sectional distributions $\{g_{i,t}(a,z)\}$ such that: households optimize, unions optimize and labor is rationed, firms optimize, and markets clear.*

2.5 GDP, Network Objects, and iMPCs

To conclude our description of the model, we introduce several accounting objects, as well as sufficient statistics that govern our analytical results in Section 5.

GDP, aggregate inflation, and monetary policy rules. Nominal GDP is the sum of all expenditures on goods for final use, that is

$$\text{nominal GDP } \equiv Y_t^n = \sum_j p_{j,t}(C_{j,t} + G_{j,t}).$$

Final expenditure shares are then defined as the share of a good j in nominal GDP,

$$b_{j,t} = \frac{p_{j,t}C_{j,t} + p_{j,t}G_{j,t}}{Y_t^n}.$$

Defining aggregate real GDP in levels is conceptually ambiguous since our environment features heterogeneous households and expenditure heterogeneity. We instead characterize our results in terms of changes in real GDP, using the standard Divisia index definition

$$d\log Y_t = \sum_j b_{j,t}\chi_{Cj,t} d\log C_{j,t} + \sum_j b_{j,t}\chi_{Gj,t} d\log G_{j,t},$$

where $\chi_{Cj,t} = \frac{C_{j,t}}{C_{j,t}+G_{j,t}}$ and $\chi_{Gj,t} = \frac{G_{j,t}}{C_{j,t}+G_{j,t}}$ are the shares of private and public expenditures in the consumption of good j. Changes in the GDP deflator are similarly defined as

$$d\log P_t = \sum_j b_{j,t} d\log p_{j,t},$$

so that $d\log Y_t^n = d\log Y_t + d\log P_t$. We define inflation in the GDP deflator around a zero-inflation steady state as $d\pi_t = d\log P_t - d\log P_{t-1}$. The usual Fisher relation then defines changes in the real interest rate as $dr_t = di_t - d\pi_t$ to first order.[13]

Given these measures of changes in real GDP and prices, we study two alternative monetary policy rules. Under a standard Taylor rule, the monetary authority responds to exogenous perturbations to first order by setting

$$di_t = \phi_\pi d\pi_t + \phi_y d\log Y_t + d\log\theta_t, \tag{12}$$

where $d\log\theta_t$ is an exogenous monetary policy shock.[14] As is now standard in the HANK literature (Auclert et al., 2023), we also consider an alternative policy rule that stabilizes the real interest rate, $dr_t = 0$, in response to exogenous perturbations. We refer to this as the "constant-real-rate rule".

[13] With slight abuse of notation, we abbreviate $d\log(1+i_t)$ as di_t and $d\log(1+r_t)$ as dr_t. Around a zero-inflation steady state, $d\log(1+\pi_t) = d\pi_t$ to first order.

[14] In an economy with a representative household that has homothetic preferences, policy rule (12) would be equivalent to a standard Taylor rule in levels, $1+i_t = (1+r_{ss})(\frac{P_t}{P_{t-1}})^{\phi_\pi}(\frac{Y_t - Y_{ss}}{Y_{ss}})^{\phi_y}\theta_t$, where P_t denotes the ideal price index of the representative consumer, Y_t is real GDP defined using P_t, and r_{ss} and Y_{ss} denote the real interest rate and real output in the deterministic zero-inflation steady state.

Domar weights, income shares, and relative price inflation. We define the revenue-based Domar weight or sales share of sector j as

$$\lambda_{j,t} = \frac{p_{j,t} y_{j,t}}{Y^n_t}.$$

As in Baqaee and Farhi (2020), the vector of Domar weights also satisfies $\lambda_t = \Psi'_t b_t$, where b_t is the $N \times 1$ vector of final expenditure shares and $\Psi_t = (I - \Omega_t)^{-1}$ is the $N \times N$ standard Leontief inverse matrix. Here, Ω_t is the standard (revenue-based) intput-output matrix, with $\Omega_{jk,t} = \frac{p_{k,t} x_{jk,t}}{p_{j,t} y_{j,t}}$. We also define cost-based Domar weights as $\tilde{\lambda}_t = \tilde{\Psi}'_t b_t$, where $\tilde{\Psi}_t = (I - \tilde{\Omega}_t)^{-1}$ is the cost-based Leontief inverse. Here, $\tilde{\Omega}_t$ is the cost-based intput-output matrix, with $\tilde{\Omega}_{jk,t} = \frac{\partial \log C_j}{\partial \log p_k}$.

An important measure of earnings heterogeneity is the non-financial income share of households of type i, which we define as

$$\Lambda_{i,t} = \frac{W_{i,t} N_{i,t} + \zeta_i \Pi_t}{Y^n_t},$$

which is the sum of factor i's income share in nominal GDP and its ownership share ζ_i. Notice that $\mathbb{E}_i \Lambda_{i,t} = 1$, so $\Lambda_{i,t}$ is a cross-sectional dispersion measure of households' non-financial income.

Finally, we introduce relative price inflation as a measure of expenditure heterogeneity. We define

$$d \log \rho_{i,t} = d \log P_{i,t} - d \log P_t$$

as the change in household type i's relative price (over the GDP deflator) in response to a shock. Similarly, we define

$$d\pi_{i,t} = d \log \rho_{i,t} - d \log \rho_{i,t-1}$$

as relative price inflation faced by households of type i.

Aggregate consumption function and intertemporal MPCs. Only two aggregate sequences matter for the household's dynamic optimization problem (1): the type-specific rate of return $\boldsymbol{R}_i = \{R_{i,s}\}_{s=0}^{\infty}$ on savings, and $\boldsymbol{e}_i = \{e_{i,s}\}_{s=0}^{\infty}$ where $ze_{i,t} = z\frac{1}{P_{i,t}} \tau_t Y^n_t \Lambda_{i,t}$ denotes nominal post-tax income adjusted for purchasing power for a household of type i with labor productivity z at time t. Aggregating households' consumption policy functions then yields an aggregate consumption function of the form

$$C_{i,t} = \mathcal{C}_{i,t}\left(\left\{R_{i,s}, e_{i,s}\right\}_{s=0}^{\infty}\right), \tag{13}$$

which maps the sequences $\boldsymbol{R}_i$ and $\boldsymbol{e}_i$ to aggregate consumption of households of type i.[15]

Following Auclert et al. (2023), we linearize (13) around the deterministic steady state of our economy. We consider bounded perturbations in the exogenously given sequences $\{d\boldsymbol{A}_j\}_j, d\boldsymbol{\theta}, d\boldsymbol{G}, d\boldsymbol{T} \in \ell^{\infty}$, where for example $d\boldsymbol{\theta} = \{d\theta_s\}_{s=0}^{\infty}$, and assume that the aggregate consumption function

[15] Intertemporal consumption functions of the form (13) are also used in Auclert et al. (2023), Kaplan et al. (2018), Auclert et al. (2021a), and many other papers. We use bold-faced notation in this context to denote infinite-dimensional sequences that are elements of ℓ^{∞}, the space of bounded sequences with the sup norm.

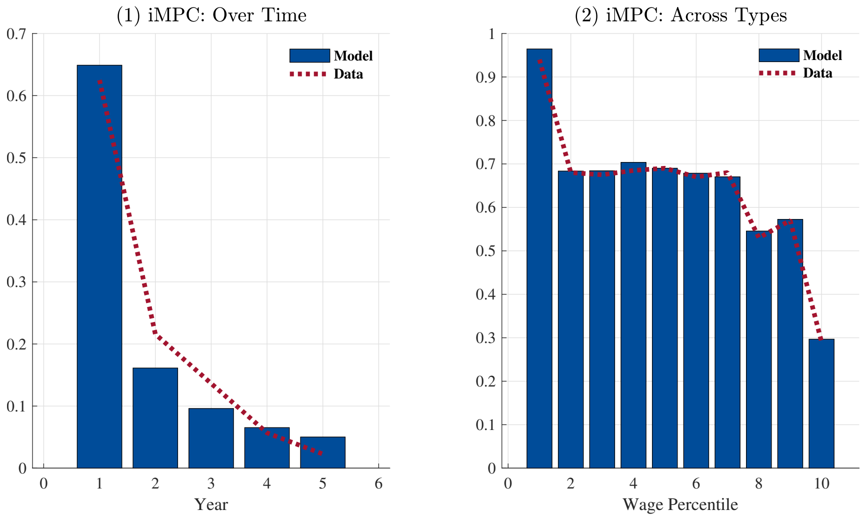

Figure 1. iMPC: Model and Data

Note. We compare the iMPC generated by the model and the data. The blue bar charts in panel (1) show the annual iMPC over time. We first calculate the quarterly iMPC by type as $\frac{\partial \log C_{i,t}}{\partial \log e_{i,1}}$ after an income shock at impact $t = 1$. The annual iMPC is calculated according to $\mathbb{E}_i \left(\sum_{s=1}^{4} \left(\frac{1}{1+r^*} \right)^s \frac{\partial \log C_{i,s}}{\partial \log e_{i,1}} \right)$. The data in red dashed line in panel (1) is reconstructed using Auclert et al. (2023) Figure 1. The blue bar charts in panel (2) show the first year iMPC of different household types. We compare them with the data from Auclert et al. (2021b) Figure 7.

$\mathcal{C}_i : \ell^\infty \times \ell^\infty \to \ell^\infty$ is Fréchet-differentiable. We collect its partial log-derivatives in two matrices, $\boldsymbol{M}_i$ and $\boldsymbol{M}_i^r$, with elements

$$M_{i,ts} = \frac{\partial \log \mathcal{C}_{i,t}}{\partial \log e_{i,s}} \qquad \text{and} \qquad M^r_{i,ts} = \frac{\partial \log \mathcal{C}_{i,t}}{\partial \log R_{i,s}}. \tag{14}$$

$\boldsymbol{M}_i$ and $\boldsymbol{M}_i^r$ are sequence-space Jacobians. We refer to $M_{i,ts}$ and $M^r_{i,ts}$ as type i's average intertemporal marginal propensity to consume (iMPC) and interest rate response elasticity.

Figure 1 illustrates iMPCs over time and across household types for a calibration of our baseline model that we discuss in Section 4. We compare model-implied iMPCs against empirical measures presented in Auclert et al. (2023) and Auclert et al. (2021b). Our model is calibrated to match the distribution of at-impact iMPCs across household types at steady state but also matches the shape of iMPCs over time as an untargeted moment.

3 Analytical Results

This section presents our analytical results. In Section 3.1, we develop a novel neutrality result by characterizing an *as-if benchmark* in which household and sectoral heterogeneity are decoupled. Away from this benchmark, novel earnings and expenditure channels govern the transmission of monetary and fiscal policy as we show in Section 3.2. Finally in Section 3.3, we develop an aggregation result for sectoral shocks in HANK-IO.

Our results in this section characterize first-order perturbations around a deterministic stationary equilibrium with zero government spending, $G_{ss} = 0$, where we use $_{ss}$ subscripts to denote variables in steady state. We also assume that the consumption aggregators of households and the government, $\mathcal{D}_i(\cdot)$ and $\mathcal{G}(\cdot)$, are of the homothetic CES form.Our results make reference to two instructive comparison benchmarks that are nested by the model of Section 2. We briefly describe these now.

Heterogeneous-agent New Keynesian model ("HANK"). Our multi-sector model nests a canonical heterogeneous-agent New Keynesian model with a single production sector, $N = 1$, and a single labor factor, which we refer to as the "HANK benchmark". With a single good, all households face the same price index $P_{i,t} = P_t = p_{1,t}$, which is also equal to the price at which the good is sold by retailers. Similarly with a single labor factor, all households work the same hours, $N_{i,t} = N_t$. We present this model in detail in the appendix and show there that it admits an aggregate consumption function $C_t = \mathcal{C}_t^{\text{HANK}}(\{R_s, \tau_s Y_s\}_{s=0}^{\infty})$. Combining it with the goods market clearing condition yields

$$Y_t = \mathcal{C}_t^{\text{HANK}}(\{R_s, \tau_s Y_s\}_{s=0}^{\infty}) + G_t. \tag{15}$$

Equation (15) represents a fixed point in output, mapping sequences of real interest rates $\boldsymbol{R}$ and exogenous fiscal policy $(\boldsymbol{\tau}, \boldsymbol{G})$ to a path for output $\boldsymbol{Y}$. It is akin to the dynamic IS equation of the standard New Keynesian model, except that $\mathcal{C}_t^{\text{HANK}}(\cdot)$ captures rich household heterogeneity.

Multi-sector representative-agent New Keynesian model ("RANK-IO"). The model of Section 2 similarly nests a canonical multi-sector representative-agent New Keynesian model, which we refer to as the "RANK-IO benchmark" (Bouakez et al., 2009; Pasten et al., 2020; Baqaee et al., 2021; Rubbo, 2023). With a representative household, there is only one labor factor and a single consumption price index P_t. Aggregate consumption satisfies a standard Euler equation of the form $u'(C_t) = \beta R_{t+1} u'(C_{t+1})$. Finally, we show in the appendix that any competitive equilibrium of this benchmark satisfies the fixed-point equation

$$R_t = \mathcal{R}_t^{\text{RANK-IO}}(\{Y_s; G_s, \theta_s, A_{j,s}\}_{s=0}^{\infty}). \tag{16}$$

Equation (16) captures a fixed point in the path of real interest rates. It maps sequences of output $\boldsymbol{Y}$ and exogenous shocks $(\boldsymbol{G}, \boldsymbol{\theta}, \{\boldsymbol{A}_j\}_j)$ to the path of real interest rates $\boldsymbol{R}$. Its interpretation is that of a dynamic LM equation: Just like its static counterpart maps output to the real interest rate in the standard IS-LM model, equation (16) maps an infinite-dimensional sequence of output to a time path of real interest rates.

3.1 An "As-If" Benchmark

We start our discussion of policy transmission in the HANK-IO model by considering an instructive *as-if* benchmark that obtains when we shut down earnings and expenditure heterogeneity across types. The as-if benchmark highlights starkly under which conditions the interaction between household and sectoral heterogeneity matters and household-sector linkages affect policy transmission. Formally, we make two assumptions in this subsection.

Assumption A1. The consumption preferences of all household types are represented by the same homothetic consumption aggregator. That is, $u_i(\cdot) = u(\cdot)$ and $\mathcal{D}_i(\cdot) = \mathcal{D}(\cdot)$.

Under assumption A1, all households aggree on the same consumption bundle price index, which we denote by $P_{i,t} = P_t$. There is no dispersion in relative prices or relative price inflation. This also implies an unambiguous definition of real GDP in levels, $Y_t = Y_t^n / P_t$.

Assumption A2. Households are endowed with a single labor factor, and $\zeta_i = \zeta$ for all i.

Under assumption A2, households of all types work the same hours, which we denote $N_{i,t} = N_t$, and face the same nominal wage, $W_{i,t} = W_t$. Similarly, ownership of corporate equity and consequently dividend flows are uniform across households. There is consequently no earnings heterogeneity and $\Lambda_{i,t} = \Lambda_t = 1$.

Importantly, assumptions A1 and A2 do not shut off either household or sectoral heterogeneity. Households still differ in terms of labor productivities and wealth. Likewise, the N production sectors are still heterogenous with respect to their production functions, labor shares, price rigidities, and markups. And there is still a rich network of production input-output linkages. However, our first result shows that household and sectoral heterogeneity are decoupled under assumptions A1 and A2.

Proposition 1 (As-If Benchmark)**.** *Under assumptions A1 and A2, any competitive equilibrium of the HANK-IO model satisfies the system of intertemporal equations*

$$\boldsymbol{Y} = \mathcal{C}^{HANK}(\boldsymbol{R}, \boldsymbol{\tau Y}) + \boldsymbol{G} \tag{17}$$

$$\boldsymbol{R} = \mathcal{R}^{RANK\text{-}IO}(\boldsymbol{Y}; \boldsymbol{G}, \boldsymbol{\theta}, \boldsymbol{A}_j), \tag{18}$$

where $\boldsymbol{\tau Y} = \{\tau_s Y_s\}_{s=0}^{\infty}$*. Equations* (17) *and* (18) *map exogenous sequences* $(\boldsymbol{\tau}, \boldsymbol{G}, \{\boldsymbol{A}_j\}_j)$ *to paths for output* $\boldsymbol{Y}$ *and real interest rates* $\boldsymbol{R}$*.*

Our as-if benchmark features a complete decoupling between household and sectoral heterogeneity in the following sense: Household heterogeneity only matters for the determination of aggregate demand $\boldsymbol{Y}$, while sectoral heterogeneity only affects the determination of real interest rates $\boldsymbol{R}$.

Under assumptions A1 and A2, the HANK-IO model's competitive equilibrium admits an intertemporal IS-LM representation as a system of two forward-looking equations: the dynamic IS curve (17) and the dynamic LM curve (18). According to Proposition 1, the dynamic IS curve is shaped by household but not by sectoral heterogeneity: Given a path of real interest rates $\boldsymbol{R}$, it takes the same form as the IS equation of the one-sector HANK model (15). Conversely, the dynamic LM curve is shaped by sectoral but not by household heterogeneity: It maps a given path of aggregate demand $\boldsymbol{Y}$ to the same path of real interest rates as the representative-household multi-sector RANK-IO model. In this sense, the determination of aggregate demand is unaffected by sectoral heterogeneity taking as given a path of real interest rates, while the evolution of real interest rates is independent from household heterogeneity taking as given a path of aggregate activity. In other words, household and sectoral heterogeneity respectively shape the demand and supply sides of the as-if economy but do not interact with each other beyond that.

We now leverage Proposition 1 to characterize the transmission of stabilization policy and the aggregate effects of sectoral shocks in the as-if economy. Corollaries 2 through 4 will serve as important reference points for our results in Sections 3.2 and 3.3.

Corollary 2 (As-If Benchmark: Fiscal Policy)**.** *Consider a bounded fiscal policy shock $\{d\boldsymbol{G}, d\log\boldsymbol{\tau}\}$ under a monetary policy rule that stabilizes the real interest rate, $d\boldsymbol{r} = 0$. The impulse response of output must satisfy*

$$(\boldsymbol{I} - \boldsymbol{M})\, d\log\boldsymbol{Y} = \boldsymbol{M}\, d\log\boldsymbol{\tau} + \frac{P^G_{ss}}{Y^n_{ss}} d\boldsymbol{G}. \tag{19}$$

where $\boldsymbol{M}$ has entries $M_{ts} = \frac{\partial \log \mathcal{C}^{HANK}_t}{\partial \log(\tau_s Y_s)}$.

The as-if benchmark of our economy admits the same Intertemporal Keynesian Cross characterization of fiscal policy shocks as the canonical one-sector HANK model (Auclert et al., 2023).[16] In particular, the iMPC matrix $\boldsymbol{M}$ remains a sufficient statistic for the effects of fiscal policy. It is observationally equivalent to that in the one-sector HANK model.

This result obtains in spite of substantial sectoral heterogeneity that is masked by the dynamic LM equation. When monetary policy neutralizes indirect effects through the real interest rate, untargeted fiscal policy is entirely unaffected by sectoral heterogeneity. In particular, fiscal multipliers in our as-if benchmark are identical to any HANK economy without sectoral heterogeneity that admits a representation of the aggregate consumption function as in (17).

Sectoral heterogeneity does, however, affect the monetary policy response $d\boldsymbol{i}$ that is necessary to neutralize real rates since this policy rule is governed by the dynamic LM equation. In

[16] In particular, equation (19) is equivalent to $(\boldsymbol{I} - \tilde{\boldsymbol{M}})d\boldsymbol{Y} = d\boldsymbol{G} - \tilde{\boldsymbol{M}}d\boldsymbol{T}$, where $\tilde{\boldsymbol{M}}$ is the matrix with elements $\tilde{M}_{ts} = \partial\mathcal{C}^{\text{HANK}}_t / \partial(\tau_s Y_s)$. Presenting Corollaries 2 and 3 in terms of log-derivatives will make it easier to compare to our main result in Section 3.2.

fact, the nominal interest rate rule $d\boldsymbol{i}$ is determined solely by the dynamic LM curve, taking as given aggregate demand $\boldsymbol{Y}$, which implies that it is governed by sectoral but not by household heterogeneity.

Corollary 3 (As-If Benchmark: Monetary Policy)**.** *Consider a monetary shock $d\log\boldsymbol{\theta}$ that induces a bounded perturbation in real interest rates $d\log\boldsymbol{R}$. The impulse response of output must satisfy*

$$(\boldsymbol{I}-\boldsymbol{M})\,d\log\boldsymbol{Y}=\boldsymbol{M}^r d\log\boldsymbol{R}, \tag{20}$$

where $\boldsymbol{M}^r$ has entries $M^r_{ts}=\frac{\partial\log C_t^{HANK}}{\partial\log R_s}$.

For a given change in the real interest rate, $d\boldsymbol{R}$, the effect of monetary policy on aggregate activity is again independent of sectoral heterogeneity and solely shaped by household heterogeneity. In particular, the iMPC and interest rate response matrices that appear in Corollary 3, $\boldsymbol{M}$ and $\boldsymbol{M}^r$, are equivalent to their counterparts in the one-sector HANK economy.

Sectoral heterogeneity does, however, affect the transmission of nominal interest rate shocks, $d\boldsymbol{i}$, to the real interest rate, $d\boldsymbol{r}$. In particular, heterogeneity in price rigidities and other sectoral variables does shape the strength of interest rate policy, i.e., monetary non-neutrality, but only through its transmission to real rates. Corollay 3 demonstrates that the path of real interest rates is a summary statistic for the effect of monetary policy on aggregate activity. Once the monetary authority has implemented a desired path of real rates, the production network structure of the economy no longer matters for transmission.

Corollary 4 (As-If Benchmark: Aggregating Sectoral Technology Shocks)**.** *Consider a bounded perturbation in sectoral technology $d\log\boldsymbol{A}_j$ and denote $d\log\boldsymbol{A}=(d\log\boldsymbol{A}_1,\ldots,d\log\boldsymbol{A}_N)$. The impulse response of output must satisfy*

$$d\log\boldsymbol{Y}=\underbrace{\frac{1+\eta}{\gamma+\eta}\,\tilde{\lambda}\,d\log\boldsymbol{A}}_{\text{Pure Technology Effect}}\underbrace{-\frac{1+\eta}{\gamma+\eta}\,\tilde{\lambda}\,d\log\boldsymbol{\mu}-\frac{\eta}{\gamma+\eta}\,d\log\boldsymbol{\Lambda}^L}_{\text{Change in Allocative Efficiency}} \tag{21}$$

where $\gamma=-\frac{C_{ss}u''(C_{ss})}{u'(C_{ss})}$ and $\eta=\frac{N_{ss}v''(N_{ss})}{v'(N_{ss})}$.

Corollary 4 is an aggregation result that traces the macroeconomic effects of microeconomic sectoral technology shocks. As in Baqaee and Farhi (2020), equation (21) decomposes the impact on real GDP into two effects. The pure technology effect holds fixed the resources employed by firms and workers, and measures the change in output that results from the increased productivity of given resources. Changes in allocative efficiency summarize the effect on output from a reallocation of resources across firms and workers. We denote by $\mu_{j,t} = \frac{p_{j,t}}{mc_{j,t}}$ the time-varying markup in sector j, so that $d \log \boldsymbol{\mu} = (d \log \boldsymbol{\mu}_1, \ldots, d \log \boldsymbol{\mu}_N)$ captures the endogenous response of sectoral markups across time and sectors. Finally, since there is a single labor factor under assumption A2, we denote by $\boldsymbol{\Lambda}^L$ the sequence of labor income shares.

It is remarkable that equation (21) is virtually identical to the aggregation result of Baqaee and Farhi (2020).[17] This is despite our economy featuring rich and dynamically evolving household heterogeneity, whereas theirs is a static representative-household setting. Corollary 4 underscores that our as-if benchmark features a strict decoupling of household and sectoral heterogeneity: For given changes in markups and factor shares, the aggregate consequences of microeconomic technology shocks do not directly interact with household heterogeneity. In other words, changes in sectoral markups $d \log \boldsymbol{\mu}$ and the labor income share $d \log \boldsymbol{\Lambda}^L$ are sufficient statistics for the implications of household heterogeneity. We will show in Section 3.3 that this simple benchmark is no longer valid in the presence of systematic household-sector linkages: With earnings and expenditure heterogeneity, the Baqaee and Farhi (2020) aggregation result must be augmented to account for household heterogeneity.

The importance of allocative efficiency is tightly linked to nominal rigidities. In the flexprice limit of our as-if benchmark, where markups are positive but constant, equation (21) becomes

$$d \log \boldsymbol{Y}^{\text{flex}} = \frac{1+\eta}{\gamma+\eta} \tilde{\lambda}' \, d \log \boldsymbol{A}. \tag{22}$$

This flexprice aggregation result still features two key departures from the canonical Hulten's theorem, according to which the macroeconomic effect of a sectoral technology shock is proportional to that sector's revenue-based Domar weight, λ. First, the importance of sectoral technology shocks is governed by the cost-based Domar weight $\tilde{\lambda}_j = \mu_j \lambda_j = \frac{\epsilon_j}{\epsilon_j - 1} \lambda_j$. Second, the standard Hulten's theorem applies in settings with inelastic factor supply. The multiplier $\frac{1+\eta}{\gamma+\eta}$ accounts for elastic labor supply, with the elasticities η and γ governing the household labor supply curve. When we allow for appropriate sectoral employment subsidies to offset steady state markups, $\mu_j = 1$ for all j, a variant of Hulten's theorem extended to elastic labor supply applies in the as-if benchmark with flexible prices. In this case, production is efficient and aggregation on the production side of the economy is again governed by revenue-based Domar weights despite rich household heterogeneity.

[17] In their baseline model, Baqaee and Farhi (2020) derive the aggregation result $d \log Y = \tilde{\lambda} \, d \log A - \tilde{\lambda} \, d \log \mu - \tilde{\Lambda} \, d \log \Lambda$. There are minor differences between our result and theirs. First, our setting is dynamic and equation (21) solves for the sequence of output changes. Second, we allow for elastic factor supply, which accounts for the presence of the elasticities η and γ. Baqaee and Farhi (2020) extend their main result to elastic factor supply in Appendix H.2 and equation (21) mirrors their extended formula. Finally, markups in our setting are endogenous and result from nominal rigidities. Nonetheless, Corollary 4 underscores that the importance of changes in markups is captured in reduced form by $\frac{1+\eta}{\gamma+\eta} \tilde{\lambda}$, as in Baqaee and Farhi (2020). Also notice that cost-based factor shares always sum to 1, and so $\tilde{\Lambda}^L = 1$ with a single factor.

3.2 An Intertemporal Keynesian Cross with Earnings and Expenditure Heterogeneity

Our next result characterizes an Intertemporal Keynesian Cross for monetary and fiscal policy in the baseline HANK-IO model of Section 2.

Proposition 5 (Intertemporal Keynesian Cross in HANK-IO)**.** *Consider a bounded perturbation in fiscal policy $(d\log\boldsymbol{\tau}, d\boldsymbol{G})$ and monetary policy $d\log\boldsymbol{R}$. The impulse response of output must satisfy*

$$\overbrace{\left(\boldsymbol{I} - \mathbb{E}_i\left[b_{i,ss}\boldsymbol{M}_i\right]\right)}^{\text{Multiplier}} d\log\boldsymbol{Y} = \overbrace{\mathbb{E}_i\left[b_{i,ss}\boldsymbol{M}_i\right]d\boldsymbol{\tau} + \frac{P_{G,ss}}{Y^n_{ss}}d\boldsymbol{G} + \mathbb{E}_i\left[b_{i,ss}\boldsymbol{M}^r_i\right]d\log\boldsymbol{R}}^{\text{As-If Benchmark}}$$
$$+ \underbrace{\mathbb{C}ov_i\left(\frac{b_{i,ss}}{\Lambda_{i,ss}}\boldsymbol{M}_i,\, d\boldsymbol{\Lambda}_i\right)}_{\text{Earnings Heterogeneity}} - \underbrace{\mathbb{C}ov_i\left(\boldsymbol{M}_i,\, b_{i,ss}d\log\boldsymbol{\rho}_i\right)}_{\substack{\text{Expenditure Heterogeneity}\\ \text{(Static)}}} - \underbrace{\mathbb{C}ov_i\left(\boldsymbol{M}^r_i,\, b_{i,ss}d\boldsymbol{\pi}_i\right)}_{\substack{\text{Expenditure Heterogeneity}\\ \text{(Intertemporal)}}}$$

where $\mathbb{E}_i(b_{i,ss}\boldsymbol{M}_i)$ and $\mathbb{E}_i(b_{i,ss}\boldsymbol{M}^r_i)$ are the cross-sectional weighted average iMPC and interest rate response matrices.

Proposition 5 characterizes the implications of household-sector linkages for policy transmission. Three new effects emerge relative to the transmission channels already operative in our as-if benchmark.[18] Figure 2 visualizes the decomposition of Proposition 5.

The first new effect is an *earnings heterogeneity channel* that emerges when households supply different labor factors. It is captured by a cross-sectional covariance across household types between iMPCs $\frac{b_{i,ss}}{\Lambda_{i,ss}}\boldsymbol{M}_i$—weighted by the ratio of expenditure and income shares—and changes in income shares $d\boldsymbol{\Lambda}_i$. Earnings heterogeneity therefore dampens the aggregate effects of contractionary monetary and fiscal policy when policy redistributes income to households with large expenditure-weighted iMPCs. Panel (2) of Figure 2 displays a strong positive correlation between iMPCs and earnings incidence. Panel (1) shows that earnings heterogeneity (the green dashed line) dampens the aggregate effect of contractionary monetary policy (solid blue line).

The next two terms of our decomposition capture the implications of *expenditure heterogeneity*. A change in the price of household type i's consumption basket elicits a static and an intertemporal effect. Holding fixed nominal income, an increase in the relative price $d\log\boldsymbol{\rho}_i$ will reduce a household's effective purchasing power. Households respond to this income effect in proportion to their intertemporal marginal propensities to consume. This first new transmission channel due to expenditure heterogeneity is therefore a covariance across household types between iMPCs $\boldsymbol{M}_i$

[18] We refer to Proposition 5 as an Intertemporal Keynesian Cross because the impulses on the RHS are amplified by a Keynesian-cross-like multiplier (Auclert et al., 2023). As in our as-if benchmark, this multiplier is governed by households' iMPCs. With multiple household types, the relevant aggregate iMPC is given by $\mathbb{E}_i(b_{i,ss}\boldsymbol{M}_i)$, using expenditure shares as aggregation weights. This Keynesian multiplier is not directly shaped by sectoral heterogeneity.

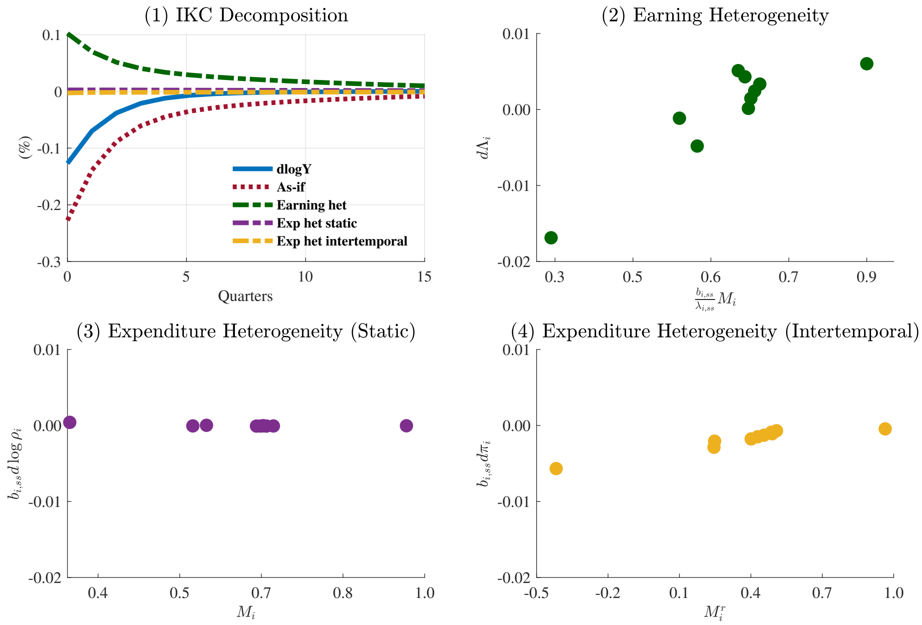

Figure 2. IKC Decomposition

Note. Panel (1) displays the Intertemporal Keynesian Cross decomposition of Proposition 5 in response to a 25bps contractionary monetary policy shock. We multiply both sides of the equation with $inv(M) = (\boldsymbol{I} - \mathbb{E}_i[b_{i,ss}\boldsymbol{M}_i])^{-1}$. In Panel (2)-(4), for the iMPC M_i and M_i^r we use the annual iMPC by each type i in response to a one-time quarterly shock to type i in either income or interest rate.

and changes in relative bundle prices $d\boldsymbol{\rho}_i$ weighted by expenditure shares. When policy leads to a relative price increase in the consumption basket of households (and in states of the world) with large iMPCs, then aggregate consumer spending falls.

A change in the path of relative prices also elicits an intertemporal effect. The relative bundle price governs a household's intertemporal decision between consuming in period t and saving for consumption in future periods. Notice that $-\mathbb{C}ov_i(\boldsymbol{M}_i^r, b_{i,ss} d \log \boldsymbol{\pi}_i) = \mathbb{C}ov_i(\boldsymbol{M}_i^r, b_{i,ss} d \log \boldsymbol{R}_i)$, where $\boldsymbol{R}_i$ is the path of effective savings rates for households of type i. When households experience *relative* price inflation, their effective real rate of return on savings $d \log \boldsymbol{R}_i$ falls disproportionately. And when relative price inflation is especially large for those households with strong intertemporal interest rate responses $\boldsymbol{M}_i^r$, then aggregate demand decreases.

Quantitatively, the expenditure heterogeneity covariances are small in our baseline model, which assumes homothetic CES consumption preferences. The expenditure-share-weighted changes in relative bundle prices are consequently small, as displayed in Panel (3) Figure 2. Panel (4) shows that the expenditure-share-weighted changes in rates of return are also small.

3.3 Beyond Hulten: An Aggregation Result for Sectoral Shocks

We now derive an aggregation result for the macroeconomic effects of sectoral technology shocks in HANK-IO. Unlike in our as-if benchmark, allocative efficiency is now also shaped by earnings and expenditure heterogeneity.

Proposition 6 (Aggregation Result for Sectoral Technology Shocks in HANK-IO)**.** *Assuming symmetric $\gamma_i = \gamma$ and $\eta_i = \eta$, the aggregate effect of sectoral technology shocks is given by*

$$
\begin{aligned}
d\log \boldsymbol{Y} = & \overbrace{\frac{1+\eta}{\gamma+\eta}\tilde{\lambda}\, d\log \boldsymbol{A} - \frac{1+\eta}{\gamma+\eta}\tilde{\lambda}\, d\log \boldsymbol{\mu} - \frac{\eta}{\gamma+\eta}\, d\log \boldsymbol{\Lambda}^L}^{\text{As-If Benchmark}} \\
& \underbrace{-\frac{1}{\gamma+\eta}\mathbb{C}ov_i\left(\tilde{\Lambda}_{i,ss},\, d\log \boldsymbol{\rho}_i\right)}_{\text{Expenditure Heterogeneity}} \underbrace{-\frac{\eta}{\gamma+\eta}\mathbb{C}ov_i\left(\tilde{\Lambda}_{i,ss},\, d\log \boldsymbol{\Lambda}_i^L\right)}_{\text{Earnings Heterogeneity}} \underbrace{-\frac{\gamma}{\gamma+\eta}\mathbb{C}ov_i\left(\tilde{\Lambda}_{i,ss},\, d\log \boldsymbol{\delta}_i\right)}_{\text{Income Effect on Labor Supply}}
\end{aligned}
$$

where $d\log \boldsymbol{\delta}_i = d\log \boldsymbol{C}_i - d\log \boldsymbol{C}$ is a measure of consumption dispersion.

Proposition 6 generalizes the aggregation result we derived for our as-if benchmark (Corollary 4). The three terms in the first line correspond to that benchmark.

The aggregate output response is again governed by a pure technology effect and changes in allocative efficiency. The pure technology effect, given by $\frac{1+\eta}{\gamma+\eta}\tilde{\lambda}\, d\log \boldsymbol{A}$, summarizes the increased productivity of resources holding fixed the current allocation. It is remarkable that pure technology gains are again unaffected by household heterogeneity. They are proportional to sectors' cost-based Domar weights, $\tilde{\lambda}$, and to the elasticities governing household labor supply, γ and η. Our assumption that these elasticities are symmetric across household types is critical: When the income and substitution effects governing labor supply are heterogeneous at a given allocation, then gains from pure technology are also shaped by household heterogeneity.

The remaining terms of our decomposition summarize changes in allocative efficiency. As in the as-if benchmark, changes in endogenous markups and factor shares have implications for allocative efficiency. These terms are exactly as in Corollary 4, and we discuss their economic intuition there.

More surprisingly, household heterogeneity and, in particular, cross-sectional household-sector linkages now also shape allocative efficiency. Three additional channels emerge that all work through the factor supply equations given by $d\log \boldsymbol{W}_i - d\log \boldsymbol{P}_i = \eta\, d\log \boldsymbol{N}_i + \gamma\, d\log \boldsymbol{C}_i$. Households of type i adjust their labor supply either in response to changes in their real wage, $d\log \boldsymbol{W}_i - d\log \boldsymbol{P}_i$, or when their income and consequently their consumption changes, $d\log \boldsymbol{C}_i$.

The first novel determinant of allocative efficiency is the effect of expenditure heterogeneity on household labor supply. Firms' cost structure is governed by nominal factor prices $W_{i,t}$. In other words, what matters for the production side is the marginal cost at which firms can hire additional labor. Labor supply, however, is governed by real wages. When the price of a household's consumption bundle increases, $d \log \rho_{i,t} > 0$, a given nominal wage for household i has less purchasing power and the real wage falls. At a given nominal wage, this household now supplies fewer hours and labor becomes more expensive for firms. The new effect on allocative efficiency is proportional to the cross-sectional covariance across household types or labor factors $\mathbb{C}ov_i(\tilde{\Lambda}_i, d \log \rho_{i,t})$: If consumer prices *increase* for those households whose labor factors have *large* cost-based Domar weights, then output falls and allocative efficiency deteriorates. Intuitively, labor factors that play a dominant role in firms' cost structures become relatively more expensive because those household types experience a relative drop in their purchasing power. Crucially, the relevant covariance is with respect to *cost-based* factor shares $\tilde{\Lambda}_i$ because markups drive a wedge between prices and marginal costs.

Earnings heterogeneity also shapes the transmission of technology shocks to allocative efficiency. The overall effect of changes in factor shares on allocative efficiency is captured by

$$-\frac{\eta}{\gamma+\eta} \sum_i \tilde{\Lambda}_i \, d \log \Lambda_{i,t} = \underbrace{-\frac{\eta}{\gamma+\eta} d \log \Lambda_t^L}_{\text{Labor Income Share}} - \underbrace{\frac{\eta}{\gamma+\eta} \mathbb{C}ov_i(\tilde{\Lambda}_i, d \log \Lambda_{i,t})}_{\text{Earnings Heterogeneity}}$$

where we denoted the average change in factor share by $d \log \Lambda_t = \mathbb{E}_i(d \log \Lambda_{i,t})$. Notice that the aggregate labor income share is not weighted by cost-based Domar weights because $\mathbb{E}_i \tilde{\Lambda}_i = 1$: All production costs must ultimately be accounted for by factor costs. The aggregate effect is also operative in the as-if benchmark. Intuitively, when the total labor income share falls, $d \log \Lambda_t < 0$, then households receive less labor income. This elicits a positive labor supply response governed by the elasticities $\frac{\eta}{\gamma+\eta}$. Heterogeneity in factor share responses can amplify or dampen the resulting change in allocative efficiency. When factors with the *largest* cost-based Domar weights $\tilde{\Lambda}_i$ experience a relative *decrease* in their income share, then resources are reallocated towards the more monopolized and distorted sectors. The strength of this effect is again proportional to $\frac{\eta}{\gamma+\eta}$, which governs the endogenous labor supply response to the fall in income.

Finally, there may be a heterogeneous income effect on labor supply across factors. This effect is summarized by a covariance across household types between cost-based factor shares $\tilde{\Lambda}_i$ and changes in consumption dispersion $d \log \delta_{i,t}$. Intuitively, if households i experience a relative drop in consumption, $d \log \delta_{i,t} < 0$, then they supply more labor for a given real wage. This income effect is governed by the elasticities $\frac{\gamma}{\gamma+\eta}$. As a result, firms can hire a given amount of labor factor i more cheaply and marginal cost falls. If the factors that become relatively cheaper also have high cost-based Domar weights, then output increases due to gains in allocative efficiency. Intuitively, marginal costs then fall in the most monopolized sectors.

Our discussions in Sections 3.2 and 3.3 point to an important conceptual takeaway. What matters for the aggregation of sectoral technology shocks—in particular their transmission through changes in allocative efficiency—are covariances with respect to cost-based factor shares $\tilde{\Lambda}_i$. Gains from allocative efficiency result from a reallocation of resources to relatively more distorted and monopolized sectors. The cost-based factor share $\tilde{\Lambda}_i$ precisely captures those markups that drive a wedge between prices and marginal costs. The covariance terms with respect to $\tilde{\Lambda}_i$ that appear in Proposition 6 therefore indicate whether resources and labor find more productive uses. The transmission of monetary and fiscal policy (demand) shocks, on the other hand, is determined by covariances with respect to iMPCs $\boldsymbol{M}_i$. Intuitively, demand propagation is governed by spending propensities in response to changes in income and prices. Mechanically, this distinction emerges because our results in Section 3.2 take as their starting point the goods market clearing condition and the aggregate consumption function, whereas our derivations in Section 3.3 focus on the supply and production equations.

4 Taking a Quantitative HANK-IO Model to the Data

This section develops a quantitative HANK-IO model that matches key empirical regularities on household-sector linkages. After discussing our data sources in Section 4.1, we present these empirical regularities in Section 4.2. To match these motivating empirical moments, we enrich the baseline HANK-IO model of Section 2 along two dimensions. The quantitative model we present in Section 4.3 features capital as an additional production factor and nonhomothetic CES consumption preferences. Allowing households to trade a second, illiquid asset is important to match the intertemporal marginal propensities to consume that govern the new earnings and expenditure heterogeneity transmission channels of monetary policy (Auclert et al., 2023), which we characterized in Section 3 . Adding capital as a factor of production also allows us to capture the importance of the investment network that is an important part of the economy's network structure (Vom Lehn and Winberry, 2022). Finally, nonhomothetic preferences allow us to match empirical expenditure patterns across the income distribution (Comin et al., 2021). Appendix C presents a self-contained description of our quantitative model.

4.1 Data

In this section, we assemble our dataset, which draws on six sources, and explain the construction of key variables. The details of data construction are reported in Appendix D. We conduct both our empirical and quantitative analysis at the 2-digit NAICS level corresponding to the Bureau of Economic Analysis' (BEA) Detailed Input-Output (I-O) Tables, which is also the number of sectors for our estimation of the nonhomothetic CES demand system.

Input-output linkages. We use the BEA I-O Use Tables to measure intput-output linkages across 71 industries from 1997 to 2015. The Use Tables specifies the nominal amount of inputs used by each industry. After eliminating industry categories related to federal, state, and local government, we are left with 66 private sectors. We aggregate the BEA sectors to the 2-digit NAICS level, arriving at 22 sectors as our baseline.

Household expenditure characteristics: CEX-IO data. We use the Consumer Expenditure Survey (CEX) conducted by the U.S. Bureau of Labor Statistics from 1997 to 2015 to obtain expenditure shares for households across the income distribution. The survey respondents report their consumption expenditures for the full consumption basket of goods and services, across 668 detailed categories called "UCCs". Sample selection and other data treatment details can be found in Appendix D.10. After the sample construction, we match the CEX spending categories to 22 industries in the IO table, constructing a dataset with household expenditure shares across the income distribution in 22 final IO industries for each year in the sample period. The mapping is based on a manual concordance assembled by Levinson and O'Brien (2019). The dataset contains 948,049 households across the sample period, each with nominal expenditure data in each IO industry and household characteristics.

Household earnings characteristics: ACS-IO data. We obtain cross-sectional household occupation and payroll data from the American Community Survey (ACS). The ACS is a survey administered by the U.S. Census Bureau and answered by a random 1% sample of the U.S. population each year. The dataset is made available by IPUMS (Ruggles et al., 2015) and offers demographical and labor information about all survey respondents. In particular, the ACS provides consistent industry identifiers for 320 industries in the private sector from 2000 to 2015. Matching ACS's 6-digit NAICS industry categories to industries in the IO table, we obtain a dataset with sector-specific payroll shares for households in various income quantiles.

Sectoral price rigidity. To measure price rigidity across industries, we map the sector-specific monthly price adjustment frequency from Pasten et al. (2017) to the 22 final IO industries. Pasten et al. (2017) use the data underlying the Producer Price Index (PPI) for 754 industries (defined by 6-digit NAICS codes) from the U.S. Bureau of Labor Statistics, from 2005 to 2011.The PPI measures changes in selling prices from the perspective of producers and covers all industrial and service sectors, including the product of intermediate inputs. Compared with earlier estimates by Nakamura and Steinsson (2008) with a focus on CPI data, the PPI measures are more suitable both for our sample period and our emphasis on intermediate inputs. We also show robustness using other price rigidity measures in Appendix E.

Factor shares. We compute the sector-specific factor shares in production from the BEA's GDP by Industry dataset with 66 private industries. The labor share in primary factors is computed as the compensation of employees as a percentage of value added, adjusted for taxes and subsidies, and averaged over 1997-2015. The intermediate inputs share is computed as total intermediate inputs as a percentage of gross output, averaged over the same sample period. We then map factor share data for 66 sectors into the 2-digit NAICS industry specification, where we weight the concordance by sector-specific gross output levels.

Capital investment shares. In our baseline, we use the Investment Flows Data for 41 Sector Partition from Vom Lehn and Winberry (2022) from 1997-2015 to calculate the share of capital investment inputs from each sector. We first calculate the share of total capital purchase from each of the 41 sectors each year in the Investment Flow tables. We then crosswalk the 41 sectors based on their NAICS codes to 22 sectors. Finally we take the average of each year's share.

Government spending shares. The BEA's industry input-output "Use" table allows us to compute the share of government spending on goods and services from different sectors. Government spending shares are calculated as total government expenditure in sector j as a percentage of total overall government spending. "Government" includes the federal government, federal government enterprises, state and local government, and state and local government enterprises. We average the annual share across 1997-2015, and then map the 66 sectors in which the government spends to the 2-digit NAICS level.

Markups. We use sectoral markup data from Baqaee and Farhi (2020) in our model calibration. They estimate three alternative measures of markups across 66 sectors from 1997 to 2015. The average markup for each sector in any particular year is computed as the harmonic sales-weighted average of firm markups, which are taken from Compustat and assigned to BEA sectors. In our baseline model, we use the average of their benchmark estimates following the accounting profits approach because the average markup is then around 10% and thus closer to the standard markup assumed in the HANK literature. Then we map the 66 industries to the 22-sector 2-digit IO categories, weighting the concordance by sector-specific gross output levels.

Network centrality. A reduced-form measure of a sector's centrality in the input-output production network is the Katz-Bonacich centrality measure discussed by Carvalho (2014). This measure of network centrality is defined as $c = \eta(\boldsymbol{I} - \lambda\alpha^x_{jk})^{-1}\mathbf{1}$, where we set $\eta = \frac{1-\theta}{N} = \frac{1-0.5}{22}$ and $\lambda = 0.5$. We denote by α^x_{jk} the share of sector j's spending on inputs from sector k in sector j's total value of the intermediate input bundle. We also calculated the "outdegree" of sectors, defined as $d_k = \sum_j \alpha^x_{jk}$, that is, the sum over all the weights of the network in which sector k appears as an input-supplying sector.

4.2 Empirical Regularities

We start by documenting empirical evidence of systematic household-sector linkages at the micro level. These empirical regularities suggest that households with different income levels are system-

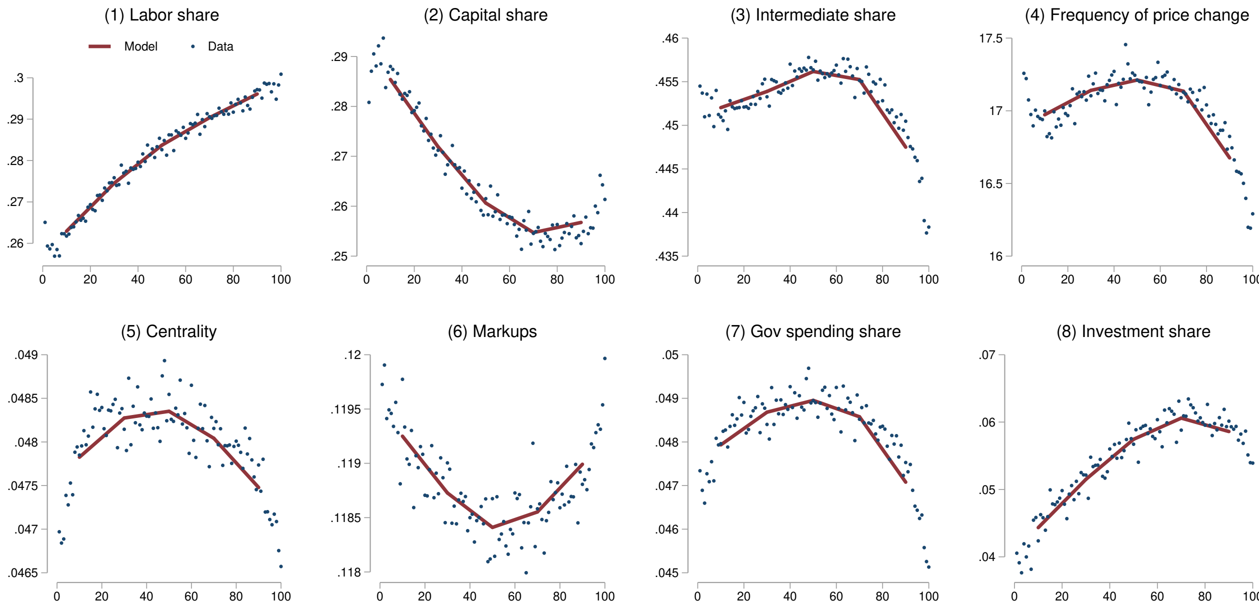

Figure 3. Expenditure-share Weighted Sectoral Heterogeneity for Income Percentiles

Note. The blue dots represent expenditure-weighted sectoral features as a function of households' after-tax income percentiles in the CEX-IO data, averaged over the sample period. The horizontal axis in each panel corresponds to household income percentiles, each bin representing 1% of the population. The vertical axis reports average sectoral features weighted by expenditure shares across household income percentiles. The red line summarizes the fit of our quantitative HANK-IO model, plotting the same measure of average sectoral features across the household income distribution using model-generated data.

atically exposed to different sectors through both expenditure and earnings patterns. Our results draw on much previous empirical work, which we review at the end of this subsection.

Expenditure heterogeneity. For each sectoral feature θ, we compute an expenditure-share-weighted measure for each household percentile hp in each year t as

$$\theta_t^{exp,hp} = \sum_{j \in S} w_{j,t}^{exp,hp} \theta_{j,t}$$

where $\theta_{j,t}$ corresponds to our empirical measure of sectoral feature θ for sector j in year t, and $w_{j,t}^{exp,hp}$ denotes the share of expenditures on goods in sector j accounted for by households of income percentile hp in year t.

The blue dots in Figure 3 represent expenditure-weighted sectoral features in the data, plotted across the household income distribution by linking cross-sectional household spending data from the CEX to the BEA IO table. We find that higher-income households consume relatively more in sectors that are more labor-intensive, less capital-intensive, and contribute more to capital production. Middle-income households consume more in sectors that are more intermediates-

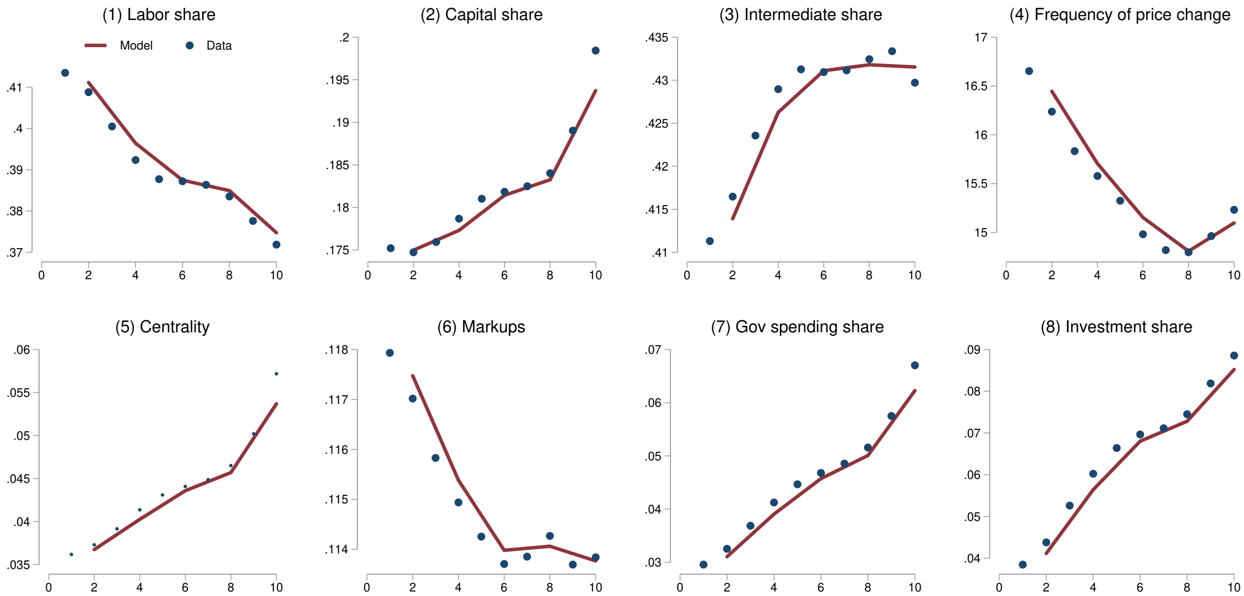

Figure 4. Earnings-share Weighted Sectoral Heterogeneity for Income Percentiles

Note. The blue dots represent payroll-weighted sectoral features as a function of households' income percentiles in the ACS-IO data, averaged over the sample period. The horizontal axis in each panel corresponds to household income percentiles, each dot representing 10% of the population. The vertical axis reports average sectoral features weighted by payroll shares across household income percentiles. The red line summarizes the fit of our quantitative HANK-IO model, plotting the same measure of average sectoral features across the household income distribution using model-generated data.

intensive, more price-flexible, more central in the production network, have lower markups, and have larger government spending shares. The red lines in Figure 3 demonstrate the fit of our quantitative model, which specifically targets these empirical regularities.

The five largest sectors in households' consumption baskets are (1) housing, (2) non-durable manufacturing goods such as food, beverages, apparels, and textile products, (3) retail trade, (4) durable manufacturing goods such as appliances, cars, electronic devices, furniture, and (5) utilities. Table 1 shows that higher-income households spend relatively less on non-durable manufacturing goods, utilities, retail and wholesale trades, and information products such as broadcasting and telecommunications. They spend relatively more on other services, hotels and restaurants, and home construction. Middle-income households have the smallest expenditure shares for housing and education services, and the largest for durable manufacturing, finance, and insurance.

Earnings heterogeneity. Next, we document systematic household-sector linkages on the earnings side. We link cross-sectional household occupation and payroll data from the American Community Survey (ACS) with data from the BEA IO Tables. Similar to expenditure heterogeneity, we define payroll-share-weighted sectoral features as

$$\theta_t^{pay,hp} = \sum_{j \in S} w_{j,t}^{pay,hp} \theta_{j,t}$$

where $w_{j,t}^{pay,hp}$ is the payroll share in sector j in year t accounted for by households in income percentile hp.

The blue dots in Figure 4 plot earnings-weighted sectoral features across 20 income bins. We find that higher-income households earn relatively more in sectors that are less labor-intensive, more capital-intensive, more central in the production network, have lower markups, have larger government spending shares, and contribute more to capital formation. Middle-income households work relatively more in sectors that are more price-rigid.

The sectors that account for the largest share of household earnings are accomodation and food services, retail trade, and healthcare for the bottom 10% of earners; healthcare, retail trade, and education services for the middle-income group; and professional and technical services, healthcare, and durable manufacturing for the top 10% of the income distribution. Table 2 presents summary statistics. Lower-income households earn more from sectors such as accomodation and food services, retail trade and other services, which are less capital-intensive and more labor-intensive. Higher-income households earn more from sectors such as professional and technical services, finance, and manufacturing, which have higher network centrality and government spending shares.

4.2.1 Relation to the Literature

Many of these empirical regularities are already well known. Hubmer (2023) documents that higher-income households spend relatively more on sectors with high labor shares. In related work, Faber and Fally (2022) show that higher-income households spend relatively more on high-quality goods, which is in turn positively correlated with sectoral labor intensity (Jaimovich et al., 2019). Relative to these papers, we show that higher-income households spend relatively more on sectors that also have high intermediates shares and work relatively more in sectors that have high intermediates and capital shares, but low labor shares.

Cravino et al. (2020) document that higher-income households spend relatively more in sectors with stickier prices. Similarly, Clayton et al. (2018) show that more educated households both spend and work more in sectors with more price rigidity. Our results suggest that the expenditure-weighted frequency of price changes is hump-shaped in household income, while higher-income households earn more in price-rigid sectors.

Sectors	Capital share	Labor share	Intermediates share	Freq. price change	Centrality	Markups	Gov spending share	Investment share	1th percentile	41-60th percentile	100th percentile	Diff 100th-1th
	Sectoral Features								Expenditure Share			
Manufacturing I	0.139	0.150	0.711	18.429	0.038	0.178	0.060	0.001	15.73%	11.63%	8.18%	-7.55%
Utilities	0.374	0.188	0.438	38.806	0.040	0.024	0.024	0.000	7.73%	5.62%	4.08%	-3.65%
Retail trade	0.187	0.468	0.345	21.172	0.023	0.085	0.000	0.022	11.88%	11.62%	9.20%	-2.68%
Housing	0.648	0.062	0.290	6.983	0.065	0.115	0.044	0.005	16.88%	13.55%	14.42%	-2.46%
Information	0.330	0.234	0.436	18.675	0.045	0.217	0.076	0.059	6.25%	4.89%	4.01%	-2.24%
Wholesale trade	0.257	0.397	0.346	8.879	0.026	0.033	0.000	0.054	5.75%	5.64%	4.11%	-1.64%
Manufacturing II	0.214	0.144	0.641	19.775	0.084	0.139	0.200	0.004	3.47%	3.43%	2.40%	-1.06%
Warehousing	0.120	0.468	0.413	6.933	0.029	0.115	0.003	0.004	0.05%	0.05%	0.04%	-0.01%
Agriculture	0.287	0.116	0.597	52.065	0.041	0.130	0.007	0.000	0.07%	0.09%	0.12%	0.05%
Mining	0.385	0.183	0.433	38.806	0.048	0.045	0.029	0.035	0.14%	0.21%	0.19%	0.06%
Professional and technical	0.174	0.467	0.359	7.133	0.105	0.113	0.148	0.189	1.27%	1.11%	1.50%	0.23%
Administrative and waste	0.155	0.458	0.386	13.926	0.059	0.096	0.068	0.000	0.52%	0.58%	1.07%	0.55%
Arts and entertainment	0.240	0.385	0.375	5.025	0.029	0.099	0.004	0.002	0.36%	0.69%	1.13%	0.77%
Healthcare and social assistance	0.102	0.515	0.384	6.887	0.023	0.124	0.010	0.000	2.46%	3.33%	3.28%	0.82%
Transportation	0.177	0.316	0.507	21.376	0.035	0.151	0.030	0.004	2.15%	2.18%	3.28%	1.13%
Rental and leasing	0.518	0.119	0.363	18.132	0.032	0.202	0.009	0.000	0.67%	0.96%	1.92%	1.25%
Finance and insurance	0.230	0.316	0.454	31.699	0.077	0.115	0.076	0.001	3.89%	5.76%	5.51%	1.61%
Manufacturing III	0.155	0.250	0.595	13.870	0.085	0.111	0.098	0.297	7.76%	12.39%	9.64%	1.88%
Education services	0.126	0.552	0.322	5.516	0.025	0.099	0.010	0.000	1.79%	1.58%	4.32%	2.53%
Construction	0.171	0.336	0.493	20.221	0.030	0.105	0.062	0.325	1.36%	2.75%	4.52%	3.16%
Accomodation and food services	0.154	0.395	0.451	27.266	0.032	0.163	0.015	0.000	5.01%	6.39%	8.37%	3.36%
Other services	0.164	0.453	0.384	4.643	0.031	0.114	0.026	0.000	4.84%	5.60%	8.73%	3.89%
Mean	0.241	0.317	0.442	18.464	0.045	0.117	0.045	0.045				
Max	0.648	0.552	0.711	52.065	0.105	0.217	0.200	0.325				
Min	0.102	0.062	0.290	4.643	0.023	0.024	0.000	0.000				

Table 1. Expenditure Share Differences and Sectoral Features, 22 Sectors

Note. Table 1 reports summary statistics for sectoral features across all 22 production sectors. Sectors are listed in ascending order according to the difference in expenditure shares between the top 1% income percentile and the bottom 1% income percentile. The light blue shade helps visualize each sector's sectoral features relative to the respective range across all sectors.

Sectors	Capital share	Labor share	Intermediates share	Freq. price change	Centrality	Markups	Gov spending share	Investment share	1-10th percentile	41-60th percentile	90-100th percentile	Diff
	Sectoral Features								Earnings Share			
Accomodation and food services	0.154	0.395	0.451	27.266	0.032	0.163	0.015	0.000	17.18%	3.78%	1.25%	-15.92%
Retail trade	0.187	0.468	0.345	21.172	0.023	0.085	0.000	0.022	18.35%	10.58%	6.13%	-12.23%
Other services	0.164	0.453	0.384	4.643	0.031	0.114	0.026	0.000	7.14%	5.46%	2.87%	-4.27%
Administrative and waste	0.155	0.458	0.386	13.926	0.059	0.096	0.068	0.000	5.46%	3.83%	2.07%	-3.39%
Education services	0.126	0.552	0.322	5.516	0.025	0.099	0.010	0.000	9.54%	10.15%	6.29%	-3.25%
Arts and entertainment	0.240	0.385	0.375	5.025	0.029	0.099	0.004	0.002	3.74%	1.96%	1.07%	-2.67%
Agriculture	0.287	0.116	0.597	52.065	0.041	0.130	0.007	0.000	2.32%	1.71%	1.58%	-0.74%
Manufacturing I	0.139	0.150	0.711	18.429	0.038	0.178	0.060	0.001	1.56%	2.47%	1.34%	-0.21%
Rental and leasing	0.518	0.119	0.363	18.132	0.032	0.202	0.009	0.000	0.39%	0.37%	0.24%	-0.15%
Warehousing	0.120	0.468	0.413	6.933	0.029	0.115	0.003	0.004	0.21%	0.32%	0.09%	-0.12%
Construction	0.171	0.336	0.493	20.221	0.030	0.105	0.062	0.325	5.29%	8.23%	5.26%	-0.03%
Transportation	0.177	0.316	0.507	21.376	0.035	0.151	0.030	0.004	3.25%	4.78%	3.61%	0.36%
Healthcare and social assistance	0.102	0.515	0.384	6.887	0.023	0.124	0.010	0.000	11.25%	14.59%	12.03%	0.79%
Mining	0.385	0.183	0.433	38.806	0.048	0.045	0.029	0.035	0.17%	0.50%	1.03%	0.86%
Utilities	0.374	0.188	0.438	38.806	0.040	0.024	0.024	0.000	0.12%	0.45%	1.50%	1.38%
Housing	0.648	0.062	0.290	6.983	0.065	0.115	0.044	0.005	1.12%	1.80%	2.75%	1.63%
Information	0.330	0.234	0.436	18.675	0.045	0.217	0.076	0.059	1.77%	2.27%	4.15%	2.39%
Wholesale trade	0.257	0.397	0.346	8.879	0.026	0.033	0.000	0.054	1.75%	3.83%	4.47%	2.72%
Manufacturing II	0.214	0.144	0.641	19.775	0.084	0.139	0.200	0.004	1.34%	3.68%	4.06%	2.72%
Manufacturing III	0.155	0.250	0.595	13.870	0.085	0.111	0.098	0.297	2.88%	8.45%	10.10%	7.22%
Finance and insurance	0.230	0.316	0.454	31.699	0.077	0.115	0.076	0.001	1.82%	5.64%	9.90%	8.08%
Professional and technical	0.174	0.467	0.359	7.133	0.105	0.113	0.148	0.189	3.34%	5.16%	18.20%	14.86%
Mean	0.241	0.317	0.442	18.464	0.045	0.117	0.045	0.045				
Max	0.648	0.552	0.711	52.065	0.105	0.217	0.200	0.325				
Min	0.102	0.062	0.290	4.643	0.023	0.024	0.000	0.000				

Table 2. Earnings Share Differences and Sectoral Features, 22 Sectors

Note. Table 2 reports summary statistics for sectoral features across all 22 production sectors. Sectors are listed in ascending order according to the difference in payroll shares between the top 10% income percentile and the bottom 10% income percentile. The light blue shade helps visualize each sector's sectoral features relative to the respective range across all sectors.

Government spending is granular and concentrated in sectors with relatively sticky prices (Cox et al., 2020). We show that government spending also tends to be high in those sectors where high-income households earn and middle-income households spend relatively more. Jaravel (2019) documents that higher-income households spend relatively more in high-markup sectors. Finally, Vom Lehn and Winberry (2022) document that the capital investment network is granular and dominated by a small set of concentrated investment hubs.

4.3 Key Model Elements and Calibration

In this section, we present the key new elements of our quantitative model and discuss its calibration. A detailed and self-contained model description is relegated to Appendix C.

4.3.1 Production Network

We calibrate the production network of our quantitative model to match data on 22 sectors. As described in Section 2.2, each sector is modeled as comprising a retailer and intermediate firms that face price adjustment costs. While this structure allows us to derive sectoral Phillips curves (4), we otherwise focus on a symmetric equilibrium in which all firms within a sector are identical. We can therefore proceed as if sectoral production decisions are taken by a representative firm.

The production function of sector j is CES over intermediate inputs and a primary factor that combines capital and labor, given by

$$y_{j,t} = A_{j,t}\left((1-\theta_j)^{\frac{1}{\eta_{f,j}}} f_{j,t}^{\frac{\eta_{f,j}-1}{\eta_{f,j}}} + \theta_j^{\frac{1}{\eta_{f,j}}} x_{j,t}^{\frac{\eta_{f,j}-1}{\eta_{f,j}}}\right)^{\frac{\eta_{f,j}}{\eta_{f,j}-1}}, \tag{23}$$

where $A_{j,t}$ is a Hicks-neutral technology shifter. We denote by θ_j the CES weight on intermediate inputs in sector j's production and by $\eta_{f,j}$ the elasticity of substitution between the primary factor and intermediate inputs.

The primary factor is a Cobb-Douglas aggregate of capital and labor, given by

$$f_{j,t} = K_{j,t}^{\alpha_j} N_{j,t}^{1-\alpha_j}, \tag{24}$$

where α_j is the share of capital in total factors.[19] We denote by $K_{j,t}$ the capital rented by sector j in period t and by $N_{j,t}$ a CES aggregate over all I labor factors used by sector j in production, given by

$$N_{j,t} = \left(\sum_i \left(\Gamma_{ji}^w\right)^{\frac{1}{\eta_{w,j}}} N_{ji,t}^{\frac{\eta_{w,j}-1}{\eta_{w,j}}}\right)^{\frac{\eta_{w,j}}{\eta_{w,j}-1}}, \tag{25}$$

[19] Horvath (2000), Carvalho (2014), Atalay (2017), Carvalho et al. (2021) and Ferrante et al. (2022) all aggregate primary factors using a Cobb-Douglas calibration. Vom Lehn and Winberry (2022) use Cobb-Douglas in their main calibration and explore deviations from Cobb-Douglas in a sensitivity analysis, where they show there are no sizable quantitative implications.

where $N_{ji,t}$ is sector j's demand for labor factor i, Γ^w_{ji} is the relative CES weight on factor i, and $\eta_{w,j}$ is a sector-specific elasticity of substitution across labor factors in production.

Sector j uses a CES basket of intermediate inputs, $x_{j,t}$, given by

$$x_{j,t} = \left(\sum_k \left(\Gamma^x_{jk} \right)^{\frac{1}{\eta_{x,j}}} x_{jk,t}^{\frac{\eta_{x,j}-1}{\eta_{x,j}}} \right)^{\frac{\eta_{x,j}}{\eta_{x,j}-1}}, \tag{26}$$

where $\eta_{x,j}$ is sector j's constant elasticitiy of substitution across intermediate inputs. Γ^x_{jk} denotes how important is good k in sector j's intermediate input bundle production function. As in Section 2, $x_{jk,t}$ is the demand for good k as an input by sector j.

The standard demand functions for intermediate inputs from sector i is given by

$$x_{jk,t} = \Gamma^x_{jk} \left(\frac{p_{k,t}}{p_{jx,t}} \right)^{-\eta_{x,j}} x_{j,t} \tag{27}$$

where p^k_t is the producer price index (PPI) in sector k and $p_{jx,t}$ is the price of intermediate input bundle in sector j. $x_{jk,t}$ is the unit of goods from sector k used by sector j, and the $x_{j,t}$ is the unit of intermediate input bundle for sector j. The relationship between intermediate input prices and the bundle price is given by

$$p_{jx,t} = \left[\sum_k \Gamma^x_{jk} (p_{k,t})^{1-\eta_{x,j}} \right]^{\frac{1}{1-\eta_{x,j}}}$$

Firms rent capital in an integrated and competitive market at the nominal rental rate i^K_t. Likewise firms hire labor of each factor in non-segmented labor markets at nominal wage rates $W_{i,t}$. Sectoral profits are now given by $\Pi_{j,t} = p_{j,t} y_{j,t} - \sum_i W_{i,t} N_{ji,t} - i^K_t K_{j,t} - p_{jx,t} x_{j,t}$. We call the share of expenditure on goods from sector k in the nominal value of the intermediate input bundle α^x_{jk}, given by

$$\alpha^x_{jk} = \frac{p_{k,t} x_{jk,t}}{p_{jx,t} x_{j,t}}. \tag{28}$$

Elasticities. We follow much of the literature and set the elasticity between the primary factor and intermediate inputs to $\eta_{f,j} = 1$ across sectors.[20] Consensus on the appropriate calibration of $\eta_{x,j}$, the elasticity of substitution across intermediate inputs, has evolved over time. While most prior work uses a Cobb-Douglas calibration, Atalay (2017) argues that this elasticity should be much smaller. We follow Atalay (2017) and set $\eta_{x,j} = 0.1$. Finally, we calibrate the elasticity of substitution between different labor factors to $\eta_{w,j} = 1$.

[20] Atalay (2017), Vom Lehn and Winberry (2022), and most prior work use this calibration.

Factor shares. Sectors differ in the share of intermediate inputs in production, θ_j, and the share of capital in primary factors, α_j. We compute 22 sector-specific factor shares from the BEA GDP-by-Industry dataset. The intermediate input share θ_j is computed as input expenditures as a percentage of gross output, averaged over 1997-2015. We compute the labor share $1 - \alpha_j$ as total compensation of employees as a percentage of value added, adjusted for taxes and subsidies, averaged over the same period. We show in Appendix D.1 that there is substantial heterogeneity in factor shares across sectors.

Input-output network. We obtain the input-output matrix Γ^x_{jk} from equations (27) and (28),

$$\Gamma^x_{jk} = \alpha^x_{jk} \left(\frac{p_{jx,t}}{p_{k,t}} \right)^{1-\eta_{x,j}}.$$

Every column of Γ^x_{jk} sums to 1.

We use data from the BEA Input Output "Use" Table to calculate the input-output share α^x_{jk} as sector j's (columns) nominal expenditure on intermediate inputs from sector k (rows) as a share of j's total expenditure on intermediate inputs. Then we average these ratios across 1997-2015. See Appendix D.7 for further discussion on input-output linkages and the IO tables.

Sectoral price rigidities. We compute the sectoral price rigidities χ_j in two steps. First, we use data made publicly available by Pasten et al. (2017) who estimate the frequency of price changes using the U.S. Bureau of Labor Statistics (BLS)'s data underlying the Producer Price Index (PPI) for 754 industries (defined by 6-digits NAICS codes) from 2005 to 2011. Since these estimates are more granular than our production network, we follow Clayton et al. (2018) and use their many-to-one merge to our 22 production sectors. From these estimates, we obtain the monthly price adjustment frequency. Additional details can be found in Appendix D.5. These estimates of price adjustment frequencies naturally map into sectoral Calvo parameters. In a second step, we analytically characterize the concordance between Calvo and Rotemberg parameters χ_j in our model to first order. See Appendix D.6 for details.

Payroll shares. To measure sectoral payroll shares—and consequently the share of salary expenditures paid to different household types i—we use data from the linked ACS-IO dataset (see Section 4.2). The details of how we link different datasets are provided in Appendix D.9. We define household earnings shares across sectors as the ratio of total earnings paid to household type i in sector j over total earnings of type i, averaged across our sample period.

Markups. Steady state markups across sectors are given by $\mu_j = \frac{\epsilon_j}{\epsilon_j - 1}$. Our baseline model calibrates ϵ_j directly to match sectoral markups using data from Baqaee and Farhi (2020) as discussed in Section 4.1. Appendix D.4 shows that there is substantial heterogeneity in markups across sectors.

4.3.2 Households

Our quantitative model features multiple household types indexed by i. To match the empirical regularities documented in Figures 3 and 4, we allow types to differ in their permanent income levels, their earnings shares across sectors, as well as their consumption preferences.

Expenditure heterogeneity. Households consume a generalized nonhomothetic CES basket of goods, $c_{i,t} = \mathcal{D}^{NH}(\{c_{ij,t}\}_j)$, implicitly defined via

$$1 = \sum_j \left(\Omega_{ij} c_{i,t}^{\varepsilon_j}\right)^{\frac{1}{\eta_c}} c_{ij,t}^{\frac{\eta_c - 1}{\eta_c}},$$

where $c_{ij,t}$ denotes consumption of good j by a household type i (Comin et al., 2021). η_c is the elasticity of substitution across consumer goods produced in different sectors and ε_j is the relative income elasticity for goods produced in sector j. Ω_{ij} is the taste parameter for good produced in sector j by household type i.

Following Comin et al. (2021) and Hubmer (2023), we estimate a nonhomothetic CES demand system for our HANK-IO model to obtain the parameters $\{\eta_c, \varepsilon_j\}_j$, using our linked CEX-IO dataset we described Section 4.1. We calibrate the taste parameters Ω_{ij} so that expenditure shares $\omega_{ij,t}$ for each household type in our model match those in the data. We walk through the details of the estimation exercise in Appendix F.

Earnings heterogeneity. As in our benchmark model, unions intermediate households' labor supply decisions and ration labor across all households within a type. Earnings heterogeneity emerges because production sectors differ in their demand for different labor factors, as discussed in Section 4.3.1.

Two accounts. We allow households to hold two accounts, a liquid checking account a and an illiquid investment account b held with a bank. Households can move funds between these two accounts subject to a transaction cost paid out of the liquid account. The liquid account bears a relatively low real rate of return $r_{i,t}^a$. Households can accumulate liquid debt up to a borrowing constraint, $a_{i,t} \geq \underline{a}$. The illiquid account bears a higher return $r_{i,t}^b$ and is subject to a short-sale constraint $b_{i,t} \geq 0$. As in our baseline model, real rates of return (in terms of households' purchasing power) depend on the relative rates of inflation households face in their type-specific consumption bundles.

When transferring funds $\iota_{i,t}$ from the liquid to the illiquid account, households incur a transaction cost $\psi(\iota_{i,t}, b_{i,t})$ that may be proportional to the size of the illiquid account. We adopt the functional form for $\psi(\cdot)$ used in Kaplan et al. (2018) and calibrate this transaction cost to match important moments of the household wealth distribution.

4.3.3 Financial Sector

We model a financial sector that consists of a representative financial intermediary, the "bank", which has two activities: (1) a banking activity, performing maturity transformation by collecting real liquid assets from households and investing them in government bonds, subject to an intermediation spread; and (2) a mutual fund activity, collecting illiquid funds and intermediating them in the form of physical capital to firms.

Banking activity. The bank fully passes through the intermediation cost to households. In addition, the bank applies a borrowing wedge to the prevailing after-intermediation-cost interest rate. We choose the intermediation spread and the borrowing wedge for the model's steady state to match the aggregate and distributional moments in Kaplan et al. (2018) and Auclert et al. (2020).

Mutual fund activity. Illiquid assets are equity claims on the bank. The bank owns the economy's capital stock and makes capital investments. It rents capital to firms in a competitive rental market. We assume the bank operates an investment technology that transforms sectoral goods into gross capital investment. The capital stock then evolves according to $K_{t+1} = I_t + (1-\delta)K_t$, where investment is given by the CES aggregator

$$I_t = \left(\sum_j (\Gamma_j^{inv})^{\frac{1}{\eta_I}} I_{j,t}^{\frac{\eta_I - 1}{\eta_I}} \right)^{\frac{\eta_I}{\eta_I - 1}},$$

where η_I is the elasticity of substitution across sectoral goods used for capital investment. Our quantitative model takes seriously the sectoral and network implications of investment spending, as emphasized by Vom Lehn and Winberry (2022). We calibrate the "investment use" parameter Γ_j^{inv} to match data from the BEA 1997 Capital Flow table, where we compute sector j's total contribution to capital production as a share of the economy's total inputs to capital production.

4.3.4 Government

Government spending takes the form of a homothetic CES aggregate over sectoral goods given by

$$G_t = \left(\sum_k \left(\Gamma_j^g\right)^{\frac{1}{\eta_g}} G_{j,t}^{\frac{\eta_g - 1}{\eta_g}} \right)^{\frac{\eta_g}{\eta_g - 1}}.$$

We calibrate Γ_j^g to match the share of government spending across sectors using the BEA industry input-output "use" table, computing government expenditures in sector j as a percentage of total government spending. Government expenditures include spending by the federal government, federal government enterprises, state and local government, and state and local government enterprises.

Table 3. List of Calibrated Parameters of HANK-IO Full

	Parameters	Value	Target / Source
	Preferences		
$\bar{\rho}$	Average discount rate (p.q.)	6.876 %	Internally calibrated
γ	Relative risk aversion	2	Standard
ϕ	Inverse Frisch elasticity	1	Standard
η_c	Elasticity of substitution between sectors in consumption	0.349	Estimation using CEX-IO
η_x	Elasticity of substitution between sectors in intermediate inputs	0.1	Atalay (2017)
η_l	Elasticity of substitution between types in labor supply	1	Standard
η_f	Elasticity of substitution between factors in production	1	Standard
η_i	Elasticity of substitution between sectors in capital investment	1	Standard
	Household portfolio choice		
$\underline{a}$	Borrowing constraint	-0.25	0.631 quarter of average income
δ	Capital depreciation (p.q.)	1 %	Internally calibrated
ψ_0	Linear adjustment cost	0.044	Internally calibrated
ψ_1	Convex adjustment cost 1	0.956	Internally calibrated
ψ_2	Convex adjustment cost 2	1.402	Internally calibrated
	Financial Intermediary		
ω	Intermediation cost	0.5 %	Internally calibrated
ϑ	Borrowing wedge	3.106 %	Internally calibrated
	Firms		
κ	Aggregate capital adjustment cost	20	Δy after shock
	Nominal rigidities		
ϵ	Average elasticity of substitution for goods in retailer bundling	12.347	Baqaee & Farhi (2020)
χ_j	Average price adjustment cost	96.08	Pasten (2017)
ϵ^w	Elasticity of substitution for labor	10	CEE (2005)
χ^w	Avg. duration of wage contracts	0	Flexible-wage limit
	Government		
λ_π	Taylor rule weight on inflation	1.5	Standard
λ_Y	Taylor rule weight on output	0	Standard
τ^{lab}	Income tax rate	27.512 %	Standard

We assume the government balances its budget at steady state. Any surplus or deficit is rebated to households according to a rescaling rule that is designed to neutralize the quantitative implications of potentially counterfactual lump-sum transfers. The proportion of the aggregate rebate distributed to households of type i is equal to their income share in stationary equilibrium.

The monetary authority follows a standard Taylor rule with weights λ_π and λ_y on inflation and output, respectively. We assume the monetary authority uses the counterpart of the empirical CPI to measure inflation in this context.

4.3.5 Remaining Calibration Parameters

In addition to the parameters already discussed in this section, we calibrate the household discount rate ρ to match average MPCs, and we calibrate the capital depreciation rate δ to match the aggregate capital-output ratio. We set the capital adjustment cost parameter κ so that our baseline model matches VAR evidence on the relative response of investment and output to a monetary

Table 4. Calibrating Alternative Models

	Parameters	Data	Source	HANK-IO Full	HANK-IO A1+A2	HANK
	Calibration					
$\bar{\rho}$	Average discount rate (p.q.)			6.876	6.876	6.876
r	Real interest rate (%)			4.96	5.28	5.66
η_c	Elasticity of substitution			0.349	0.349	0.349
ε	Relative income elasticity			Het	Homo	Homo
Ω^c	Taste preference			By-type	Symmetric	Symmetric
Γ^w	Wage share			By-type	Symmetric	Symmetric
Γ^m	Input-output structure			Yes	Yes	No
	Sectoral heterogeneity			Yes	Yes	No
	Moments					
K/Y	Mean illiquid assets to GDP ratio	2.92	KMV(2018)	2.92	2.55	2.26
B/Y	Mean liquid assets to GDP ratio	0.23	KMV (2018)	0.23	0.23	0.23
G/Y	Government spending to GDP ratio	0.16	ARS (2021)	0.16	0.16	0.16
Borrower	Fraction with $a < 0$	0.15	Bayer (2020)	0.15	0.16	0.16
r_t^{borrow}	Borrow rate (%)	8.00	KMV(2018)	7.56	7.89	8.27
HtM	Fraction of HtM	0.30	KMV(2018)	0.35	0.32	0.35

policy shock. We take the remaining parameters—including Taylor rule coefficients and the labor income tax rate—from the literature. We calibrate the borrowing wedge to match the share of borrowers. We summarize the calibration of our baseline HANK-IO model in Table 3. We show comparison of moments from our quantitative model to the targets in Table 4.

4.4 Alternative Models

In addition to our main quantitative model ("HANK-IO full"), we also calibrate two instructive comparison benchmarks to illustrate our quantitative results in Section 5. Our first comparison benchmark ("HANK-IO A1+A2") deviates from the full model by only shutting off earnings and expenditure heterogeneity, exactly as under assumptions A1 and A2, which gave rise to the analytical as-if benchmark of Section 3.1.[21] Our second comparison benchmark ("HANK") corresponds to the standard one-sector one-type HANK limit of our model. This benchmark not only shuts off earnings and expenditure heterogeneity, but also all other dimensions of sectoral heterogeneity. We calibrate households' discount rate to be the same across models and let the real interest rate adjust to clear the bond market.

[21] To turn off expenditure heterogeneity, we assume that consumer demand takes a homothetic CES structure where the relative income elasticity becomes homogeneous across sectors $\varepsilon_j = 1 - \eta_c$. All household types share the same CES weights, which are symmetric across sectors. To turn off earnings heterogeneity, we assume that all sectors allocate an equal share of total wage expenditure across household types.

5 Policy Transmission in HANK-IO

In this section, we use our quantitative HANK-IO model to revisit the transmission of monetary and fiscal policy.

5.1 Monetary Policy

Figure 5 plots impulse responses of macroeconomic aggregates to a 25bps contractionary monetary policy shock in the full quantitative model ("HANK-IO full", solid blue line) as well as the two comparison benchmarks, whose calibration we summarize in Table 4. Figure 5 illustrates two quantitative results: First, sectoral heterogeneity amplifies the aggregate effects of monetary policy. Panel (1) shows that the at-impact response of real GDP in the multi-sector variant of the standard HANK model ("HANK-IO A1 + A2", dashed green line) is roughly twice as large as in the standard model ("HANK", dashed red line). Second, as in Section 3, household-sector linkages dampen aggregate monetary policy transmission. In the full quantitative model, the at-impact real GDP response is roughly 30% smaller.

Accounting for sectoral heterogeneity increases monetary non-neutrality. This is a well-understood result from the literature on multi-sector New Keynesian models (Pasten et al., 2017; Rubbo, 2023). Our analysis highlights that household-sector linkages push against this amplification. The remainder of this subsection discusses the forces underlying this result.

iMPCs. Our quantitative model matches intertemporal marginal propensities to consume (iMPCs) over time, as well as at-impact MPCs across household types. These are illustrated in Panel (1) of Figure 6, which compares model MPCs against data from Auclert et al. (2023).

Earnings heterogeneity. In our model, contractionary monetary policy decreases the income of low-income households the most, as depicted in Figure 6. This pattern is consistent with empirical studies by Amberg et al. (2022) using Swedish administrative data, Faia et al. (2022) using U.S. data, and Hubert and Savignac (2023) using French data. They collectively underscore the U-shaped impact of monetary policy shocks on labor income, which is most pronounced for low-income households.

Expenditure heterogeneity. Our model predicts that monetary policy shocks exhibit a U-shaped effect on purchasing power across income groups, with mid-income earners least affected. This outcome largely stems from the non-homothetic CES demand function embedded in our full model. In Panel (3) of Figure 6, we instead illustrate the cross-sectional incidence on purchasing power under homothetic CES preferences.

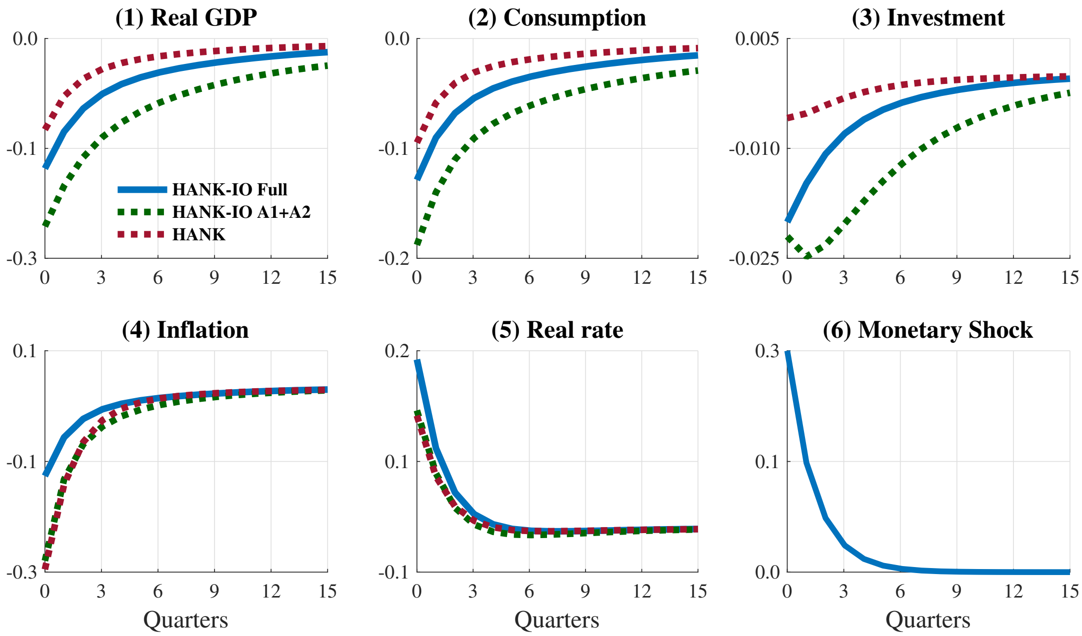

Figure 5. Monetary Policy Shock

Note. Figure 5 plots aggregate impulse responses to a contractionary monetary policy shock in HANK-IO Full (solid blue line) as well as the comparison benchmarks HANK-IO A1+A1 (dashed red line) and HANK (dashed green line). The calibration of these alternative models is reported in Table 4. Panels (1) through (3) plot $d\log Y_t$, $d\log C_t$, and $d\log I_t$ constructed using an expenditure-share-weighted Divisia index. Panel (4) plots inflation in the GDP deflator, $d\pi_t$. Panels (5) and (6) plot changes in the real interest rate and the monetary policy shock in %. The monetary shock has a half life of 1 quarter.

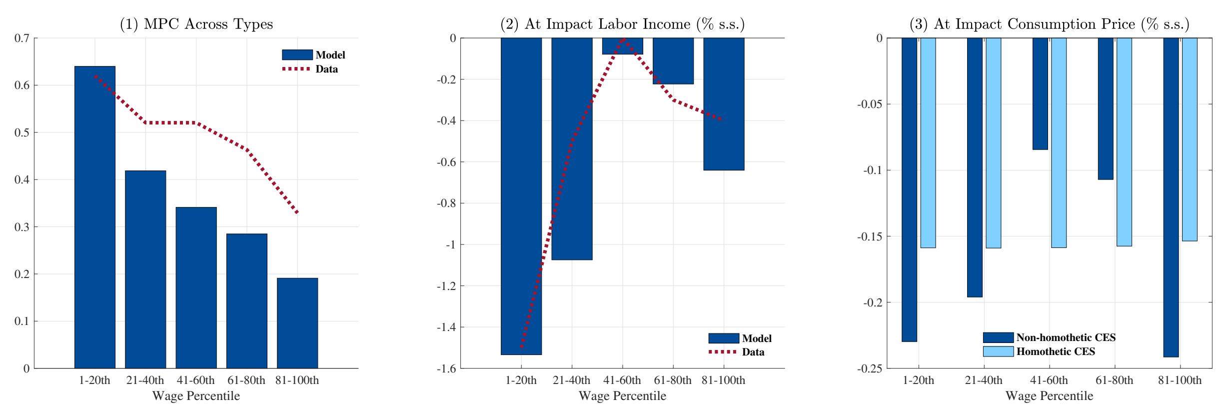

Figure 6. By-type Responses

Note. Figure 6 plots the annual MPC, at-impact changes in labor income, and at-impact changes in consumption price for each household type across the income distribution. The blue bars are generated by the HANK-IO Full model. The data in Panel (1) is reconstructed from Auclert et al. (2023), and data in Panel (2) comes from Amberg et al. (2022).

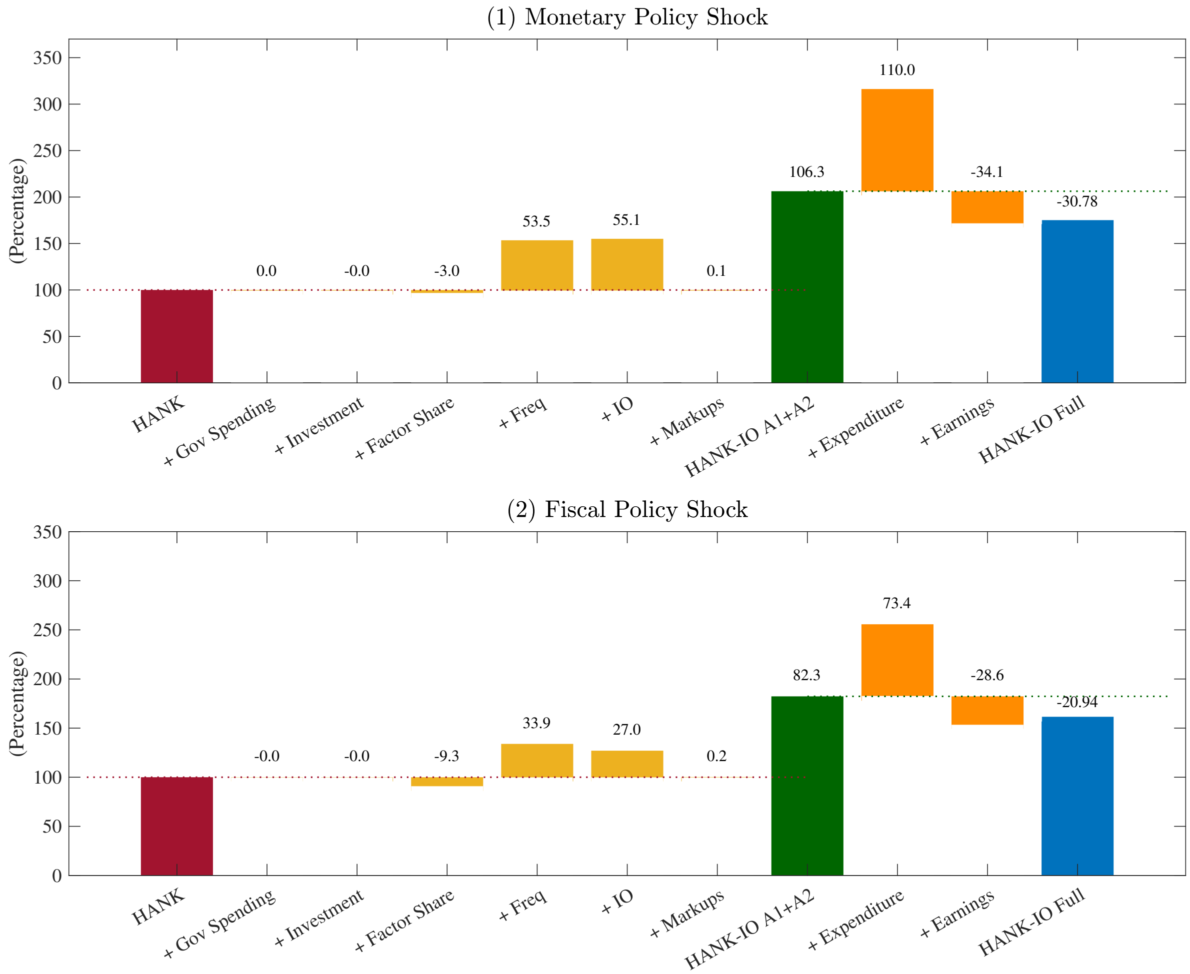

Figure 7. Alternative Model Results

Note. We plot alternative models' at-impact $d \log Y$ in response to a 25bps contractionary monetary policy shock (panel (1)) and a current-allocation deficit-financed fiscal shock.

The role of sectoral heterogeneity. Next, we dissect the impact of sectoral heterogeneity on the at-impact response of real GDP. By adding each heterogeneous feature to the HANK model one at a time, we observe their individual contributions. The yellow bars in Figure 7 illustrate the comparison between an alternative model with one sectoral heterogeneity added to the HANK model with no sectoral heterogeneity. Heterogeneous price rigidity amplifies monetary policy transmission by 54%. The HANK-IO model's departure from monetary neutrality is also notably more pronounced, enlarging the aggregate economic response by 55% with the integration of the BEA input-output structure. This amplification aligns with the theory that production networks exacerbate the overall nominal rigidity, thereby reinforcing the responses of real consumption and investment. Conversely, we find that other sectoral heterogeneity such as government spending share, capital investment network, factor share heterogeneity, and markup variations contribute little to the aggregate effect of policy transmission. Sectoral heterogeneity all together amplifies monetary policy transmission by 106%.

The role of household-sector linkages. Building on HANK-IO A1+A2, we incorporate earning heterogeneity and expenditure heterogeneity separately to discern their individual impacts. Earning and expenditure heterogeneity push changes in aggregate economic activities in opposite directions. Expenditure heterogeneity significantly intensifies the model's response to policy shocks, leading to a 110% increase in the change of aggregate economic activity. In contrast, earning heterogeneity exhibits a dampening effect, attenuating the transmission of monetary policy. When considering the interplay of household-sector linkages, the net influence appears to be a tempering one, with an overall dampening of policy transmission by 31%.

5.2 Fiscal Policy

Figure 8 reports our quantitative results for fiscal policy shocks. We allow for different assumptions on the responsiveness of government debt and the expenditure weights of public spending. Across all scenarios, we consider a 1% increase in real government spending $d \log G_t$ on impact, with a half-life of 1 quarter. Panel (a) plots the impulse responses of real GDP (a.1) and consumption (a.2)—both measured using changes in expenditure-share weighted Divisia indices, $d \log Y_t$ and $d \log C_t$—as well as government debt dB_t (a.3) and the fiscal spending shock $d \log G_t$ (a.4). Panel (b) shows the on-impact fiscal multiplier across all experiments.

We consider six alternative models with variations in three dimensions: (i) the fiscal shock is either financed with a balanced-budget principle ("BB") or with bond accumulation ("DF") persistence with a half life of 8 quarters (in line with Auclert and Rognlie (2018)'s estimate) where $dBg_t = \exp(-\theta_b * t)(dB_{t-1} + dG_t)$; (ii) the government spending adopts either the current spending share allocation ("Current"), or is fully allocated to the highest at-impact multiplier sector ("Targeted"); (iii) the model is either vanilla HANK ("HANK"), or accounting for only sectoral heterogeneity ("HANK-IO A1+A2"), or the full model with household-sector linkages ("HANK-IO").

The red and orange dashed lines in panel (a) plot transition dynamics in the HANK comparison benchmark in response to a deficit-financed and balanced-budget government spending shock respectively. The associated impact multiplier is 1.42 for deficit-financed, and 1.14 for balanced-budget, in line with the findings in Auclert et al. (2023). Deficit-financed fiscal policy has a larger on-impact multiplier than balanced-budget policy because consumption is not crowded out by taxes and transfers. Under balanced-budget policy, consumers' after-tax and after-transfer income—and thus consumption—remains largely unchanged. While under deficit-financed policy, households' after-tax and after-transfer income increases as additional government spending is not fully offset by contemporaneously higher taxes. Consumption increases more in response to deficit-financed than to balanced-budget fiscal policy shocks.

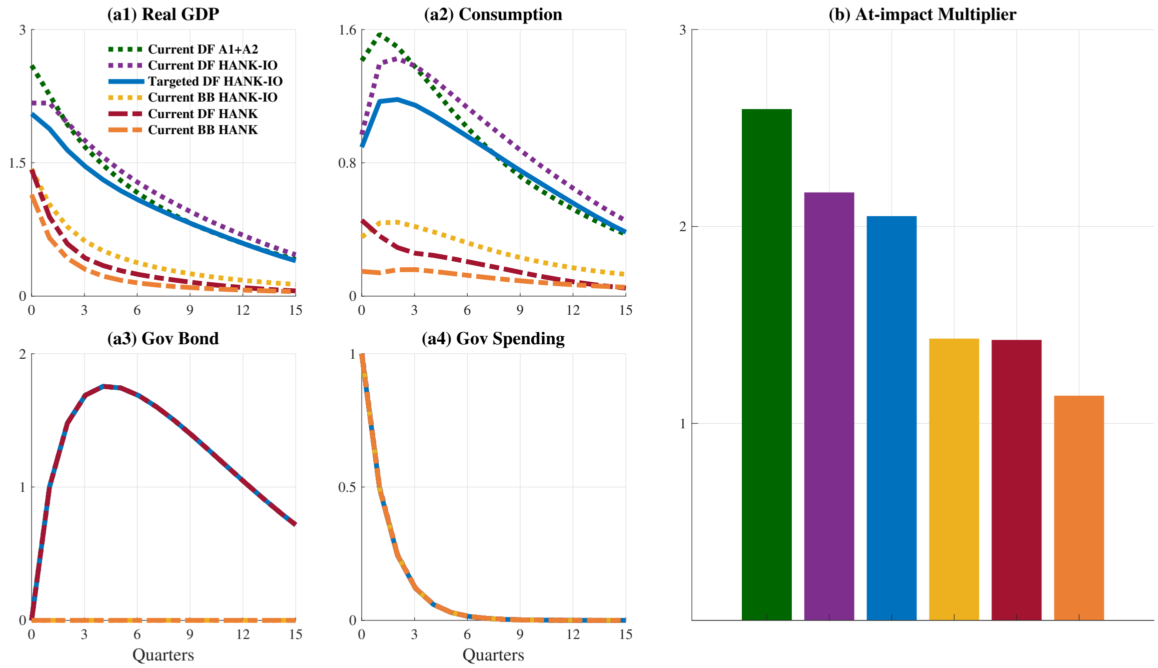

Figure 8. Fiscal Policy Shock

Note. Figure 8 plots the aggregate impulse responses to different fiscal policy shocks (panel a) and the associated on-impact multipliers (panel b). Panels (a.1) and (a.2) plot changes in output and consumption constructed using an expenditure-shares-weighted Divisia index. Panels (a.3) and (a.4) plot public debt and the government spending shock. The on-impact fiscal multiplier in panel (b) is calculated as dY_1/dG_1.

Similar to the monetary policy shock, sectoral heterogeneity amplifies the aggregate effects of monetary policy. The HANK-IO model without earnings or expenditure heterogeneity has an impact multiplier of 2.60, which is 82% larger than that in the HANK benchmark. Panel (2) of Figure 7 dissects the individual heterogeneity's effect. Heterogeneous price rigidity and the input-output structure have notable amplification implications, while heterogeneous factor share dampens transmission.

From HANK-IO A1+A2, incorporating household-sector linkages through earnings and expenditure channels dampens policy transmission. The blue solid line plots transition dynamics in response to the same deficit-financed spending shock in HANK-IO Full model. The on-impact multiplier is 2.05, which is 21% smaller than in the HANK-IO A1+A2 comparison. This compares to a fiscal multiplier of 1.43 for a balanced-budget spending shock in HANK-IO Full.

The allocation of aggregate government expenditures across sectors plays an important role in the transmission of fiscal policy to economic activity. We compare three different spending scenarios: "current" uses empirically observed expenditure weights, "uniform" assumes a uniform allocation of expenditures across sectors, and "targeted" assumes that spending is allocated to maximize the on-impact output multiplier. The baseline fiscal multiplier of 2.05 uses empirically

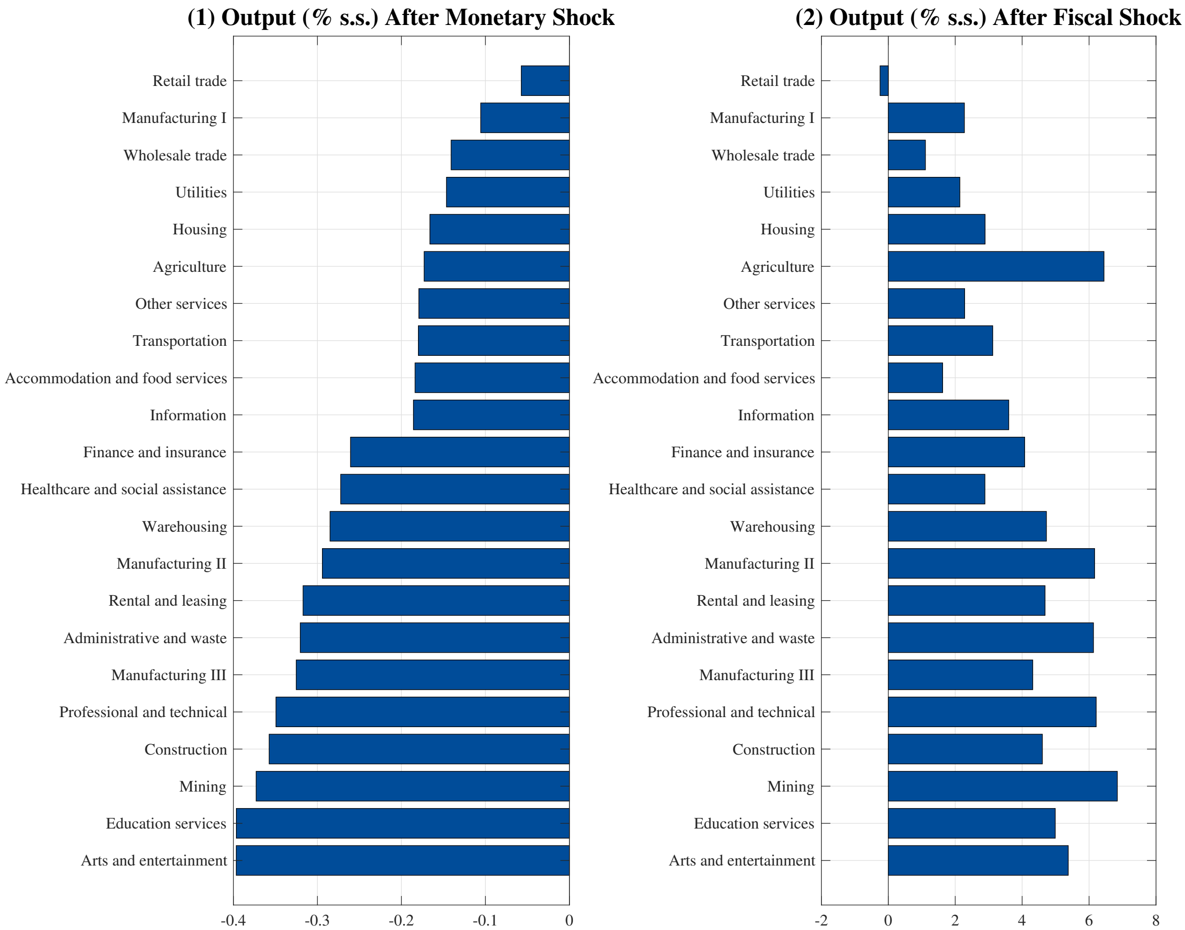

Figure 9. Sectoral Output Implications of Monetary and Fiscal Policy Shocks

Note. Figure 9 visualizes the heterogeneous incidence of stabilization policy across production sectors. Panel (1) plots the on-impact response of sectoral activity $d \log Y_{j,1}$ to a 25bps contractionary monetary policy shock with a half-life of 1 quarter, ranking sectors in ascending order. Panel (2) plots the on-impact response of sectoral activity to a 1% expansionary, deficit-financed fiscal policy shock, assuming an allocation of spending according to the empirically observed expenditure shares.

observed expenditure weights to allocate additional government spending. It is only slightly smaller than the fiscal multiplier under a uniform allocation of spending, which is 2.06. Targeted spending, on the other hand, attains a larger multiplier of 2.17.

Sectoral incidence of policy transmission. The incidence of stabilization policy varies across sectors in our model. Figure 9 plots the response of sectoral activity to a 25bps contractionary monetary policy shock (panel 1) and an expansionary, deficit-financed government spending shock (panel 2). The sectors (y-axis) are ranked in descending order according to the on-impact output response to the monetary shock. The responsiveness of sectoral activity to monetary policy shocks ranges from -6% (deviations from steady state) for the retail sector to -40% in the arts and entertainment industry. The sectoral incidence of fiscal policy—assuming an allocation of spending across sectors in proportion to current public expenditure shares—is broadly symmetric. Activity in retail trade features the smallest response while activity in mining is most responsive.

6 Conclusion

This paper is motivated by empirical evidence of systematic household-sector linkages in disaggregated micro data. We build a quantitative framework to assess their implications for policy transmission and the aggregation of sectoral shocks. Our "HANK-IO" model brings together a heterogeneous-agent New Keynesian model with a multi-sector business cycle model with input-output linkages in the tradition of Long and Plosser (1983). We analytically characterize an as-if benchmark that features a strict decoupling between household and sectoral heterogeneity. Away from this benchmark, however, novel earnings and expenditure heterogeneity channels emerge through which household-sector linkages have important implications for policy transmission.

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Online Appendix

A Additional Model Details

A.1 Labor Market Structure and Union Problem

We follow closely Erceg et al. (2000) and Auclert et al. (2023). Each household of type i provides $n_{ik,t}$ hours of work to each of a continuum of unions indexed by $k \in [0,1]$. Total labor hours supplied by this single household are

$$n_{i,t} = \int_k n_{ik,t} dk.$$

Each union aggregates effective labor units provided by each household into a union-type-specific task $N_{ik,t}$, given by

$$N_{ik,t} = n_{k,t}.$$

A labor packer then further aggregates these labor services into aggregate supply of factor i

$$N_{i,t} = \left(\int_k N_{ik,t}^{\frac{\epsilon^w - 1}{\epsilon^w}} dk \right)^{\frac{\epsilon^w}{\epsilon^w - 1}}$$

and sells it to firms at the nominal wage $W_{i,t}$. Importantly, unions ration labor so that all households of a type work the same hours.

Labor union k sets a common wage $W_{ik,t}$ for each of its members. It can adjust this wage flexibly—unlike Auclert et al. (2023), our paper focuses on price stickiness, for which we have good sectoral data. Under flexible wage adjustments, we obtain a type-specific labor supply curve

$$v'(N_{i,t}) = \frac{\epsilon^w - 1}{\epsilon^w} w_{i,t} u'(C_{i,t}),$$

where $w_{i,t} = \frac{W_{i,t}}{p_{i,t}}$ is the real wage. Crucially, we assume that unions maximize an objective specified in terms of average consumption for each household type.

B Proofs

B.1 Proof of Proposition 1

B.1.1 Dynamic IS Curve

We impose assumptions A1 and A2, and we also assume that the household consumption and government spending aggregators are the same CES, $\mathcal{D} = \mathcal{G}$. We drop household i subscripts to emphasize that all types are effectively symmetric under these assumptions.

Consumption is then given by $c_t = \left[\sum_j \kappa_j c_{j,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ and $G_t = \left[\sum_j \kappa_j G_{j,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$, where κ_j is the CES weight on good j. The ideal price index then solves $P_t = \left[\sum_j \kappa_j p_{j,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$, and the standard sectoral demand function is then given by $c_{j,t} = \kappa_j \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} c_t$.

Under assumption A2, we also have $n_t(a,z) = N_t$ for all (a,z), where N_t is aggregate labor. Using the GDP deflator P_t, the household budget constraint can then be written as $a_t = R_{t-1}a_{t-1} + e_t + T_t - c_t$, where R_t is the real interest rate. Real earnings are given by $e_t(z) = \tau_t \left(z w_t N_t + z \frac{1}{P_t} \sum_j \Pi_{j,t}\right)^{1-\lambda}$, where $w_t = \frac{W_t}{P_t}$ is the real wage.

Notice that aggregate corporate profits satisfy $\frac{1}{P_t}\Pi_t = \frac{1}{P_t}\sum_j (p_{j,t} y_{j,t} - W_t N_{j,t} - \sum_k p_{k,t} x_{jk,t})$. Given the price index P_t, we can unambiguously define real GDP in levels,

$$Y_t^n = P_t Y_t.$$

Therefore, we are left with

$$\begin{aligned}
\frac{1}{P_t}\Pi_t &= \frac{1}{P_t}\left(\sum_j p_{j,t} y_{j,t} - \sum_j \sum_k p_{k,t} x_{jk,t}\right) - w_t N_t \\
&= \frac{1}{P_t}\sum_j \left(p_{j,t} y_{j,t} - \sum_k p_{j,t} x_{kj,t}\right) - w_t N_t \\
&= Y_t - w_t N_t
\end{aligned}$$

where we used the labor market clearing condition $N_t = \sum_j N_{j,t}$ and the sectoral goods market clearing conditions. Household earnings can then be written in terms of real GDP as

$$e_t = \tau_t z_t^{1-\lambda} Y_t^{1-\lambda}.$$

Sequence-space recursive representation. The household problem admits a recursive representation. The consumption policy function can be written as $c_t(a,z)$, and likewise the post-tax earnings function of a household is given by $e_t(a)$.

As in Auclert et al. (2023), we now define

$$Z_t = \int e_t(a,z) g_t(a,z)\, da\, dz = \tau_t Y_t^{1-\lambda} \int z^{1-\lambda} g_t(a,z)\, da\, dz,$$

and we define the government's tax revenue as $T_t = Y_t - \int e_t(a,z) g_t(a,z)\, da\, dz$. Total income is split into tax revenue and post-tax payouts to households via labor income and dividends according to $Y_t = Z_t + T_t$. Finally, notice that we can define a given household's post-tax income as

$$e_t(a,z) = e_t(z) = z^{1-\lambda} \tau_t Y_t^{1-\lambda} = \frac{z^{1-\lambda}}{\int \tilde{z}^{1-\lambda} g_t(a,\tilde{z})\, da\, d\tilde{z}} Z_t,$$

where we used

$$\tau_t Y_t^{1-\lambda} = \frac{Z_t}{\int z^{1-\lambda} g_t(a,z)\, da\, dz}$$

from above. Crucially, the normalization here $\int \tilde{z}^{1-\lambda} g_t(a,\tilde{z})\, da\, d\tilde{z}$ is a constant even though $g_t(\cdot)$ moves around. Therefore, we have

$$e_t(z) = e(z; Z_t).$$

Summarizing: the household problem. In conclusion, the household problem can be written as

$$\max_{\{c_t\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[u(c_t) - v(N_t) \Big],$$

subject to

$$c_t + a_{t+1} = (1 + r_{t-1}) a_t + \frac{z_t^{1-\lambda}}{\int \tilde{z}^{1-\lambda} g_t(a,\tilde{z})\, da\, d\tilde{z}} Z_t$$

$$a_{t+1} \geq \underline{a},$$

and taking as given the law of motion for idiosyncratic earnings risk z_t. It then follows that the household's consumption policy function at time t can be represented as

$$c_t = c\Big(a, z; \Big\{ Z_s, r_s \Big\}_{s\geq t}\Big).$$

Similarly, we define the aggregate consumption function as

$$\mathcal{C}_t = \int c\Big(a, z; \Big\{ Z_s, r_s \Big\}_{s\geq t}\Big)\, g_t(a,z)\, da\, dz,$$

which consequently also admits the representation

$$\mathcal{C}_t\Big(\Big\{ Z_s, r_s \Big\}_{s\geq t}\Big),$$

taking as given the cross-sectional distribution at time t. Crucially, g_t is invariant to shocks (since it's pre-determined at time t), so we don't explicitly need to take it into account.

Sectoral demand and goods market clearing. The key observation so far is that the *aggregate consumption function* of this economy, $\mathcal{C}_t(\cdot)$, admits the same sequence space representation as in the standard HANK model. Now using household sectoral demand, we can also characterize an aggregate sectoral consumption function, given by

$$\begin{aligned}\mathcal{C}_{j,t} &= \int c_{j,t}(a,z)\, g_t(a,z)\, da\, dz \\ &= \int \kappa_j \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} c\left(a,z;\left\{Z_s,r_s\right\}_{s\geq t}\right) g_t(a,z)\, da\, dz \\ &= \kappa_j \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} \mathcal{C}_t\left(\left\{Z_s,r_s\right\}_{s\geq t}\right).\end{aligned}$$

The sectoral demand equations implied by CES preferences aggregate conveniently. In particular, sectoral demand is a function of aggregate consumption and the *relative price*. Only the contemporaneous relative price matters, while the forward-looking sequences $\{Z_s, r_s\}$ matter to pin down the aggregate consumption level.

We can now use this in goods market clearing, which becomes

$$y_{j,t} = \kappa_j \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} \mathcal{C}_t\left(\left\{Z_s,r_s\right\}_{s\geq t}\right) + \kappa_j \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} G_t.$$

Plugging into the definition of real GDP, we now have

$$\begin{aligned}Y_t &= \sum_j \frac{p_{j,t}}{P_t} \kappa_j \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} \left(\mathcal{C}_t\left(\left\{Z_s,r_s\right\}_{s\geq t}\right) + G_t\right) \\ &= \sum_j \kappa_j p_{j,t}^{1-\eta} P_t^{-(1-\eta)} \left(\mathcal{C}_t\left(\left\{Z_s,r_s\right\}_{s\geq t}\right) + G_t\right)\end{aligned}$$

Using the definition of the ideal CPI $P_t = \left[\sum_j \kappa_j p_{j,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$, we arrive at

$$\begin{aligned}Y_t &= \sum_j \kappa_j p_{j,t}^{1-\eta} \left(\sum_j \kappa_j p_{j,t}^{1-\eta}\right)^{-1} \left(\mathcal{C}_t\left(\left\{Z_s,r_s\right\}_{s\geq t}\right) + G_t\right) \\ &= \mathcal{C}_t\left(\left\{Z_s,r_s\right\}_{s\geq t}\right) + G_t.\end{aligned}$$

This concludes the proof of the dynamic IS curve representation.

B.1.2 Dynamic LM Curve

The real interest rate is given by the Fisher relation

$$r_t = i_t - \pi_t,$$

where i_t is the exogenous nominal interest rate and $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is CPI inflation in the as-if benchmark. The CPI P_t is a function of sectoral prices $p_{j,t}$, which are themselves determined by sectoral Phillips curves. In particular, sectoral Phillips curves in our setting admit a sequence-space representation

$$p_{j,t} = \mathcal{P}_{j,t}\Big(\Big\{mc_{j,s}\Big\}_{s\geq 0}\Big).$$

Furthermore, it follows directly from firm cost minimization that marginal cost takes the form

$$mc_{j,t} = mc_j\Big(A_{j,t}, \{p_{k,t}\}_k, W_t\Big).$$

Thus, sectoral prices $p_{j,t}$ depend on the time paths of all past and future sectoral prices, $\boldsymbol{p}_j = \{p_{j,s}\}_{s\geq 0}$, as well as the time paths of sectoral technology shocks and nominal wages.

Finally, the nominal wage of the single labor factor (assumption A2) is determined by the (union) labor supply schedule

$$v'(N_t) = \frac{\epsilon^w - 1}{\epsilon^w}\frac{W_t}{P_t}u'(C_t)$$
$$v'(\sum_j N_{j,t}) = \frac{\epsilon^w - 1}{\epsilon^w}\frac{W_t}{P_t}u'(Y_t - G_t).$$

This equation solves for the nominal wage W_t as a function of real GDP and government spending, $Y_t - G_t$, sectoral prices $\{p_{k,t}\}_k$, and sectoral labor demand $N_{j,t}$. It then follows directly from the firm's cost minimization problem that optimal labor demand $N_{j,t}$ must be a function of the wage W_t, input prices $\{p_{k,t}\}_k$, technology $A_{j,t}$ and output $y_{j,t}$. Output $y_{j,t}$ is determined by demand, which is again pinned down by prices and aggregate demand.

The real interest rate r_t then admits a sequence-space representation as a function of the time paths of aggregate demand $\{Y_s\}_{s\geq 0}$, technology shocks $\{A_{j,s}\}_{j,s\geq 0}$, and policy $\{G_s, T_s, i_s\}_{s\geq 0}$. This sequence-space representation is equivalent to its analog in the representative-household RANK-IO model, taking as given a path of aggregate demand. The assumption of labor rationing is key for this.

B.1.3 Proof of Corollary 2

Starting with the dynamic IS equation, we have

$$dY_t = dG_t + \sum_{s=t}^{\infty} \frac{\partial \mathcal{C}_t}{\partial (Y_s - T_s)} (dY_s - dT_s),$$

where monetary policy holds the real interest rate constant, $dr_t = 0$. Notice that

$$\frac{\partial \mathcal{C}_t}{\partial (Y_s - T_s)} = \frac{\partial \mathcal{C}_t}{\partial e_s} = M_{ts}$$

using our notation in the main text: this is the spending propensity with respect to a marginal increase in unearned income. In matrix form,

$$d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M} d\boldsymbol{T} + \boldsymbol{M} d\boldsymbol{Y},$$

which concludes the proof.

B.1.4 Proof of Corollary 3

We again start with the dynamic IS equation. Differentiating,

$$dY_t = \sum_{s=t}^{\infty} \left[\frac{\partial \mathcal{C}_t}{\partial (Y_s - T_s)} dY_s + \frac{\partial \mathcal{C}_t}{\partial r_s} dr_s \right].$$

Recalling our definition of M^r_{ts} from the main text and stacking, the matrix form becomes

$$d\boldsymbol{Y} = \boldsymbol{M} d\boldsymbol{Y} + \boldsymbol{M}^r d\boldsymbol{r}.$$

B.1.5 Proof of Corollary 4

From the definition of factor shares (revenue-based Domar weights), $\Lambda_t = \frac{W_t N_t}{P_t Y_t}$, we have

$$d \log \Lambda_t = d \log W_t + d \log N_t - d \log P_t - d \log Y_t.$$

Also notice that $d \log w_t = d \log W_t - d \log P_t$. Also, from the labor supply schedule, we have

$$d \log W_t - d \log P_t = \eta d \log N_t + \gamma d \log Y_t.$$

Next, we define the time-varying markup $\mu_{j,t}$ to represent the sectoral Phillips curve in reduced form,

$$p_{j,t} = \mu_{j,t} MC_{j,t}.$$

This yields $d \log p_{j,t} = d \log \mu_{j,t} + d \log MC_{j,t}$. From cost minimization (and Shephard's lemma) it follows that marginal cost can be unpacked as

$$d\log p_{j,t} = d\log \mu_{j,t} + \sum_{k=1}^{N} \tilde{\Omega}_{jk}\, d\log p_{k,t} + \tilde{\Omega}_{jN+1}\, d\log W_t - d\log A_{j,t}$$
$$d\log P_t = \left(\boldsymbol{I} - \tilde{\Omega}^p\right)^{-1} \left(\tilde{\Omega}_{[:,L]}\, d\log W_t + d\log \mu_t - d\log A_t\right)$$

Now notice that

$$\tilde{\Psi}_{jL} = \sum_k \tilde{\Psi}_{jk} \tilde{\Omega}_{kL} = 1$$

because $\tilde{\Omega}_{jL} + \sum_k \tilde{\Omega}_{jk} = 1$. Therefore, we have

$$d\log P_t = \mathbf{1}\, d\log W_t + \tilde{\Psi}\, d\log \mu_t - \tilde{\Psi}\, d\log A_t.$$

Now, notice that we can write

$$d\log P_t = b' d\log P_t = d\log W_t + \tilde{\lambda}\left(d\log \mu_t - d\log A_t\right)$$

And we have

$$d\log \Lambda_t = d\log W_t - d\log P_t + d\log N_t - d\log Y_t$$
$$d\log W_t - d\log P_t = \eta d\log N_t + \gamma d\log Y_t.$$

Thus, $d\log N_t = d\log P_t + d\log Y_t + d\log \Lambda_t - d\log W_t$. Plugging in for real wages

$$d\log W_t - d\log P_t = \tilde{\lambda}\left(d\log A_t - d\log \mu_t\right),$$

we have

$$d\log \Lambda_t = \tilde{\lambda}\left(d\log A_t - d\log \mu_t\right) + d\log N_t - d\log Y_t$$
$$\tilde{\lambda}\left(d\log A_t - d\log \mu_t\right) = \eta d\log N_t + \gamma d\log Y_t,$$

and solving out for $d\log N_t$ yields

$$\tilde{\lambda}\left(d\log A_t - d\log \mu_t\right) = \eta\Big(d\log Y_t + d\log \Lambda_t - \tilde{\lambda}\left(d\log A_t - d\log \mu_t\right)\Big) + \gamma d\log Y_t.$$

Rearranging, we arrive at our result.

B.2 Proof of Proposition 5

We start from the definition of nominal GDP $Y_t^n = \sum_j p_{j,t}(C_{j,t} + G_{j,t})$. We then decompose changes in nominal GDP into real GDP and changes in the GDP deflator according to

$$dY_t^n = \sum_j p_{j,t}(dC_{j,t} + dG_{j,t}) + \sum_j (C_{j,t} + G_{j,t}) dp_{j,t}.$$

Notice that we can write

$$\begin{aligned} d\log Y_t^n &= \frac{1}{Y_t^n}\sum_j p_{j,t}(dC_{j,t} + dG_{j,t}) + \frac{1}{Y_t^n}\sum_j p_{j,t}(C_{j,t} + G_{j,t}) d\log p_{j,t} \\ &= \sum_j b_{j,t}\frac{dC_{j,t} + dG_{j,t}}{C_{j,t} + G_{j,t}} + \sum_j b_{j,t} d\log p_{j,t} \\ &= \sum_j b_j d\log C_{j,t} + \sum_j \frac{b_j}{C_j} dG_{j,t} + \sum_j b_j d\log p_{j,t}. \end{aligned}$$

where the last line evaluates around a steady state with $G_{j,ss} = 0$. Notice that

$$\frac{b_j}{C_j} = \frac{p_j(C_j + G_j)}{Y^n C_j} = \frac{p_j}{Y^n} = p_j,$$

where the last equality uses our numeraire assumption that $Y_{ss}^n = 1$. Thus, we arrive at the following expression for real GDP changes:

$$d\log Y_t = \sum_j b_j d\log C_{j,t} + \sum_j p_j dG_{j,t}.$$

Fiscal policy. Under our CES assumption, we have

$$G_{j,t} = \kappa_{Gj}\left(\frac{p_{j,t}}{P_{G,t}}\right)^{-\eta_G} G_t,$$

where

$$P_{G,t} = \left[\sum_j \kappa_{Gj} p_{j,t}^{1-\eta_G}\right]^{\frac{1}{1-\eta_G}}.$$

Thus, we have

$$dG_{j,t} = \kappa_{Gj}\left(\frac{p_{j,t}}{P_{G,t}}\right)^{-\eta_G}\left[dG_t - \eta_G\left(d\log p_{j,t} - d\log P_{G,t}\right)G_t\right].$$

We evaluate our result to first order around a steady state with $G_{ss} = 0$. So we are simply left with, to first order,

$$dG_{j,t} = \kappa_{Gj} \left(\frac{p_j}{P_G} \right)^{-\eta_G} dG_t.$$

Sectoral consumption. Next, we unpack sectoral consumption. Recall that

$$C_{j,t} = \sum_i \mu_i \kappa_{ij} \left(\frac{p_{j,t}}{P_{i,t}} \right)^{-\eta_i} \mathcal{C}_{i,t}\left(\left\{ e_{i,s}, R_{i,s} \right\}_{s \geq t} \right).$$

We have

$$\begin{aligned} d \log C_{j,t} &= \frac{1}{C_{j,t}} \sum_i \mu_i d \left[\kappa_{ij} \left(\frac{p_{j,t}}{P_{i,t}} \right)^{-\eta_i} C_{i,t} \right] \\ &= \frac{1}{C_{j,t}} \sum_i \mu_i \kappa_{ij} \left(\frac{p_{j,t}}{P_{i,t}} \right)^{-\eta_i} \left[dC_{i,t} - \eta_i \left(d \log p_{j,t} - d \log P_{i,t} \right) C_{i,t} \right] \end{aligned}$$

Notice that real GDP is weighted by

$$b_j d \log C_{j,t} = p_j \sum_i \mu_i \kappa_{ij} \left(\frac{p_j}{P_i} \right)^{-\eta_i} dC_{i,t} - p_j \sum_i \mu_i \kappa_{ij} \left(\frac{p_j}{P_i} \right)^{-\eta_i} \eta_i \left(d \log p_{j,t} - d \log P_{i,t} \right) C_i$$

where we now evaluated around steady state and used our numeraire assumption $Y^n = 1$, which implies

$$b_j = \frac{p_j (C_j + G_j)}{Y^n} = p_j C_j.$$

Next, we work out the contribution to real GDP. Evaluating around steady state, the consumption is given by

$$\begin{aligned} \sum_j b_j d \log C_{j,t} &= \sum_j p_j \left[\sum_i \mu_i \kappa_{ij} \left(\frac{p_j}{P_i} \right)^{-\eta_i} dC_{i,t} - \sum_i \mu_i \kappa_{ij} \left(\frac{p_j}{P_i} \right)^{-\eta_i} \eta_i \left(d \log p_{j,t} - d \log P_{i,t} \right) C_i \right] \\ &= \sum_j \sum_i \mu_i \kappa_{ij} p_j \left(\frac{p_j}{P_i} \right)^{-\eta_i} dC_{i,t} - \sum_j \sum_i \mu_i \kappa_{ij} p_j \left(\frac{p_j}{P_i} \right)^{-\eta_i} \eta_i \left(d \log p_{j,t} - d \log P_{i,t} \right) C_i. \end{aligned}$$

The first term becomes

$$
\begin{aligned}
\sum_j \sum_i \mu_i \kappa_{ij} p_j \left(\frac{p_j}{P_i}\right)^{-\eta_i} dC_{i,t} &= \sum_i \mu_i P_i^{\eta_i} \sum_j \kappa_{ij} p_j^{1-\eta_j} dC_{i,t} \\
&= \sum_i \mu_i P_i^{\eta_i} P_i^{1-\eta_i} dC_{i,t} \\
&= \sum_i \mu_i P_i dC_{i,t}.
\end{aligned}
$$

Now notice that

$$
d \log P_{i,t} = \sum_j \kappa_{ij} \left(\frac{p_{j,t}}{P_{i,t}}\right)^{1-\eta_i} d \log p_{j,t}
$$

Thus, we get

$$
\begin{aligned}
& - \sum_j \sum_i \mu_i \kappa_{ij} p_j \left(\frac{p_j}{P_i}\right)^{-\eta_i} \eta_i \left(d \log p_{j,t} - d \log P_{i,t} \right) C_i \\
=& - \sum_j \sum_i \mu_i \kappa_{ij} p_j \left(\frac{p_j}{P_i}\right)^{-\eta_i} \eta_i \left(d \log p_{j,t} - \sum_j \kappa_{ij} \left(\frac{p_j}{P_i}\right)^{1-\eta_i} d \log p_{j,t} \right) C_i \\
=& - \sum_i \mu_i \eta_i C_i \left[\sum_j \kappa_{ij} p_j \left(\frac{p_j}{P_i}\right)^{-\eta_i} d \log p_{j,t} - \sum_j \kappa_{ij} p_j \left(\frac{p_j}{P_i}\right)^{-\eta_i} \sum_j \kappa_{ij} \left(\frac{p_j}{P_i}\right)^{1-\eta_i} d \log p_{j,t} \right] \\
=& - \sum_i \mu_i \eta_i C_i \left[\sum_j \kappa_{ij} p_j \left(\frac{p_j}{P_i}\right)^{-\eta_i} d \log p_{j,t} - P_i \sum_j \kappa_{ij} \left(\frac{p_j}{P_i}\right)^{1-\eta_i} d \log p_{j,t} \right] \\
=& - \sum_i \mu_i \eta_i C_i \left[\sum_j \kappa_{ij} p_j^{1-\eta_i} P_i^{\eta_i} d \log p_{j,t} - \sum_j \kappa_{ij} p_j^{1-\eta_i} P_i^{\eta_i} d \log p_{j,t} \right] \\
=& 0.
\end{aligned}
$$

Under CES, there is no composition effect.

Real GDP. We are thus left with

$$
\begin{aligned}
d \log Y_t &= \sum_i \mu_i P_i C_i d \log C_{i,t} + P_G dG_t \\
&= \sum_i \mu_i P_i C_i \sum_{s \geq t} \left(\frac{\partial \log \mathcal{C}_{i,t}}{\partial \log e_{i,s}} d \log e_{i,s} + \frac{\partial \log \mathcal{C}_{i,t}}{\partial \log R_{i,s}} d \log R_{i,s} \right) + P_G dG_t.
\end{aligned}
$$

Next, we have $e_{i,s} = d(\frac{1}{P_{i,s}}\tau_s(zY_s^n\xi_{i,s})^{1-\lambda})$, and so

$$d\log e_{i,s} = -d\log P_{i,s} + d\log\tau_s + (1-\lambda)d\log Y_s^n + (1-\lambda)d\log\xi_{i,s}.$$

Notice that around the steady state with $G = T = 0$ and numeraire $Y^n = 1$, we have

$$\begin{aligned}
\tau_t Y_t^n &= Y_t^n - T_t \\
d\log\tau_t + d\log Y_t^n &= d\log(Y_t^n - T_t) \\
d\log\tau_t + d\log Y_t^n &= \frac{dY_t^n - dT_t}{Y_t^n - T_t} \\
d\log\tau_t + d\log Y_t^n &= \frac{dY_t^n}{Y_t^n} - \frac{dT_t}{Y_t^n} \\
d\log\tau_t &= -\frac{dT_t}{Y_t^n} \\
d\log\tau_t &= -dT_t.
\end{aligned}$$

Finally, using $\lambda = 0$, we have

$$d\log e_{i,s} = -dT_t + d\log Y_s - d\log\rho_{i,s} + d\log\xi_{i,s}.$$

Thus, we have

$$\begin{aligned}
d\log Y_t &= \sum_i \mu_i P_i dC_{i,t} + P_G dG_t \\
&= \sum_i \mu_i P_i C_i d\log C_{i,t} + P_G dG_t \\
&= \sum_i \mu_i P_i C_i \sum_{s\geq t}\left(M_{i,ts}\Big[-dT_t + d\log Y_s - d\log\rho_{i,s} + d\log\xi_{i,s}\Big] + M_{i,ts}^r dr_{i,s}\right) + P_G dG_t.
\end{aligned}$$

where $M_{i,ts} = \frac{\partial \log C_{i,t}}{\partial \log e_{i,s}}$. Now notice that by definition

$$
\begin{aligned}
d\log P_t &= \sum_j p_{j,t} C_{j,t} d\log p_{j,t} \\
&= \sum_j p_j \sum_i \mu_i \kappa_{ij} \left(\frac{p_j}{P_i}\right)^{-\eta_i} C_i d\log p_{j,t} \\
&= \sum_i \mu_i P_i C_i \sum_j \kappa_{ij} \left(\frac{p_j}{P_i}\right)^{1-\eta_i} d\log p_{j,t} \\
&= \sum_i \mu_i P_i C_i \, d\log P_{i,t}.
\end{aligned}
$$

We next show that under our (steady state) nominal GDP numeraire, we have

$$
\sum_i \mu_i P_i C_i = 1.
$$

This follows because we have $1 = Y_n = \sum_j p_j C_j$, and

$$
\sum_i \mu_i P_i C_i = \sum_i \mu_i \sum_j p_j c_{ij} = \sum_j p_j \sum_i \mu_i c_{ij} = \sum_j p_j C_j.
$$

Our result follows after applying a covariance decomposition using the operator $\mathbb{E}_i = \sum_i \mu_i$. Note that

$$
\begin{aligned}
\sum_i \mu_i P_i C_i \boldsymbol{M}_i \, d\log \boldsymbol{\rho}_i &= \sum_i \mu_i \boldsymbol{M}_i \sum_i \mu_i P_i C_i d\log \boldsymbol{\rho}_i - \mathbb{C}ov_i\left(\boldsymbol{M}_i, P_i C_i \, d\log \boldsymbol{\rho}_i\right) \\
&= \bar{\boldsymbol{M}}_i \sum_i \mu_i P_i C_i (d\log P_{i,t} - d\log P_t) - \mathbb{C}ov_i\left(\boldsymbol{M}_i, P_i C_i \, d\log \boldsymbol{\rho}_i\right) \\
&= \bar{\boldsymbol{M}}_i \left(\underbrace{\sum_i \mu_i P_i C_i d\log P_{i,t}}_{=d\log P_t} - d\log P_t \underbrace{\sum_i \mu_i P_i C_i}_{=1}\right) - \mathbb{C}ov_i\left(\boldsymbol{M}_i, P_i C_i \, d\log \boldsymbol{\rho}_i\right)
\end{aligned}
$$

so the average term cancels.

B.3 Proof of Proposition 6

Firms. Firm cost minimization implies that

$$
\begin{aligned}
d\log y_{j,t} &= d\log A_{j,t} + \sum_{k=1}^{N} \tilde{\Omega}_{jk}\, d\log x_{jk,t} + \sum_{i=1}^{I} \tilde{\Omega}_{jN+i}\, d\log N_{ji,t} \\
\Gamma_j\, d\log \tilde{x}_j &= d\log \tilde{p} - d\log MC_j - d\log A_j \\
d\log MC_{j,t} &= \sum_{k=1}^{N} \tilde{\Omega}_{jk}\, d\log p_{k,t} + \sum_{i=1}^{I} \tilde{\Omega}_{jN+i}\, d\log W_{i,t} - d\log A_{j,t}
\end{aligned}
$$

We still have the Domar weight definition

$$\mu_i \Lambda_{i,t} = \frac{\mu_i W_{i,t} N_{i,t}}{P_t Y_t},$$

now accounting for household mass. From here, we have

$$d\log \Lambda_{i,t} = d\log W_{i,t} + d\log N_{i,t} - d\log P_t - d\log Y_t.$$

Next, we write

$$
\begin{aligned}
d\log p_{j,t} &= d\log \mu_{j,t} + \sum_{k=1}^{N} \tilde{\Omega}_{jk}\, d\log p_{k,t} + \sum_{i=1}^{I} \tilde{\Omega}_{jN+i}\, d\log W_{i,t} - d\log A_{j,t} \\
d\log P_t &= \left(\boldsymbol{I} - \tilde{\Omega}^p\right)^{-1} \left(\sum_{i=1}^{I} \tilde{\Omega}_{[:,\,N+i]}\, d\log W_{i,t} + d\log \mu_t - d\log A_t \right)
\end{aligned}
$$

Now we have

$$d\log p_{j,t} = \sum_k \sum_i (\boldsymbol{I} - \tilde{\Omega}^p)^{-1}_{ji} \tilde{\Omega}_{jN+i}\, d\log W_{i,t} + \sum_k \tilde{\Psi}_{jk} \Big(d\log \mu_{k,t} - d\log A_{k,t} \Big)$$

or simply

$$d\log p_{j,t} = \sum_i \tilde{\Psi}_{jN+i}\, d\log W_{i,t} + \sum_k \tilde{\Psi}_{jk} \Big(d\log \mu_{k,t} - d\log A_{k,t} \Big)$$

Network objects and Domar weights. The goods market clearing condition for good j is

$$p_{j,t} y_{j,t} = \sum_i \mu_i p_{j,t} C_{ij,t} + \sum_k p_{j,t} x_{kj,t}.$$

Now aggregating households, we have

$$\begin{aligned} p_{j,t}y_{j,t} &= p_{j,t}C_{j,t} + \sum_k p_{j,t}x_{kj,t} \\ &= p_{j,t}\sum_i \mu_i b_{ij,t}C_{i,t} + \sum_k p_{j,t}x_{kj,t}. \end{aligned}$$

where

$$C_{j,t} = \sum_i \mu_i \kappa_{ij}\left(\frac{p_{j,t}}{P_{i,t}}\right)^{-\eta_i} \mathcal{C}_{i,t}\left(\left\{e_{i,s}, R_{i,s}\right\}_{s\geq t}\right).$$

Now define

$$b_{j,t} = \frac{p_{j,t}C_{j,t}}{\sum_j p_{j,t}C_{j,t}} = \frac{p_{j,t}\sum_i \mu_i b_{ij,t}C_{i,t}}{\sum_j p_{j,t}\sum_i \mu_i b_{ij,t}C_{i,t}}$$

as the final (consumption) expenditure share of good j. Now we can rewrite the goods market clearing condition as

$$p_{j,t}y_{j,t} = b_{j,t}\left(\sum_j p_{j,t}\sum_i \mu_i b_{ij,t}C_{i,t}\right) + \sum_k \Omega_{kj,t}p_{k,t}y_{k,t}$$

Now notice that $\sum_j p_{j,t}C_{j,t} = Y_t^n$ is also nominal GDP. So defining the revenue-based Domar weight

$$\lambda_{j,t} = \frac{p_{j,t}y_{j,t}}{Y_t^n},$$

we have

$$\begin{aligned} \lambda_{j,t} &= b_{j,t} + \sum_k \Omega_{kj,t}\lambda_{k,t} \\ \lambda_{j,t} &= b_{j,t} + \Omega'_{[:,j],t}\lambda_t \\ \lambda_t &= b_t + \Omega'_t\lambda_t \\ \lambda'_t &= b'_t + \lambda'_t\Omega_t, \end{aligned}$$

and thus

$$\lambda'_t = b'_t\Psi_t.$$

Similarly, we define

$$\tilde{\lambda}'_t = b'_t\tilde{\Psi}_t.$$

Putting it all together, we can write

$$d\log P_t = b'_t\, d\log p_{j,t} = \sum_i \tilde{\Lambda}_{i,t}\, d\log W_{i,t} + \sum_k \lambda_{k,t}\left(d\log \mu_{k,t} - d\log A_{k,t}\right)$$

Wages. The key change is that there is now a labor supply equation for each labor type. We have

$$\frac{W_{i,t}}{P_{i,t}} = \frac{\epsilon_i^w}{\epsilon_i^w - 1} \frac{v'(N_{i,t})}{u'(C_{i,t})}$$

and therefore

$$d \log W_{i,t} - d \log P_{i,t} = \eta_i \, d \log N_{i,t} + \gamma_i \, d \log C_{i,t}.$$

Now, crucially, we can no longer use the goods market clearing condition to just solve out for $d \log C_{i,t}$. Instead, we will use our sufficient statistics on the household side. We have

$$C_{i,t} = \mathcal{C}_{i,t}\Big(\Big\{e_{i,s}, r_{i,s}\Big\}_{s \geq t}\Big).$$

Thus, we have

$$\begin{aligned} d \log C_{i,t} &= \frac{1}{C_{i,t}} \sum_{s \geq t} \Bigg(M_{i,ts} de_{i,s} + M^r_{i,ts} dr_{i,s} \Bigg) \\ &= \frac{1}{C_{i,t}} \sum_{s \geq t} M_{i,ts} \Bigg(- dT^n_s + d \log \xi_{i,s} + d \log Y_s - d \log \rho_{i,s} \Bigg) + \frac{1}{C_{i,t}} \sum_{s \geq t} M^r_{i,ts} dr_{i,s} \end{aligned}$$

Domar weights still solve

$$d \log \Lambda_{i,t} = d \log W_{i,t} + d \log N_{i,t} - d \log P_t - d \log Y_t.$$

We now introduce our two sufficient statistics

$$\xi_{i,t} = 1 + \Lambda_{i,t} - \mathbb{E}_i \Lambda_{i,t}$$

$$d \log \rho_{i,t} = d \log P_{i,t} - d \log P_t$$

This yields

$$d \log N_{i,t} = d \log P_t + d \log Y_t + d \log \Lambda_{i,t} - d \log W_{i,t}$$

and plugging in we get

$$\begin{aligned} d \log W_{i,t} - d \log P_{i,t} &= \eta_i \, d \log P_t + \eta_i \, d \log Y_t + \eta_i \, d \log \Lambda_{i,t} - \eta_i \, d \log W_{i,t} + \gamma_i \, d \log C_{i,t} \\ (1 + \eta_i) d \log W_{i,t} &= (1 + \eta_i) \, d \log P_{i,t} - \eta_i \, d \log \rho_{i,t} + \eta_i \, d \log Y_t + \eta_i \, d \log \Lambda_{i,t} + \gamma_i \, d \log C_{i,t} \\ d \log W_{i,t} &= d \log P_{i,t} - \frac{\eta_i}{1 + \eta_i} d \log \rho_{i,t} + \frac{\eta_i}{1 + \eta_i} d \log Y_t + \frac{\eta_i}{1 + \eta_i} d \log \Lambda_{i,t} + \frac{\gamma_i}{1 + \eta_i} d \log C_{i,t}. \end{aligned}$$

Now I actually want to go in the other direction! I want to solve out for $d \log P_{i,t}$! So this yields

$$d \log W_{i,t} = \frac{1}{1+\eta_i} d \log \rho_{i,t} + d \log P_t + \frac{\eta_i}{1+\eta_i} d \log Y_t + \frac{\eta_i}{1+\eta_i} d \log \Lambda_{i,t} + \frac{\gamma_i}{1+\eta_i} d \log C_{i,t}$$

Now plugging into the pricing equation and noting that $\sum_i \tilde{\Lambda}_{i,t} = 1$, we have

$$d \log P_t = \sum_i \tilde{\Lambda}_{i,t} \left(\frac{1}{1+\eta_i} d \log \rho_{i,t} + d \log P_t + \frac{\eta_i}{1+\eta_i} d \log Y_t + \frac{\eta_i}{1+\eta_i} d \log \Lambda_{i,t} + \frac{\gamma_i}{1+\eta_i} d \log C_{i,t} \right)$$
$$+ \sum_k \lambda_{k,t} \Big(d \log \mu_{k,t} - d \log A_{k,t} \Big)$$

and simplifying, we arrive at

$$\sum_i \tilde{\Lambda}_{i,t} \frac{\eta_i}{1+\eta_i} d \log Y_t = - \sum_i \tilde{\Lambda}_{i,t} \frac{1}{1+\eta_i} d \log \rho_{i,t} - \sum_i \tilde{\Lambda}_{i,t} \frac{\eta_i}{1+\eta_i} d \log \Lambda_{i,t} - \sum_i \tilde{\Lambda}_{i,t} \frac{\gamma_i}{1+\eta_i} d \log C_{i,t}$$
$$+ \sum_k \lambda_{k,t} \Big(d \log A_{k,t} - d \log \mu_{k,t} \Big)$$

This concludes the proof.

C Quantitative HANK-IO: Model Details

This Appendix provides a self-contained description of the quantitative HANK-IO model that we implement and take to the data in Section 4.

C.1 Production Network

Production in this economy takes place in N distinct production sectors. Within each sector, we adopt the standard New Keynesian structure in which a retailer bundles intermediate varieties produced by a continuum of intermediate goods producers. Dimensions of heterogeneity include share of intermediate input bundle in production factors $\mu_{x,j}$, and share of capital in primary factor α_j. We detonate all variable in sector j with a underscript j.

C.1.1 Retailer

The retailer produces the final consumption good by bundling intermediate varieties according to the CES aggregation technology

$$y_{j,t} = \left(\int_0^1 y_{j,t}(k)^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $y_{j,t}$ denotes sectoral production output and $y_{j,t}(k)$ is the output produced by intermediate firm k in sector j. ϵ denotes the elasticity of substitution across intermediate inputs. Each retailer demands intermediate input j according to the standard demand function

$$y_{j,t}(k) = \left(\frac{p_{j,t}(k)}{p_{j,t}} \right)^{-\epsilon} y_{j,t},$$

where $P_{j,t}(k)$ is the price of intermediate good produced by firm k in sector j, and $P_{j,t}$ is the producer price index (PPI) in sector j,

$$p_{j,t} = \left(\int_0^1 p_{j,t}(k)^{1-\epsilon} dk \right)^{\frac{1}{1-\epsilon}}.$$

C.1.2 Intermediate Goods Producers

Production function. Firms in each industry employ CES technology to transform intermediate inputs, capital and labor into final products.

$$y_{j,t} = A_{j,t} \cdot \left[(1-\mu_{x,j})^{\frac{1}{\eta_{f,j}}} \left(f_{j,t}\right)^{\frac{\eta_{f,j}-1}{\eta_{f,j}}} + (\mu_{x,j})^{\frac{1}{\eta_{f,j}}} \left(x_{j,t}\right)^{\frac{\eta_{f,j}-1}{\eta_{f,j}}} \right]^{\frac{\eta_{f,j}}{\eta_{f,j}-1}}$$

where $A_{j,t}$ is the factor-neutral total factor productivity of sector j at time t. $\mu_{x,j}$ is the share of intermediate inputs factor in sector j's production function. $\eta_{f,j}$ is the elasticity between primary factor and intermediate input factor. $f_{j,t}$ is the primary factor, and $x_{j,t}$ is the aggregate intermediate input bundle.

Primary factor. $f_{j,t}$ is aggregated by a Cobb-Douglas technology,

$$f_{j,t} = \left(K_{j,t}\right)^{\alpha_j} \left(N_{j,t}\right)^{1-\alpha_j}$$

α_j is the share of capital in the primary factor production in sector j. $K_{j,t}$ is capital in use for sector j, $N_{j,t}$ is the effective labor of sector j.

Intermediate inputs bundle. $x_{j,t}$ is aggregated by a CES technology,

$$x_{j,t} = \left(\sum_{k=1}^{S} \left(\Gamma^x_{jk}\right)^{\frac{1}{\eta_{x,j}}} \left(x_{jk,t}\right)^{\frac{\eta_{x,j}-1}{\eta_{x,j}}} \right)^{\frac{\eta_{x,j}}{\eta_{x,j}-1}}$$

where $\eta_{x,j}$ parameterizes the elasticity of goods used in the intermediate input bundle from different sectors i. Γ^x_{jk} indicates the share of importance of industry k in the production of sector j's intermediate input bundle. $x_{jk,t}$ is the unit of final goods from sector k used in j's intermediate input bundle.

The standard demand functions for intermediate inputs from sector i is given by

$$x_{jk,t} = \Gamma^x_{jk} \left(\frac{p_{k,t}}{p_{jx,t}} \right)^{-\eta_{x,j}} x_{j,t}$$

where $p_{k,t}$ is the producer price index (PPI) in sector k and $p_{jx,t}$ is the price of intermediate input bundle in sector j. The relationship between intermediate input prices and the bundle price is given by

$$p_{jx,t} = \left[\sum_k \Gamma^x_{jk} (p_{k,t})^{1-\eta_{x,j}} \right]^{\frac{1}{1-\eta_{x,j}}}$$

Nominal profit. There is an integrated and competitive market in which firms rent capital. The nominal rental rate of capital is i^K_t. Market clearing in the labor markets reallocation gives rise to a nominal wage rate $W_{j,t}$. The nominal price of the intermediate inputs bundle is $p_{jx,t}$. Firms in sector j have a nominal profit

$$\Pi_{j,t} = \underbrace{p_{j,t} y_{j,t}}_{\text{Revenue from sales}} - \underbrace{(1-\tau^{empl}) W_{j,t} N_{j,t}}_{\text{Cost of labor}} - \underbrace{i_t^K K_{j,t}}_{\text{Cost of capital}} - \underbrace{p_{jx,t} x_{j,t}}_{\text{Cost of intermediate bundle}}$$

where τ^{empl} is the employment subsidy from the government to address the distortion resulted from monopolistic competition.

Optimization. The sector-specific salary expenditure is aggregated through the CES technology. The optimal composition between wage for different household types and effective labor is given by

$$N_{ij,t} = \left(\frac{W_{i,t}}{W_{j,t}}\right)^{-\eta_{l,j}} \Gamma_{ij}^w N_{j,t}$$

$$W_{j,t} = \left[\sum_i \Gamma_{ij}^w (W_{i,t})^{1-\eta_{l,j}}\right]^{\frac{1}{1-\eta_{l,j}}}$$

where $N_{ij,t}$ is the effective labor of household type i in sector j, $\eta_{l,j}$ is the elasticity of substitution between labor supply by different types of workers for sector j, $W_{i,t}$ is the type-specific wage, $W_{j,t}$ is the sector-specific wage, Γ_{ij}^w is the share of salaries earned by type i worker in sector j, and $N_{j,t}$ is the effective labor in sector j.

In our baseline model, we calibrate $\eta_l^s = 1$ across sectors. The optimal labor factor composition for each sector can be re-written as

$$N_{ij,t} W_{i,t} = \Gamma_{ij}^w W_{j,t} N_{j,t}$$

$$W_{j,t} = \Pi_i (W_{i,t})^{\Gamma_{ij}^w}.$$

Furthermore, the optimization yields the relationship between marginal product of intermediate bundle and its nominal price as

$$p_{jx,t} = MC_{j,t} \frac{\partial y_{j,t}}{\partial x_{j,t}}$$

$$= MC_{j,t} (A_{j,t})^{\frac{\eta_{f,j}-1}{\eta_{f,j}}} (\mu_{x,j} y_{j,t})^{\frac{1}{\eta_{f,j}}} (x_{j,t})^{-\frac{1}{\eta_{f,j}}};$$

$$(p_{jx,t})^{\eta_{f,j}} = (MC_{j,t})^{\eta_{f,j}} (A_{j,t})^{\eta_{f,j}-1} \mu_{x,j} y_{j,t} (x_{j,t})^{-1}.$$

Similarly, for the primary factor, we have

$$(i_t^K)^{\eta_{f,j}} = (MC_{j,t})^{\eta_{f,j}} (A_{j,t})^{\eta_{f,j}-1} (1-\mu_{x,j}) y_{j,t} (f_{j,t})^{-1} \left(\alpha_j \frac{f_{j,t}}{K_{j,t}} \right)^{\eta_{f,j}}$$

$$\left((1-\tau^{empl}) W_{j,t} \right)^{\eta_{f,j}} = (MC_{j,t})^{\eta_{f,j}} (A_{j,t})^{\eta_{f,j}-1} (1-\mu_{x,j}) y_{j,t} (f_{j,t})^{-1} \left((1-\alpha_j) \frac{f_{j,t}}{N_{j,t}} \right)^{\eta_{f,j}}$$

In our baseline model, we calibrate $\eta_{f,j} = 1$ across sectors. Denote $\mu_{k,j} = (1-\mu_{x,j})\alpha_j$ to be the share of capital in total production, and $\mu_{l,j} = (1-\mu_{x,j})(1-\alpha_j)$. The production function is therefore given by

$$y_{j,t} = A_{j,t} \left(x_{j,t} \right)^{\mu_{x,j}} \left(K_{j,t} \right)^{\mu_{k,j}} \left(N_{j,t} \right)^{\mu_{l,j}}$$

The nominal marginal cost of production is given by

$$MC_{j,t} = \frac{1}{A_{j,t}} \frac{1}{(\mu_{x,j})^{\mu_{x,j}} (\mu_{k,j})^{\mu_{k,j}} (\mu_{l,j})^{\mu_{l,j}}} \left(p_{jx,t} \right)^{\mu_{x,j}} \left(i_t^K \right)^{\mu_{k,j}} \left((1-\tau^{empl}) W_{j,t} \right)^{\mu_{l,j}}$$

The optimization yields the relationship between marginal product of factors and their nominal prices

$$p_{jx,t} = MC_{j,t} \mu_{x,j} \frac{y_{j,t}}{x_{j,t}}$$

$$i_t^K = MC_{j,t} \mu_{k,j} \frac{y_{j,t}}{K_{j,t}}$$

$$(1-\tau^{empl}) W_{j,t} = MC_{j,t} \mu_{l,j} \frac{y_{j,t}}{N_{j,t}}$$

We define the real marginal cost in sector j as

$$mc_{j,t} = \frac{MC_{j,t}}{p_{j,t}}.$$

Given the definition of marginal cost, the firm's nominal profit can, as usual, be expressed as $\Pi_{j,t} = (1 - mc_{j,t}) p_{j,t} y_{j,t}$.

Dynamic price-setting. We have now discussed firms' optimal composition of factors of production. In this essentially static choice, the firm takes as given its price level $p_{j,t}(k)$, which is sticky in the short run, as well as the demand it faces at this price level, $y_{j,t}(k)$. We now turn to the dynamic choice of the optimal price level subject to an adjustment cost in the spirit of Rotemberg (1982).

Define $\pi_{j,t}(k) = \dot{p}_{j,t}(k) / p_{j,t}(k)$ to be the instantaneous rate of inflation in the price of firm k. Firm k determines this rate of inflation subject to an adjustment cost (in utility units), in order to maximize an appropriately discounted sum of all future profits. The firm's problem, then, in real terms, is given by

$$\max_{\pi_{j,t}(k)} \int_0^\infty e^{-\int_0^t \rho ds} \frac{1}{p_{j,t}} \Big[(1 - mc_{j,t}) p_{j,t}(k) y_{j,t}(k) - \Lambda(\pi_{j,t}(k)) \Big] dt,$$

The cost of adjusting prices at rate $\pi_{j,t}(k)$ is given by $\Lambda(\pi_{j,t}(k))$. We assume the specific functional form

$$\Lambda(\pi_{j,t}(k)) = \frac{\chi_j}{2} \Big(\pi_{j,t}(k) \Big)^2 p_{j,t} y_{j,t}.$$

Lemma 7. *The New Keynesian Phillips Curves for each production sector of the economy can be written as*

$$\dot{\pi}_{j,t} = \rho \pi_{j,t} - (mc_{j,t} - \frac{\epsilon - 1}{\epsilon}) \frac{\epsilon}{\chi_j}$$

Proof of the Lemma is provided in Appendix C.6.

C.2 Government

C.2.1 Fiscal Policy

Employment subsidy. The government implements an employment subsidy $\tau^{empl} = \frac{1}{\epsilon}$. *On the household side*, the government pays a wage subsidy to households, and such outlays are funded by a lump-sum tax based on aggregate employment. That is, the household-side net fiscal rebate that a household with idiosyncratic labor productivity z receives is always zero, with

$$\int_0^1 \tau^{empl} (1 - \tau^{lab}) z_{i,t} W_{ik,t} n_{ik,t} dk - \tau^{empl} (1 - \tau^{lab}) z_{i,t} W_{i,t} n_{i,t} = 0$$

On the firm side, there is an employment subsidy in place to avoid distortion resulted from monopolistic competition. The government gives each firm k an employment subsidy $\tau^{empl} W_{j,t} N_{j,t}(k)$. And such subsidy to firms is funded a lump-sum tax based on aggregate subsidy. The aggregate subsidy is given by $\sum_j \tau^{empl} W_{j,t} N_{j,t}$.

Tax collected. The government's fiscal income comes from payroll tax collected, $\sum_j \tau^{lab} W_{j,t} N_{j,t}$.

Government spending. The government purchases final goods from all sectors $G_{j,t}$ according to the share of consumption Γ_j^g, therefore the aggregate government spending G_t can be written as

$$p_{j,t} G_{j,t} = \Gamma_j^g P_t G_t$$

Interest expenses. Another fiscal outflow for the government is nominal interest paid on the nominal government debt outstanding $i_t P_t A_t^g$.

Transfer. The government finances such net flow through a transfer from the household. The nominal transfer to the household's budget constraint is

$$T_t = \sum_j (\tau^{lab} - \tau^{empl}) W_{j,t} N_{j,t} - P_t G_t - i_t P_t A_t^g + d(P_t A_t^g),$$

where $d(P_t A_t^g)$ is the change of nominal government debt outstanding. The total nominal transfer to the households is

$$T_t = \sum_j (\tau^{lab} - \tau^{empl}) W_{j,t} N_{j,t} - P_t G_t - i_t P_t A_t^g + P_t \dot{A}_t^g + \dot{p}_t A_t^g,$$

which in real terms is given by

$$\tau_{i,t} = \frac{\sum_j \Pi_{j,t} + T_t}{P_t}.$$

Rebate re-scaling. The government collect all the aggregate rebate then distribute them to different household types following a re-scaling rule. The proportion of aggregate rebate distributed to type i household is equal to the ratio of such type's total income over all types' total income at steady state. Total income is defined as the sum of wage earnings and interest income from savings.

C.2.2 Monetary Policy

The central bank in our model sets the nominal interest rate, i_t, according to a Taylor rule.

$$i_t = r_{ss} + \lambda_\pi \pi_t + \lambda_Y \triangle y_t + \varepsilon_t,$$

where r_{ss} is the real interest rate in the zero-inflation steady state, $\triangle y_t = \log(y_t / y^*)$ denotes the output gap, and ε_t is the monetary shock.

C.3 Households

The economy is populated by a set of types $i \in \mathcal{I}$ of households. We denote their measure by μ_i and assume $\sum_i \mu_i = 1$. Household types differ in terms of their long-term income quantiles. In addition to ex-ante heterogeneity across types, our baseline model allows for ex-post heterogeneity in productivity (z) liquid assets (a) and illiquid assets (b) within types. We can therefore uniquely identify a household of type i with the three state variables (a, b, z), and we denote the cross-sectional income and wealth distribution for i by $g_{i,t}(a, b, z)$. All households purchase consumption goods from and supply labor across N production sectors.

Preference. The preferences of a household of type i are ordered according to

$$\mathbb{E}_0 \int_0^\infty e^{-\int_0^t (\rho_i+\xi)ds} u(c_{i,t}) - \Phi\left(\left\{n_{ik,t}, \pi^w_{ik,t}\right\}_{k\in[0,1]}\right) dt,$$

where

$$c_{i,t} = \mathcal{D}_i^{NH}\left\{c_{i1,t},\, c_{i2,t},\, \ldots,\, c_{iN,t}\right\}$$

is a generalized nonhomothetic CES aggregator and $c_{ij,t}$ denotes the household i's consumption of goods from sector j at time t. $c_{i,t}$ depends on household's liquid asset holdings a, illiquid asset holdings b, and labor productivity z.

$n_{ik,t}$ is the labor hour supplied to union k. ρ is the discount rate. $\pi^w_{ik,t}$ is union k's wage inflation. Households die at rate ξ. The expectation operator is over future realizations of idiosyncratic earnings risk. We abstract from aggregate risk in this paper. Finally, we assume CRRA preferences, with

$$u(c_{i,t}) = \frac{c_{i,t}^{1-\gamma}}{1-\gamma}.$$

The cost function of labor hour and wage inflation, $\Phi(\cdot)$, will be discussed in more detail below.

Heterogeneous consumption baskets. The type-specific consumption aggregator is implicitly defined via

$$1 = \sum_j \left(\Omega_{i,j} c_{i,t}^{\varepsilon_{j,t}}\right)^{\frac{1}{\eta_c}} c_{ij,t}^{\frac{\eta_c-1}{\eta_c}}.$$

in which $c_{ij,t}$ denotes consumption by type i household of good produced in sector j.

Nonhomothetic CES preferences still admit an ideal price index $P_{i,t}$, but it is now governed by additional economic effects. To derive this price index, consider the intratemporal cost minimization problem of the household type i

$$\min_{\{c_{ij,t}\}_j} \sum_j p_{j,t} c_{ij,t} - \phi_i \sum_j \left(\Omega_{i,j} j c_{i,t}^{\varepsilon_{j,t}}\right)^{\frac{1}{\eta_c}} c_{ij,t}^{\frac{\eta_c-1}{\eta_c}},$$

taking as given a desired level of real consumption $c_{i,t}$. The first-order condition yields

$$0 = p_{j,t} - \phi \frac{\eta_c - 1}{\eta_c} \left(\Omega_{i,j} c_{i,t}^{\varepsilon_{j,t}}\right)^{\frac{1}{\eta_c}} c_{j,t}^{\frac{-1}{\eta_c}}$$

or simply

$$c_{ij,t} = \Omega_{i,j} c_{i,t}^{\varepsilon_{j,t}} \left(\frac{\eta_c}{\eta_c - 1} \frac{p_{j,t}}{\phi_i}\right)^{-\eta_c}.$$

Plugging into the definition of $\mathcal{D}^{NH}$, we obtain

$$\phi_i^{1-\eta_c} = \sum_j \Omega_{i,j} \left(\frac{\eta_c}{\eta_c - 1} \right)^{1-\eta_c} p_{j,t}^{1-\eta_c} c_{i,t}^{\varepsilon_{j,t}}.$$

Different line of attack. The first-order condition can be written as

$$\phi_i \frac{\eta_c - 1}{\eta_c} \left(\Omega_{i,j} c_{i,t}^{\varepsilon_{j,t}} \right)^{\frac{1}{\eta_c}} c_{ij,t}^{\frac{\eta_c - 1}{\eta_c}} = p_{j,t} c_{ij,t}$$

Now summing across j, and defining the *expenditure share* $\omega_{ij,t} = (\Omega_{i,j} c_{i,t}^{\varepsilon_{j,t}})^{\frac{1}{\eta_c}} c_{ij,t}^{\frac{\eta_c - 1}{\eta_c}}$, we have

$$\phi_i \frac{\eta_c - 1}{\eta_c} = \sum_j p_{j,t} c_{ij,t} \equiv E_{i,t},$$

noting that by definition $\sum_j \omega_{ij,t} = 1$. We will also write $P_{i,t} c_{i,t} = E_{i,t}$. Thus, we have

$$c_{ij,t} = \Omega_{i,j} c_{i,t}^{\varepsilon_{j,t}} \left(\frac{p_{j,t}}{E_{i,t}} \right)^{-\eta_c} = \Omega_{i,j} \left(\frac{p_{j,t}}{P_{i,t}} \right)^{-\eta_c} c_{i,t}^{\eta_c + \varepsilon_{j,t}}.$$

This is the key equation for the intratemporal problem with nonhomothetic CES. Given parameters, sectoral prices $p_{j,t}$, and desired real consumption $c_{i,t}$, this equation defines the spending on each good that minimizes total expenditures while attaining the real consumption level.

Price index. The price level $P_{i,t}$ itself changes as real consumption $c_{i,t}$ changes due to a switching effect.

Plugging in for the optimal demand $c_{ij,t}$, we have

$$\begin{aligned} E_{i,t} = P_{i,t} c_{i,t} &= \sum_j \left\{ p_{j,t} \Omega_{i,j} \left(\frac{p_{j,t}}{P_{i,t}} \right)^{-\eta_c} c_{i,t}^{\eta_c + \varepsilon_{j,t}} \right\} \\ &= \sum_j \left\{ p_{j,t}^{1-\eta_c} P_{i,t}^{\eta_c} \Omega_{i,j} c_{i,t}^{\eta_c + \varepsilon_{j,t}} \right\} \end{aligned}$$

or simply

$$P_{i,t} = \left(\sum_j \Omega_{i,j} p_{j,t}^{1-\eta_c} c_{i,t}^{\varepsilon_{j,t} - (1-\eta_c)} \right)^{\frac{1}{1-\eta_c}}$$

which of course also implies

$$P_{i,t} c_{i,t} = \left(\sum_j \Omega_{i,j} p_{j,t}^{1-\eta_c} c_{i,t}^{\varepsilon_{j,t}} \right)^{\frac{1}{1-\eta_c}}.$$

Therefore, a household's consumption bundle price really takes the form

$$P_{i,t} = \mathcal{P}\left(\{p_{j,t}\}_j, c_{i,t}\right).$$

In particular, once we solve for the consumption policy function $c_{i,t}(a,b,z)$, the consumption basket price will take the form $P_{i,t}(a,b,z)$. In other words, every single household faces a different consumption price index!

Death and birth process. Following [Blanchard (1985)], we introduce perfect annuity markets, in which households can trade claims on their remaining wealth at time of death. They pledge this wealth to a risk-neutral insurance company that, in turn, compensates households with a flow annuity payment at a rate ξ times their current asset positions. This is exactly the payment rate that makes the insurance company break even in expectation. Introducing household death rates is a commonly used technique to ensure stationarity in the wealth distribution.

Labor market. Following Auclert et al. (2023), we assume that household labor supply decisions are intermediated by labor unions. Each household type i provides $n_{ik,t}$ hours of work to each of a continuum of unions indexed by $k \in [0,1]$. Total labor hours supplied is

$$n_{i,t} = \int_k n_{ik,t} dk.$$

Each union aggregates effective labor units provided by each household of type i, into a union-type-specific task $N_{ik,t}$, given by

$$N_{ik,t} = \bar{z} n_{ik,t},$$

where $\bar{z}$ is the average productivity of all type i households supplying labor to union k. A labor packer then further aggregates these labor services into aggregate labor supply

$$N_{i,t} = \left(\int_k N_{ik,t}^{\frac{\epsilon^w - 1}{\epsilon^w}} dk \right)^{\frac{\epsilon^w}{\epsilon^w - 1}}$$

and sell it to firms at the nominal wage $W_{i,t}$. Each household type i supply $n_{i,t}$ hours of work, which is the sum of labor hours supplied to N sectors

$$n_{i,t} = \sum_j n_{ij,t}$$

Wage subsidy. As is standard in the New Keynesian literature, we allow for an wage subsidy to avoid inefficiency resulted from monopolistic labor competition. Given union wage receipts $(1-\tau^{lab}) z_{i,t} W_{ik,t} n_{ik,t}$ to a household with labor productivity $z_{i,t}$, the government pays the household a proportional wage subsidy $\tau^{empl}(1-\tau^{lab}) z_{i,t} W_{ik,t} n_{ik,t}$ which the union internalizes when setting wages.

Wage rigidity. Labor union k sets a common wage $W_{ik,t}$ for each of its members, and regulates its members to supply the same hours of work. We assume that each union k faces a quadratic utility cost when adjusting its nominal wage $W_{ik,t}$. This cost is given by $\frac{\chi^w}{2}\left(\pi^w_{ik,t}\right)^2$, in which $\pi^w_{ik,t} = \frac{\dot{W}_{ik,t}}{W_{ik,t}}$ is the rate of nominal wage inflation by unions, χ^w modulates the strength of wage rigidity. $\Phi(\cdot)$ is given by

$$\Phi\left(\left\{n_{ik,t}, \pi^w_{ik,t}\right\}_{k\in[0,1]}\right) = v\left(\int_0^1 n_{ik,t}dk\right) + \frac{\chi^w}{2}\int_0^1 \left(\pi^w_{ik,t}\right)^2 dk,$$

where $v(\cdot)$ captures dis-utility from working, given by

$$v(n_{i,t}) = \frac{(n_{i,t})^{1+\phi}}{1+\phi}$$

Wage Phillips curve. Union k chooses wage in order to maximize stakeholder value, namely the sum of stakeholders' utilities. That is, union k solves

$$\max_{\pi^w_{ik,t}} \mathbb{E}_0 \int_0^\infty e^{-\int_0^t \rho_s ds}\left[\int [u\left(c_{i,t}(a,b,z;W_{ik,t})\right) - v\left(\int_0^1 n_{ik,t}dk\right) - \frac{\chi^w}{2}\int_0^1 \left(\pi^w_{ik,t}\right)^2 dk] g_{i,t} d(a,b,z)\right] dt,$$

We further assume that union k is small, and it takes all the macroeconomic aggregates, including the cross-sectional household distribution. We solve the dynamic wage setting problem, where we derive the wage Phillips curve in continuous time. We show that in equilibrium the solution is symmetric, that is $W_{ik,t} = W_{i,t}$, $n_{ik,t} = n_{i,t}$, and $N_{ik,t} = N_{i,t}$. The New-Keynesian wage Phillips curve is given by

$$\dot{\pi}^w_{i,t} = \rho_t \pi^w_{i,t} + \frac{\epsilon^w}{\chi^w}\left[\frac{\epsilon^w - 1}{\epsilon^w}(1+\tau^{empl})(1-\tau^{lab}) w_{i,t}\Lambda_{i,t} - v'\left(n_{i,t}\right)\right] n_{i,t}$$

where we define $\Lambda_{i,t}$ as

$$\Lambda_{i,t} = \int z_{i,t} u'\left(c_{i,t}(a,b,z)\right) g_{i,t} d(a,b,z)$$

As χ^w approaches zeros when we assume flexible wage setting, for the wage Phillips curve to have stationary solution, we must have

$$v'\left(n_{i,t}\right) = \frac{\epsilon^w - 1}{\epsilon^w}(1+\tau^{empl})(1-\tau^{lab}) w_{i,t}\Lambda_{i,t}.$$

Two-account portfolio. Each household has two asset accounts, liquid $a_{i,t}$ and illiquid $b_{i,t}$. They can move funds between these two accounts, subject to a transaction cost. The liquid account has a relatively low real return of and they are subject to a borrowing constraint, $a_{i,t} \geq \underline{a}$. The illiquid account carries a higher return and is subject to a short-sale constraint $b_{i,t} \geq 0$. The household makes a real transfer decision in each period $\iota_{i,t}$, the deposit from liquid asset to illiquid account (or withdrawal from the illiquid account to the liquid account if $\iota_{i,t}$ is negative). The transfer is subject to a transaction cost $\psi(\iota_{i,t}, b_{i,t})$, which will be paid out from the liquid account. Households' two asset accounts are all held with the representative financial intermediary. We will discuss the transaction cost function below.

Portfolio adjustment costs. When households deposit funds into the illiquid account they incur adjustment costs given by $\psi(\iota_{i,t}, b_{i,t})$. We follow [Kaplan et al. (2018)] and use a functional form for adjustment costs given by

$$\psi(\iota_{i,t}, b_{i,t}) = \psi_0|\iota_{i,t}| + \psi_1 \left(\frac{|\iota_{i,t}|}{\max\{b_{i,t}, \psi_3\}} \right)^{\psi_2} \max\{b_{i,t}, \psi_3\}.$$

in which $\psi_2 > 1$, $\psi_3 \geq 0$.

Such an adjustment cost function has a kink at $\iota = 0$. With $\psi_0 > 0$ and $\psi_1 > 0$, we have

$$\psi_\iota(\iota_{i,t}, b_{i,t}) = \begin{cases} \psi_0 + \psi_1\psi_2 \left| \frac{\iota_{i,t}}{\max\{b_{i,t},\psi_3\}} \right|^{\psi_2 - 1}, & \iota_{i,t} > 0 \\ -\psi_0 - \psi_1\psi_2 \left| \frac{\iota_{i,t}}{\max\{b_{i,t},\psi_3\}} \right|^{\psi_2 - 1}, & \iota_{i,t} < 0 \end{cases}.$$

The inverse of $\psi_{\iota_{i,t}}$ with respect to $\iota_{i,t}$ has the following function

$$\iota_t\left(\psi_{\iota_{i,t}}, b_{i,t}\right) = \begin{cases} \beta\left(\psi_{\iota_{i,t}} - \psi_0\right)^{\frac{1}{\psi_2 - 1}} \max\{b_{i,t}, \psi_3\}, & \psi_{\iota_{i,t}} > \psi_0 \\ \beta\left(-\psi_{\iota_{i,t}} - \psi_0\right)^{\frac{1}{\psi_2 - 1}} \max\{b_{i,t}, \psi_3\}, & \psi_{\iota_{i,t}} < \psi_0 \end{cases}.$$

in which $\beta = (\psi_1\psi_2)^{\frac{1}{1-\psi_2}}$, and $\iota_{i,t}$ cannot exceed the limit ι^{max}.

Budget constraints. Households faces two budget constraints, one for the liquid account and the other for the illiquid account.

Illiquid account. Household's illiquid account is denominated in the unit of capital, and it evolves according to

$$\dot{b}_{i,t} = \xi b_{i,t} + \iota_{i,t},$$

in which $\iota_{i,t}$ is the deposit from the liquid account in unit of capital, and ξ is death rate.

Liquid account. In nominal terms, the household's evolution of liquid wealth is given by

$$d(P_t a_{i,t}) = (i_t^a + \xi)P_t a_{i,t} + i_t^b P_t^K b_{i,t} + (1 - \tau^{\text{lab}}) z_{i,t} n_{i,t} W_{i,t} + T_{i,t} - \sum_j p_{j,t} c_{ij,t} - P_t^K \iota_{i,t} - P_t \psi(\iota_{i,t}, b_{i,t}),$$

where i_t^a is the nominal return on liquid account holdings. $T_{i,t} = \theta_i(\sum_j \Pi_{j,t} + T_t)$ is the nominal aggregate transfer to type i. $\Pi_{j,t}$ is the nominal profits from intermediate producers in sector s which are all distributed to the liquid account. i^b_t is the nominal rate of return on illiquid account investment that is distributed to the liquid account. τ^{lab} is a constant income tax rate. P^K_t is the nominal price per unit of capital. Rewriting this, we have

$$\dot{P}_t a_{i,t} + P_t \dot{a}_{i,t} = (i^a_t + \xi) P_t a_{i,t} + i^b_t P^K_t b_{i,t} + (1 - \tau^{\text{lab}}) z_{i,t} n_{i,t} W_{i,t} + T_{i,t} - \sum_j p_{j,t} c_{ij,t} - P^K_t \iota_{i,t} - P_t \psi(\iota_{i,t}, b_{i,t}).$$

Let $\pi_t = \frac{\dot{P}_t}{P_t}$ denotes price inflation. Furthermore, we denote the household's effective real wage as $w_{i,t} = \frac{W_{i,t}}{P_t}$. Thus, the household's nominal wealth evolution equation becomes

$$\frac{\dot{P}_t}{P_t} a_{i,t} + \dot{a}_{i,t} = (i^a_t + \xi) a_{i,t} + \frac{i^b_t P^K_t}{P_t} b_{i,t} + (1 - \tau^{\text{lab}}) z_{i,t} n_{i,t} \frac{W_{i,t}}{P_t} + \frac{T_{i,t}}{P_t} - \frac{E_{i,t}}{P_t} - \frac{P^K_t}{P_t} \iota_{i,t} - \psi(\iota_{i,t}, b_{i,t}).$$

Thus, a household's liquid wealth in real terms evolves according to

$$\dot{a}_{i,t} = (r^a_t + \xi) a_{i,t} + r^b_t b_{i,t} + (1 - \tau^{\text{lab}}) z_{i,t} n_{i,t} w_{i,t} + \tau_{i,t} - \frac{E_{i,t}}{P_t} - q_t \iota_{i,t} - \psi(\iota_{i,t}, b_{i,t}).$$

in which

$$\begin{aligned} r^a_t &= i^a_t - \pi_t \\ r^b_t &= \frac{i^b_t P^K_t}{P_t} \\ w_{i,t} &= \frac{W_{i,t}}{P_t} \\ \tau_{i,t} &= \frac{\theta_i (\sum_j \Pi_{j,t} + T_t)}{P_t} \\ q_t &= \frac{P^K_t}{P_t}. \end{aligned}$$

Optimization. The first-order conditions from HJB optimization are given by

$$\begin{aligned} \rho V_{i,t}(a, b, z) = \max u(c_{i,t}) - \Phi(n_{ik,t}, \pi^w_{ik,t}) \\ &+ \Big((r^a_{i,t} + \xi) a_{i,t} + r^b_{i,t} b_{i,t} + (1 - \tau^{\text{lab}}) z_{i,t} n_{i,t} w_{i,t} + \tau_{i,t} - c_{i,t} - q_{i,t} \iota_{i,t} - \psi(\iota_{i,t}, b_{i,t}) \Big) \partial_a V_{i,t}(a, b, z) \\ &+ \left(\xi b_{i,t} + \iota_{i,t} \right) \partial_b V_{i,t}(a, b, z) \\ &+ \mu_z \partial_z V_{i,t}(a, b, z) + \frac{\sigma^2_z}{2} \partial_{zz} V_{i,t}(a, b, z). \end{aligned}$$

The first-order conditions with respect to $c_{i,t}$ are given by

$$u'(c_{i,t}) = \partial_a V_{i,t}(a,b,z)$$

The first order condition of the HJB equation with respect to $\iota_{i,t}$ is given by

$$\partial_b V_{i,t}(a,b,z) = \partial_a V_{i,t}(a,b,z)(q_{i,t} + \psi_\iota(\iota_{i,t}, b_{i,t}))$$

Plugging in ψ_ι, we obtain the optimal deposit/withdraw being

$$\iota^*_{i,t} = \psi_\iota^{-1}\left(\frac{\partial_b V_{i,t}(a,b,z)}{\partial_a V_{i,t}(a,b,z)} - q_{i,t}, b_{i,t}\right).$$

We use an semi-implicit upwind finite different method following [Ben Moll]. Our upwind method splits the drift of a into two parts s^c and s^i, and splits the drift of b into m^k and m^i, and upwind them separately.

C.4 Financial Intermediary

A representative financial intermediary (the "bank") has two activities: (1) a banking activity, performing maturity transformation by collecting real liquid assets from households $a_{i,t}$ and invests them in government bonds, subject to an intermediation rate ω (2) a mutual fund activity, collecting illiquid funds $b_{i,t}$ and intermediate the funds in the form of physical capital to intermediate producers. The representative financial intermediary faces the following flow-of-funds constraints for the liquid and illiquid account respectively.

Liquid account. The financial intermediary takes the aggregate real liquid asset and invest them into government bond, net of an intermediation cost. On the liability side, it is obligated to deliver returns i^a_t on $P_t A_t$. On the asset side, the financial intermediary would own $P_t A_t$ worth of government bond, which yields a return equal to the nominal interest rate i_t. The financial intermediary would take an intermediation cost $\omega P_t A_t$ from the return on government bonds in total. Under no arbitrage, the payment to households and the gain from owning government bonds net of intermediation cost must be equal, so we have:

$$i^a_t = i_t - \omega, a_{i,t} \geq 0$$

that is, in equilibrium, the financial intermediary fully pass through the cost of intermediation to the liquid account depositors.

Note that there is a borrowing wedge θ, that is the borrowing rate if $a_{i,t} < 0$ is,

$$i^a_t = i_t - \omega + \theta, a_{i,t} < 0$$

Given that $r^a_{i,t} = i^a_t - \pi_{i,t}$, the real return to liquid account investment $r^a_{i,t}$ is therefore given by

$$r^a_{i,t} = \mathbb{1}_{a_{i,t}\geq 0}(i_t - \omega - \pi_{i,t}) + \mathbb{1}_{a_{i,t}<0}(i_t - \omega + \theta - \pi_{i,t}).$$

Illiquid account. Illiquid assets are equity claims on the bank. Therefore, the nominal value of the bank equals households' aggregate illiquid assets $V_t = P^K_t \sum_i \int b_{i,t} d(a,b,z) = P^K_t B_t$. The bank delivers a nominal return of i^b_t to its shareholder. The bank intermediate illiquid assets through an integrated and competitive rental market in the form of physical capital to intermediate producers in sectors as $K_{j,t}$ at the rental rate i^K_t.

Investment and capital stock. The bank owns capital K_t, rents it to firms for final goods production, and makes investment decisions for capital stock replenishment.[22] The bank has a technology that aggregate $I_{j,t}$ unit of final consumption goods across sectors indexed by j, through the economy's investment network, into $GI_t = I_t + \Phi K_t$ unit of gross investment priced at P^{GI}_t, which got turned into I_t unit of capital at price P^K_t. Capital depreciates at rate δ. ΦK_t is a depreciation-offsetting quadratic adjustment costs in unit of final consumption goods

$$\Phi(\frac{I_t}{K_t})K_t = \frac{\kappa}{2}(\frac{I_t}{K_t} - \delta)^2 K_t$$

in which δ is the depreciation rate, and κ is the capital investment adjustment cost coefficient.

The capital stock evolution is given by

$$\dot{K}_t = I_t - \delta K_t$$

The total capital investment is aggregated by a CES technology,

$$GI_t = \left(\sum_{j=1}^{N} \left(\Gamma^I_j\right)^{\frac{1}{\eta_i}} \left(I_{j,t}\right)^{\frac{\eta_i - 1}{\eta_i}} \right)^{\frac{\eta_i}{\eta_i - 1}}$$

where η_i parameterizes the elasticity of goods used in the input bundle from different sectors for investment production. Γ^I_j indicates the share of importance of industry j in the production of the aggregate investment goods bundle. $I_{j,t}$ is the goods produced in sector j that is used in the production of the aggregate investment bundle.

The standard demand functions for investment goods from sector j is given by

$$I_{j,t} = \Gamma^I_j \left(\frac{p_{j,t}}{p^{GI}_t} \right)^{-\eta_i} GI_t$$

[22] The bank does not own shares in intermediate firms, households own those directly

The relationship between sectoral prices and the capital price is given by

$$p_t^{GI} = \left[\sum_j \Gamma_j^{inv} (p_{j,t})^{1-\eta_i} \right]^{\frac{1}{1-\eta_i}}$$

In our baseline model, we calibrate $\eta_i = 1$, therefore we have the relationship for every j sector

$$p_{j,t} I_{j,t} = \Gamma_j^{inv} p_t^{GI} GI_t$$
$$p_t^{GI} = \prod_j (p_{j,t})^{\Gamma_j^{inv}}$$

Optimization problem. The bank solves the problem

$$V_0 := \max_{\{I_t\}_{t\geq 0}} \int_0^\infty e^{-\int_0^t i_s^b ds} \left\{ i_t^K K_t - P_t^{GI} GI_t \right\} dt$$

subject to

$$\dot{K}_t = I_t - \delta K_t$$

Let V_t be the value of the bank at time t. It must satisfy the following HJB:

$$\begin{aligned} i_t^b V_t &= \max i_t^K K_t - P_t^{GI} GI_t + \partial_K V_t \left(I_t - \delta K_t \right) + \partial_t V_t \\ &= \max i_t^K K_t - P_t^{GI} \left(I_t + \Phi(\frac{I_t}{K_t}) K_t \right) + \partial_K V_t \left(I_t - \delta K_t \right) + \partial_t V_t \end{aligned}$$

The first order condition on investment I_t is

$$\partial_K V_t = P_t^{GI} (1 + \Phi'(\frac{I_t}{K_t}) K_t) = P_t^{GI} (1 + \kappa(\frac{I_t}{K_t} - \delta))$$

We follow the guess-and-verify approach. We guess the value of the bank is given by $V_t = P_t^K K_t$, which implies the first order condition becomes

$$P_t^K = P_t^{GI} (1 + \kappa(\frac{I_t}{K_t} - \delta))$$

Substituting the solution back to the HJB, we get

$$i_t^b P_t^K K_t = i_t^K K_t - P_t^{GI} (I_t + \Phi K_t) + P_t^K \left(I_t - \delta K_t \right) + \dot{P}_t^K K_t$$

The intuition is that the bank conduct the following activities: (1) receives $i_t^K K_t$ for rent payment; (2) pays out $P_t^{GI}\left(I_t+\Phi(\frac{I_t}{K_t})K_t\right)$ to generate new capital worth of $P_t^K I_t$, (4) capital $P_t^K K_t$ depreciates at rate δ, and (5) the capital value accretion $\dot{P}_t^K K_t$.

We eventually arrive at the return condition

$$i_t^b = \frac{i_t^K K_t - P_t^{GI}(I_t+\Phi K_t) + P_t^K\left(I_t-\delta K_t\right) + \dot{P}_t^K K_t}{P_t^K K_t}$$

$$r_{i,t}^b = \frac{i_t^b P_t^K}{P_{i,t}}$$

C.5 Equilibrium

Definition 1. (Competitive Equilibrium) Given an initial capital level K_0, household variables $\{P_{i,t}, W_{i,t}, P_t\}$, sector-specific variables $\{y_{j,t}, N_{j,t}, K_{j,t}, I_{j,t}, p_{j,t}\}$, bank-related variables $\{P_t^G I, P_t^K, K_t, i_t^K\}$, individual decision rules $\{c_{i,t}, \iota_{i,t}, n_{i,t}\}$, such that households optimize, firms optimize, bank optimizes and markets clear.

Aggregation. The following equations characterize our definition of aggregation by type.

$$A_t = \sum_i \int a g_{i,t} d(a,b,z)$$

$$B_{i,t} = \sum_i \int b g_{i,t} d(a,b,z)$$

$$C_{i,t} = \sum_i \int c_{i,t}(a,z) g_{i,t}(a,b,z) d(a,b,z)$$

$$\Lambda_{i,t} = \int z u'\left(c_{i,t}(a,b,z)\right) g_{i,t} d(a,b,z)$$

$$D_{i,t} = \sum_i \int \iota_{i,t} g_{i,t} d(a,b,z)$$

$$\Psi_{i,t} = \sum_i \int \psi_{i,t} g_{i,t} d(a,b,z)$$

Market clearing. Capital market clears when capital owned by household is equal to the sum of capital demand by firms. Bond market clears when the household budget constraint is binding. Goods market clears when production in each sector is equal to the consumption and use. Note that we are assuming that the goods market finance all miscellaneous costs, including the borrowing wedge, the bank intermediation costs, and the portfolio adjustment costs, through the same consumption network and the dynamic type-specific consumption shares. The inputs are final goods from all different sectors $oc_{j,t}$. All market clearing conditions are given by

Bond market clears.

$$A_t = A_t^g$$

Capital market clears.

$$B_t = K_t = \sum_j K_{j,t}$$

Capital production clears.

$$D_t = I_t - \delta K_t$$

Labor market clears.

$$v'(n_{i,t}) = \frac{\epsilon^w - 1}{\epsilon^w}(1 + \tau^{empl})(1 - \tau^{lab}) w_{i,t} \Lambda_{i,t}$$

Goods market clears from Walras' Law in Appendix C.7.

$$y_{j,t} = c_{j,t} + G_{j,t} + I_{j,t} + \sum_i x_{j\to i,t} + oc_{j,t}$$

Steady Equilibrium. At steady state, $\dot{c}_t = 0$, and $\dot{K}_t = 0$, $P_t = 1$, therefore we have

$$\begin{aligned} A_t &= A_t^g \\ B_t &= K_t \\ D_t &= I_t - \delta K_t = 0 \\ mc_{j,t} &= \frac{\epsilon - 1}{\epsilon} \\ v'(n_{i,t}) &= \frac{\epsilon^w - 1}{\epsilon^w}(1 + \tau^{empl})(1 - \tau^{lab}) w_{i,t} \Lambda_{i,t} \\ y_{j,t} &= c_{j,t} + G_{j,t} + I_{j,t} + + \sum_i x_{j\to i,t} + oc_{j,t} \end{aligned}$$

Transition Equilibrium. During transition, we have the following conditions

$$
\begin{aligned}
S_t &= 0 \\
B_t &= K_t \\
v'(n_{i,t}) &= \frac{\epsilon^w - 1}{\epsilon^w}(1+\tau^{empl})(1-\tau^{lab})w_{i,t}\Lambda_{i,t} \\
\dot{\pi}_{j,t} &= \rho\pi_{j,t} - (mc_{j,t} - \frac{\epsilon - 1}{\epsilon})\frac{\epsilon}{\chi_j} \\
y_{j,t} &= c_{j,t} + G_{j,t} + I_{j,t} + + \sum_i x_{j\to i,t} + oc_{j,t}
\end{aligned}
$$

C.6 Proof of Lemma 7

Proof. Since in equilibrium, all firms are symmetric, we will drop the j indexation for simplicity. Denote $p = P_t(j)$, $P = P_t$, $Y = Y_t$, $\pi = \pi_t$, $\chi = \chi$, $W = W_t$, taking P as given, the firm's problem in recursive form is

$$
\rho J(p,t) = \max_{\pi}\left\{\left(\frac{p}{P} - mc\right)\left(\frac{p}{P}\right)^{-\epsilon} Y - \frac{\chi_s}{2}\pi^2 Y + J_p(p,t)p\pi + J_t(p,t)\right\}
$$

where J is the corresponding value function of the maximization problem. The first order conditions of the recursive form are given by

$$
\begin{aligned}
J_p(p,t)p &= \chi\pi Y \\
(\rho - \pi)\, J_p(p,t) &= -\left(\frac{p}{P} - mc\right)\epsilon\left(\frac{p}{P}\right)^{-\epsilon-1}\frac{Y}{P} + \left(\frac{p}{P}\right)^{-\epsilon}\frac{Y}{P} + J_{pp}(p,t)p\pi + J_{tp}(p,t)
\end{aligned}
$$

In a symmetric equilibrium we will have $p = P$, and hence

$$
\begin{aligned}
J_p(p,t) &= \frac{\chi\pi Y}{P} \\
(\rho - \pi)\, J_p(p,t) &= -(1 - mc)\epsilon\frac{Y}{P} + \frac{Y}{P} + J_{pp}(p,t)p\pi + J_{tp}(p,t)
\end{aligned}
$$

Differentiating the first equation with respect to time, we get

$$
J_{pp}(p,t)\dot{p} + J_{pt}(p,t) = \frac{\chi Y\dot{\pi}}{P} + \frac{\chi\dot{Y}\pi}{P} - \frac{\chi\pi Y}{P}\frac{\dot{P}}{P}
$$

and plugging in the second equation, we get

$$
(\rho - \pi)\,\frac{\chi\pi Y}{P} = -(1 - mc)\epsilon\frac{Y}{P} + \frac{Y}{P} + \frac{\chi Y\dot{\pi}}{P} + \frac{\chi\dot{Y}\pi}{P} - \frac{\chi\pi Y}{P}\frac{\dot{P}}{P}
$$

Putting it together, we have

$$\dot{\pi}_t = \left(\rho - \frac{\dot{Y}_t}{Y_t}\right)\pi - (mc_t - \frac{\epsilon - 1}{\epsilon})\frac{\epsilon}{\chi}$$

■

C.7 Derivation of the Walras' Law

Illiquid account. Given the capital production clearing condition $D_t = I_t - \delta K_t$, the aggregation of the illiquid account budget constraint is

$$M_t = -\xi B_t + \int dbd(a,b,z) = -\xi B_t + \xi \sum_i \int b_{i,t} d(a,b,z) + \sum_i \int \iota_{i,t} d(a,b,z) = D_t = I_t - \delta K_t$$

Liquid account. The law of motion for households' liquid weath is given by

$$\dot{a}_{i,t} = (r^a_{i,t} + \xi)a_{i,t} + r^b_{i,t} b_{i,t} + (1-\tau^{\text{lab}}) z_t n_{i,t} w_{i,t} + rebate_{i,t} - c_{i,t} - q_{i,t}\iota_{i,t} - \psi(\iota_{i,t}, b_{i,t}).$$

Similarly, the aggregation of liquid account in real terms budget constraint is

$$\begin{aligned}
0 &= S_t \\
&= \frac{\sum_i P_{i,t} S_{i,t}}{P_t} \\
&= \frac{\sum_i P_{i,t}(-\xi A_{i,t} + \int da_{i,t} d(a,b,z))}{P_t} \\
&= -\xi A_t + \frac{1}{P_t}\sum_i P_{i,t}\left((r^a_{i,t} + \xi)A_{i,t} + r^b_{i,t} B_{i,t} + (1-\tau^{\text{lab}})\frac{\sum_j W_{j,t} N_{j,t}}{P_{i,t}} + \frac{\sum_j \Pi_{j,t} + T_t}{P_{i,t}} - C_{i,t} - q_{i,t} D_{i,t} - \Psi_{i,t}\right) \\
&= \frac{1}{P_t}\sum_i r^a_{i,t} P_{i,t} A_{i,t} + \frac{i^b_t P^K_t}{P_t} B_t + (1-\tau^{\text{lab}})\frac{\sum_j W_{j,t} N_{j,t}}{P_t} + \frac{\sum_j \Pi_{j,t} + T_t}{p_t} - C_t - \frac{P^K_t}{P_t} D_t - \Psi_t \\
&= \frac{1}{P_t}\sum_i r^a_{i,t} P_{i,t} A_{i,t} + \frac{i^K_t K_t + P^K_t I_t - P^{GI}_t GI_t - \delta P^K_t K_t}{p_t K_t} B_t + (1-\tau^{\text{lab}})\frac{\sum_j W_{j,t} N_{j,t}}{p_t} \\
&+ \frac{\sum_j (p_{j,t} y_{j,t} - (1-\tau^{empl}) W_{j,t} N_{j,t} - i^K_t K_{j,t} - p_{jx,t} x_{j,t}) + \sum_j (\tau^{lab} - \tau^{empl}) W_{j,t} N_{j,t} - P_t G_t - r_t P_t A^g_t}{P_t} - C_t - \frac{P^K_t}{P_t} D_t - \Psi_t \\
&= r_t(A_t - A^g_t) - \omega A_t + \theta A^-_t + \frac{P^K_t}{P_t}(I_t - \delta K_t - D_t) - \frac{P^{GI}_t}{P_t} GI_t + Y_t - G_t - C_t - \Psi_t - \frac{\sum_j (p_{jx,t} x_{j,t})}{P_t}
\end{aligned}$$

Government expenditure, investment expenditure, and consumption expenditure follow Γ^g_j, Γ^{inv}_j and Γ^c_{ij} respectively. We assume that all other expenditures, including the intermediate costs and the portfolio adjustment cost, follow the consumption preference and the consumption shares between different household types, Θ^c_i, are the same as that of the consumption expenditure.

Therefore the sector-specific goods market clearing condition is given by

$$y_{j,t} = c_{j,t} + G_{j,t} + I_{j,t} + + \sum_i x_{j\to i,t} + oc_{j,t}$$

in which $oc_{j,t}$ is the unit of final products from sector j which are used to finance all other costs for all types of households.

C.8 Sequence-Space Representation

A household of type i has lifetime utility

$$V_{i,0}(\cdot) = \max_{\{c_{i,t},\iota_{i,t}\}} \mathbb{E}_0 \int_0^\infty e^{-(\rho+\xi)t}\Big[u(c_{i,t}) - v(N_{i,t})\Big]dt,$$

where hours of work $N_{i,t}$ are taken as given.

The household's liquid and illiquid budget constraints are given by

$$\dot{a}_{i,t} = (r_t^a(a_{i,t}) + \xi)a_{i,t} + r_t^b b_{i,t} + (1-\tau^{\text{lab}})z_{i,t}w_{i,t}N_{i,t} + \tau_{i,t} - \frac{1}{P_t}\left(\sum_j \Omega_j p_{j,t}^{1-\sigma} c_{i,t}^{\varepsilon_{j,t}}\right)^{\frac{1}{1-\sigma}} - q_t\iota_{i,t} - \psi(\iota_{i,t}, b_{i,t})$$

$$\dot{b}_{i,t} = \xi b_{i,t} + \iota_{i,t}$$

The household also faces borrowing constraint $a_{i,t} \geq \underline{a}$ and short-sale constraint $b_{i,t}$. We will switch notation to denote by $r_t^K = r_t^b$ the illiquid return ("on capital"). The composite return on liquid assets or debt is given by

$$r_t^a(a) = \mathbb{1}_{a\geq 0}(r_t - \omega) + \mathbb{1}_{a<0}(r_t - \omega + \theta).$$

The return on the illiquid asset, the fiscal rebate (in absence of rescaling), and the capital price are respectively given by

$$r_t^K = \frac{i_t^K K_t + P_t^K I_t - P_t^{GI} GI_t - \delta P_t^K K_t}{P_t K_t}$$

$$\tau_t = \frac{\sum_j (p_{j,t}y_{j,t} - W_{j,t}N_{j,t} - i_t^K K_{j,t} - p_{jx,t}x_{j,t}) + \sum_j \tau^{lab} W_{j,t} N_{j,t}}{P_t} - G_t - r_t A_t^g$$

$$q_t = \frac{P_t^K}{P_t} = \frac{P_t^{GI}(1 + \kappa(\frac{I_t}{K_t} - \delta))}{P_t},$$

where P_t^{GI} is the price of investment.

Sequence-space representation. First, notice that the household problem does not depend directly on parameters that differ across types i. The consumption policy function can therefore be written in sequence-space representation as

$$c_{i,t}(a,b,z) = c_t\Big(a,b,z;\ \Big\{r_s, r_s^K, q_s, \tau_s, Z_s, \{\xi_{i,s}\}_i, \{\rho_{j,s}\}_j\Big\}_{s\geq 0}\Big)$$

where we denote by

$$\rho_{j,t} = \frac{p_{j,t}}{P_t}$$

the price of sector / good j relative to the aggregate consumper price index P_t, by

$$Z_t = (1-\tau^{\text{lab}})\sum_i \iiint z w_{i,t} N_{i,t} g_{i,t}(a,b,z)\, da\, db\, dz = (1-\tau^{\text{lab}})\sum_i \mu_i w_{i,t} N_{i,t}$$

aggregate private post-tax labor income, and by

$$\xi_{i,t} = \frac{w_{i,t} N_{i,t}}{\sum_i \mu_i w_{i,t} N_{i,t}}$$

household type i's labor income share, so that we can write the household budget constraint as $(1-\tau^{\text{lab}}) z_{i,t} w_{i,t} N_{i,t} = z_{i,t} Z_t \xi_{i,t}$.

D Data

This appendix provides details on the data we use to construct our empirical results.

D.1 Factor Shares

We obtain the factor shares for sectors from BEA's I-O GDP by Industry dataset from 1997 after industry classifications are based on NAICS. We then crosswalk sectors based on 2-digit NAICS level to 22 sectors in total. All concordances are weighted by gross output levels from BEA's Gross Output by Industry Table. First, we calculate the labor share in the production of primary factors $1-\alpha_j$ for each sector j. Given the Cobb-Douglas structure of our primary factor production function, the parameters are calculated as the ratio of compensation of employees to the value added adjusted for taxes and subsidies. We obtain this ratio for each year from 1997-2015, then take the average value. Second, we obtain the share of intermediate inputs in the production function $\mu_{x,j}$ for each sector j. The parameters are calculated as the intermediate input expenditures as a percentage of gross output, averaged over our sample period. We obtain this ratio for each year from 1997-2015, then take the average value. Finally, we calculate the labor share in the over production function $\mu_{l,j} = (1-\mu_{x,j})(1-\alpha_j)$, and the capital share in the over production function $\mu_{k,j} = (1-\mu_{x,j})(\alpha_j)$. Figure 10 plots the factor shares in the production for each sector j. Sectors with the highest labor share are service-related, such as Education Services, Healthcare, Professional and Technical Services. Manufacturing is particularly intermediates dependent. Housing and leasing industries are the most capital-intensive.

D.2 Capital Investment

In our baseline, we use the Investment Flows Data for 41 Sector Partition from Vom Lehn and Winberry (2022) from 1997-2015 to calculate the share of capital investment inputs from each sector Γ_j^{inv}. First, for each year from 1997 to 2015, we calculate the share of total capital investment across all purchasing sectors (by summing the row in the Investment Flows tables) and calculating the shares. We then crosswalk the 41 sectors based on 2-digit NAICS level to 22 sectors in total. Finally, we take the average of each year's share to get Γ_j^{inv}. Figure 11 plots sectoral input shares of total capital investment. The top investment hubs are Construction, Durable Manufacturing, and Professional and Technical Services. These three investment hub industries together account for nearly 80% of inputs used in the production of capital investment.

For robustness checks in Appendix E, we use BEA's Capital Flows Table in 1997 to derive the share of capital investment inputs from each of the 66 BEA sectors. The Capital Flows table includes 180 commodities with corresponding NAICS codes. Our first step is to match the commodities used to the sector categories. Most of the matching are straightforward given the NAICS codes in both the Capital Flows table and the Input-Output table. Special attention should be paid to the following: (1) "Manufactured homes, mobile homes" in commodities is categorized under the "Housing" sector; (2) "Retail trade" in commodities include both 44 and 45 by NAICS codes, we divide and assign it to "Motor vehicle and parts dealers", "Food and beverage stores", "General merchandise stores", "Other retail" according to these four sectors' sizes measured by the gross output in 1997; (3) "Offices of real estate agents and brokers" in commodities is categorized under the "Rental and leasing services and lessors of intangible assets" sector; (4) "Noncomparable imports" is excluded. The top investment hubs are "Construction", "Machinery", "Motor vehicles, bodies and trailers, and parts", and "Computer and electronic products". These four investment hub industries together account for nearly 70% of inputs used in the production of capital investment. The sparseness of the investment network is collaborated in Vom Lehn and Winberry (2022).

D.3 Government Spending

We use the BEA industry input-output "Use" table to compute the share of government spending on goods from different sectors Γ_j^g. The parameters are calculated as the government expenditures in sector j as a percentage of total government spending. "Government" includes federal government, federal government enterprises, state and local government, and state and local government enterprises. We average the annual share across 1997-2015. Figure 12 plots the government spending share, averaged across the sample period.

D.4 Mark-ups

Steady state markups across sectors are given by $\mu_j = \frac{\epsilon_j}{\epsilon_j - 1}$. We calibrate ϵ_j directly to match sectoral markups using data from Baqaee and Farhi (2020). They use three alternative approaches to estimate sectoral markups from 1997 to 2015. The average markup for each sector in any particular year is computed as the harmonic sales-weighted average of firm markups, which are taken from Compustat and assigned to BEA sectors. In our baseline model, we use the average of their benchmark estimates following the accounting profits approach because the average markup is then around 10% and thus closer to the standard markup assumed in the HANK literature. Figure 13 plots the accounting-profits markups for each of the 22 sectors. We have also run robustness checks of the empirical regularities using the other two approaches (user-cost and production function) in Appendix E. To ensure $\epsilon_j > 0$, for each approach, we replace markups below 1 with the lowest markup in all sectors above 1 using that particular approach.

D.5 Monthly Price Adjustment Frequency

To measure monthly price adjustment frequency across industries, we compare two data sources. First, in our baseline model, we map the sector-specific monthly price adjustment frequency, $1 - \ominus$, from Pasten et al. (2017) to the 22 sector categories. They use the data underlying the Producer Price Index (PPI) for 754 industries (defined by 6-digits NAICS codes) from the U.S. Bureau of Labor Statistics, from 2005 to 2011. The PPI measures covers both final consumption prices and intermediates prices. The mapping from NAICS to BEA sectors are not one-to-one. Therefore, we need to make certain inferences based on industry similarities.

Second, we estimate the frequency of price changes using the confidential micro-level price data underlying the the Consumer Price Index (CPI) from 1998 to 2005, made available by Nakamura and Steinsson (2008). The estimates are available for 272 "Entry-Level Item" (ELI) categories, in two measures, with and without sales. They cover most of consumer expenditures, but not intermediate industries. The mapping from ELI to BEA sectors are not one-to-one. Therefore, we need to make certain inferences based on industry similarities.

We use measures from Pasten et al. (2017) as our baseline because they are more recent than earlier estimates by Nakamura and Steinsson (2008), more comprehensive than estimates from [D'Acunto et al. (2018)], and account for price changes in intermediates industries. Figure 14 plots the price adjustment frequency using data from Pasten et al. (2017). We have also run robustness checks of the empirical regularities using the other two approaches (Nakamura and Steinsson (2008) with sales, and Nakamura and Steinsson (2008) without sales) in Appendix E).

D.6 Firm Price Adjustment Cost Parameters

To pin down parameter χ, We establish the relationship between our monthly price adjusting data, usually seen in Calvo models, and the adjustment cost parameter in Rotemberg-type setting. We derive the continuous-time firm adjustment cost parameters according to [Sims and Wolff (2017)].[23]

Calvo. In the Calvo model, a randomly selected fraction of firms, $1-\theta$, can adjust their price in a given period. All updating firms adjust to the same price P^*, and the adjusting inflation is $1+\pi_t^* = P_t^*/P_{t-1}$. Overall inflation π_t satisfies

$$\frac{1+\pi_t^*}{1+\pi_t} = \frac{\epsilon}{\epsilon-1}\frac{x_{1,t}}{x_{2,t}}$$

$$x_{1,t} = \frac{1}{C_t}mc_t Y_t + \theta\beta\mathbb{E}_t\left(1+\pi_{t+1}\right)^{\epsilon} x_{1,t+1}$$

$$x_{2,t} = \frac{1}{C_t}Y_t + \theta\beta\mathbb{E}_t\left(1+\pi_{t+1}\right)^{\epsilon-1} x_{2,t+1}$$

[23] There is a strand of literature that studies the difference between Rotemberg and Calvo models. [Nistic'o (2007)] and [Lombardo and Vestin (2008)] compare the welfare implications of the two models. [Ascari et al. (2011)] and [Ascari and Rossi (2012)] investigate the differences between the two models under a positive trend inflation rate. [Ascari and Rossi (2011)] study the effect of a permanent disinflation in the Rotemberg and Calvo models. More recently, [Boneva et al. (2016), Richtera and Throckmorton (2016), Eggertsson and Singh (2018), and Miao and Ngo (2018)] investigate the differences in the predictions of the Rotemberg and Calvo models with the zero lower bound for the nominal interest rate. [Sims and Wolff (2017)] study the state-dependent fiscal multipliers in the two models under a Taylor rule in addition to periods where monetary policy is passive. Moreover, [Born and Pfeifer (2020)] discuss the mapping between Rotemberg and Calvo wage rigidities.

Inflation evolves according to

$$(1+\pi_t)^{1-\epsilon} = (1-\theta)\left(1+\pi_t^*\right)^{1-\epsilon} + \theta$$

The NKPC in the Calvo setting is given by

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}\left(\ln mc_t - \ln mc_t^*\right) \tag{29}$$

Rotemberg. We use an alternative of Rotemberg setting, where we denote the adjustment cost by $\Lambda(\pi_t) = \frac{\chi}{2}\left(\pi_t\right)^2 P_t Y_t$. The NKPC in Rotemberg is given by,

$$\pi_t = \beta E_t \pi_{t+1} + \frac{\epsilon - 1}{\chi}(\ln mc_t - \ln mc_t^*) \tag{30}$$

First order equivalence. For the slopes of NKPC in equation 29 and in equation 30 to be equivalent, we would have

$$\frac{(1-\theta)(1-\theta\beta)}{\theta} = \frac{\epsilon - 1}{\chi}$$

Therefore, the adjustment cost parameter χ is given by

$$\chi_j = \frac{(\epsilon - 1)\theta}{(1-\theta)(1-\theta\beta)}$$

in which θ is the probability that price remains unchanged for a quarter (3 months), $\theta = (1-\ominus)^3$, where $\ominus$ is the monthly price adjustment frequency.

D.7 Intermediates Input-output Share

We calibrate the weights on intermediate inputs α_{jk}^x so that our model's production network is consistent with the BEA's input-output table. We use the Industry Input-output "Use" Table. For each year, we calculate the parameters of the intermediates input-output network α_{jk}^x as sector j (columns)'s nominal expenditure on intermediate inputs from sector k (rows) as a share of j's total expenditure on intermediate inputs. Then we average the ratios across 1997-2015. Figure 15 plots the heatmap of the input-output network, averaged across the sample period.

D.8 Centrality

A reduced-form measure of a sector's centrality in the input-output production network is the Katz-Bonacich centrality measure discussed by Carvalho (2014). We compute centrality as $c =$

$\eta(\boldsymbol{I} - \lambda\alpha^x_{jk})^{-1}\mathbf{1}$, where we set $\eta = \frac{1-\theta}{N} = \frac{1-0.5}{22}$ and $\lambda = 0.5$.

Figure 16 plots the centrality measure for each of the 22 sectors. The most important suppliers in the production network are Professional and Technical Services, Durable and Non-durable Manufacturing, Finance and Insurance.

We also calculated the outdegree of sectors, defined as $d_k = \sum_j \alpha^x_{jk}$, that is, the sum over all the weights of the network in which sector k appears as an input-supplying sector. The correlation between the outdegree measure and the centrality measure is 0.99, providing us with confidence with the central nodes in the production network. We have run robustness check of our empirical regularities results using both measures in Appendix E.

D.9 ACS-IO: Sectoral Payroll Shares

We build the dataset of sector-specific payroll shares for households in various income quantiles.

Data Source. We obtain cross-sectional household occupation and payroll data from the American Community Survey (ACS), made available by IPUMS (Ruggles et al., 2015) from 2000-2015.

Sample Restriction and Household Types. First, we clean up the ACS dataset by excluding those who are not in the labor force as well as outlier data points (wage income below $1,000 and beyond $9,999,998). For each year, we further exclude the extreme values (top 1% and bottom 1%), and then divide the remaining earnings data points into 10 income quantiles.

Matching. Second, we map the cleaned earnings data to the sector classification from the BEA. Finally, we follow Clayton et al. (2018) and use the "many-to-one" method to merge the sectoral earnings data. For the BEA sectors that do not have a corresponding ACS industry identifier, we borrow the variables of interest from industries closest to those.

D.10 CEX-IO: Expenditure Share

Data Source. We use the U.S. Consumer Expenditure Survey (CEX) by the BLS from 1997-2015 to obtain expenditure shares across product categories for households at different quantiles of the income distribution. The CEX is a widely used consumption survey tracking spending in all product categories, including goods, services, housing, and health. It consists of two parts, the Interview and Diary surveys. The Interview surveys collect responses from households annually for up to 4 consecutive quarters of questions, covering a wide range of purchases. The Dairy questionnaire contains more detailed questions about daily purchases, and are collected at weekly frequency. We use both for our data construction.

Sample Restriction. We follow the literature and impose a set of sample restrictions. We restrict the samples to urban households, with heads aged 25-64, have a full-year of interview coverage and complete income responses. We exclude outlier data points (households with the top 1% and bottom 1% of after-tax income, households with the top 1% and bottom 1% of total expenditure, and households whose income is below $1,000). We also eliminate data points when expenditures are negative (as a result of Medicaid and Medicare reimbursement).

Expenditure Adjustments. We follow Comin et al. (2021) and make certain adjustment to the expenditure data. We exclude any taxes and social security payments. Similar to [Aguiar and Bils (2015)], we exclude alimony, child support payments, support for college students, and occupational expenses. To avoid double counting for expenditures associated with owning a house, we follow [Hubmer (2020)] and subtract estimated monthly rental value of owned home, estimated monthly rental value of vacation home, and estimated rental value of timeshare and treat those as part of household income.

Income Adjustment and Distribution. We add estimated monthly rental value of owned home, estimated monthly rental value of vacation home, and estimated rental value of timeshare to after-tax household income as reported in the CEX (FINCATXM and FINCAEFX) to get household income levels. We divide the households into 100 quantiles according to their adjusted income levels.

Matching. We match the UCC categories in our sample to 22 BEA industries mostly based on a manual concordance assembled by [Levinson and O'Brien (2019)]. Additionally, we match Diary items and missing Interview items by hand based on our best judgment.

Treatment of Zeros. For some goods, households expenditure share in those will be zero. This is either because households forgot to record, or households don't spend in some pure intermediate sectors (such as oil and gas extraction). When there are missing values, we assume a minuscule amount of 0.0001% on household expenditure share in order to avoid dropping these households from our sample.

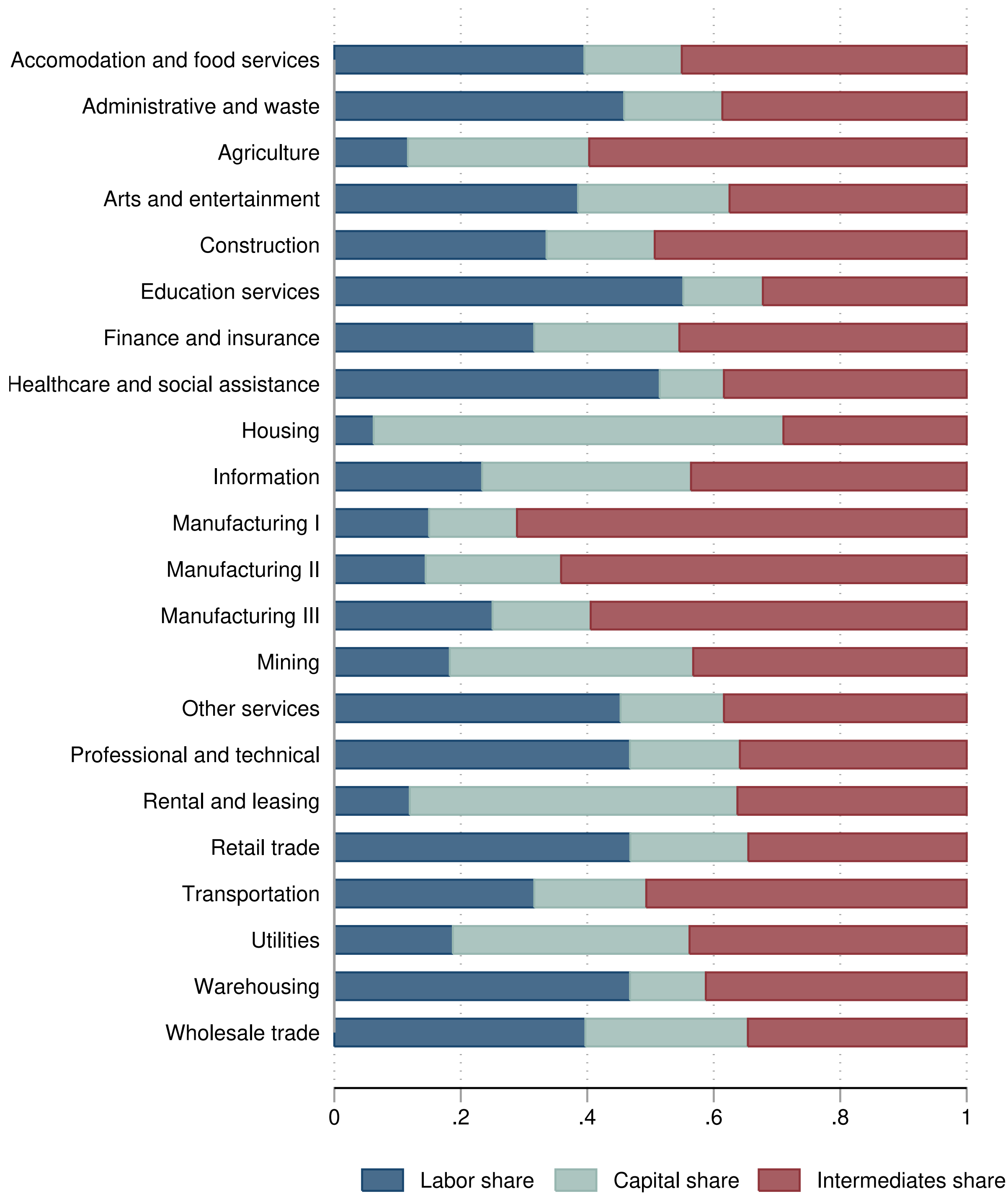

Figure 10. Factor Share in the Total Gross Output

Note. We compute the sector-specific factor shares in the total production from the BEA GDP by Industry dataset. The intermediate input share $\mu_{x,j}$ (in red) is computed as the intermediate input expenditures as a percentage of gross output. The labor share $\mu_{l,j}$ (in blue) is calculated as the product of $1 - \mu_{x,j}$ and the ratio of compensation of employees to the value added adjusted for taxes and subsidies. The remaining is the capital share $\mu_{k,j}$ (in green). We obtain each factor share for each year from 1997-2015, then take the average value.

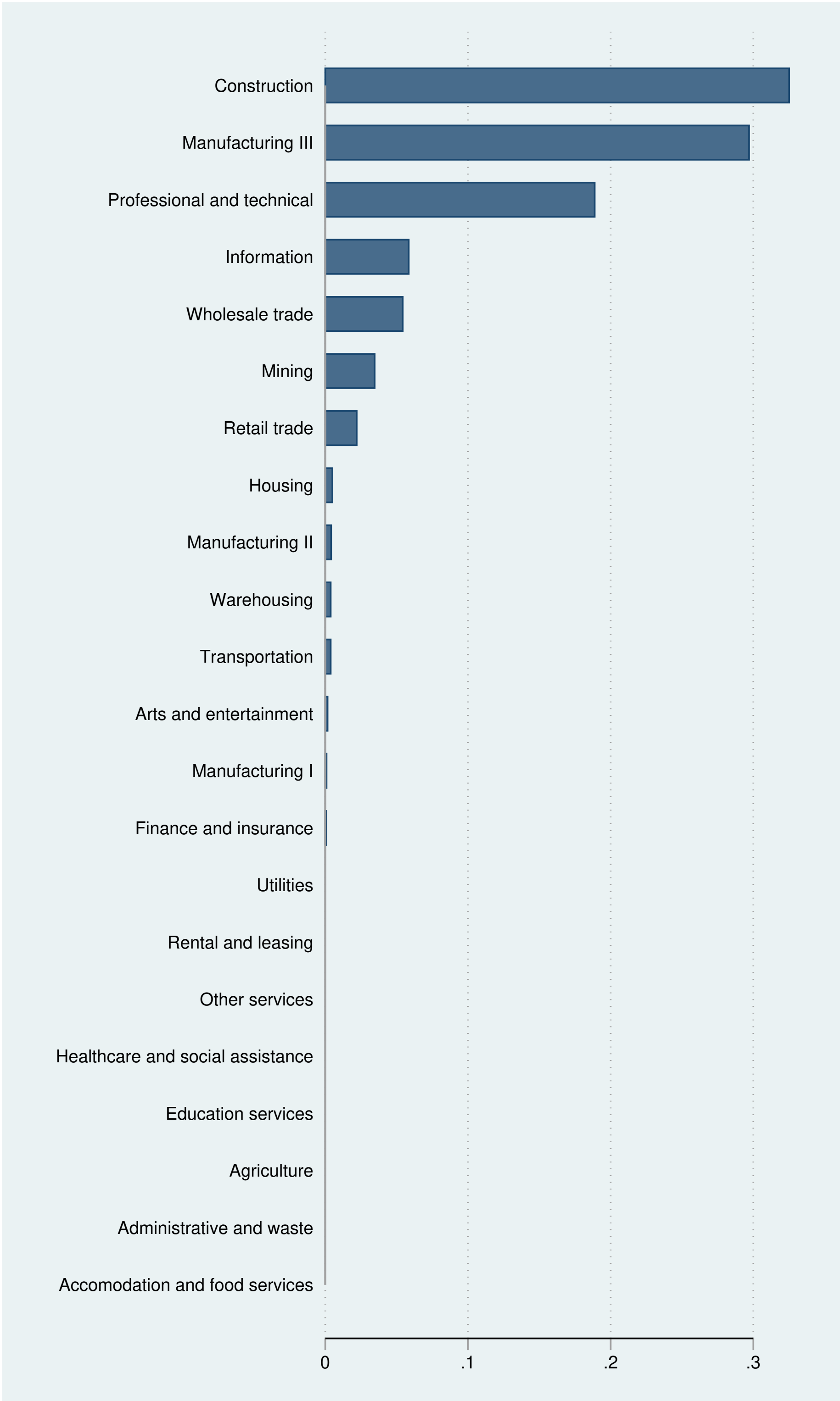

Figure 11. Sectoral Inputs Share in Capital Investment Γ_j^{inv}

Note. We compute the sectoral inputs share in the aggregate capital investment using the Investment Flows Data for 41 Sector Partition from Vom Lehn and Winberry (2022) from 1997-2015.

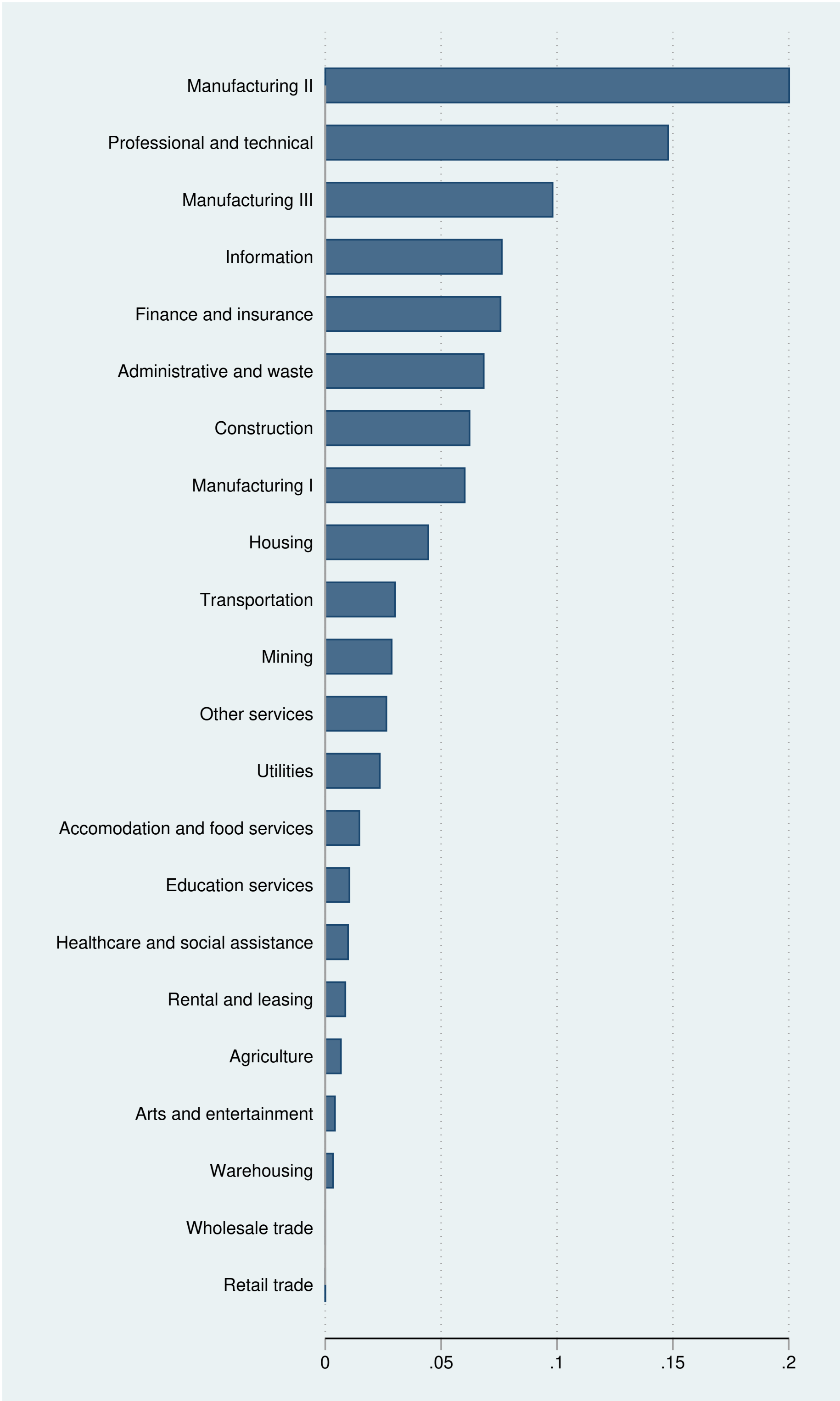

Figure 12. Government Spending Share on Sectoral Goods Γ_j^g

Note. We compute the share of government spending on goods from individual sector j in total government spending every year using the BEA industry input-output "Use" table , then we average the ratio across 1997-2015.

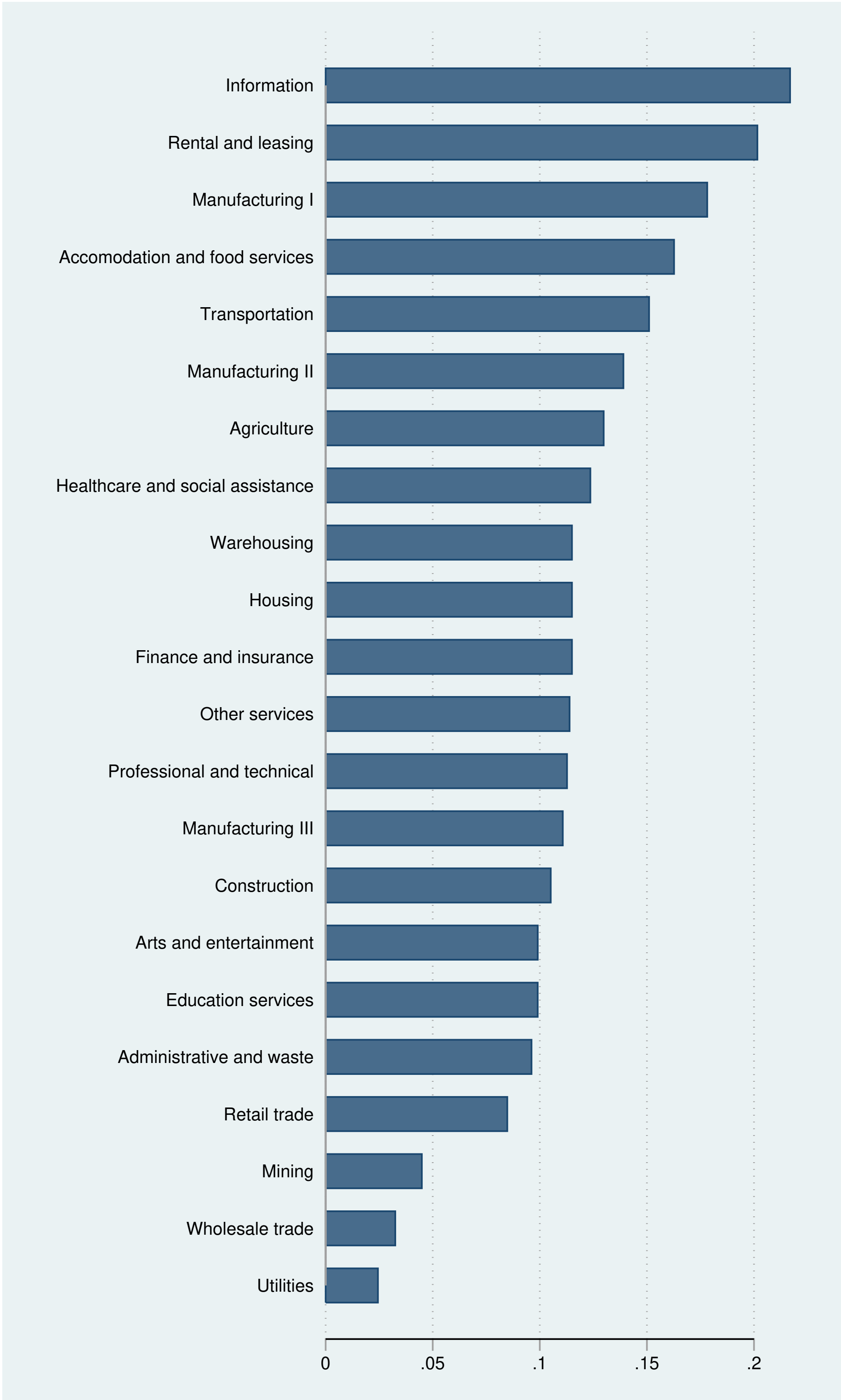

Figure 13. Sectoral Accounting-Profit Markups

Note. We compute the average accounting-profits markups over cost using data from Baqaee and Farhi (2020).

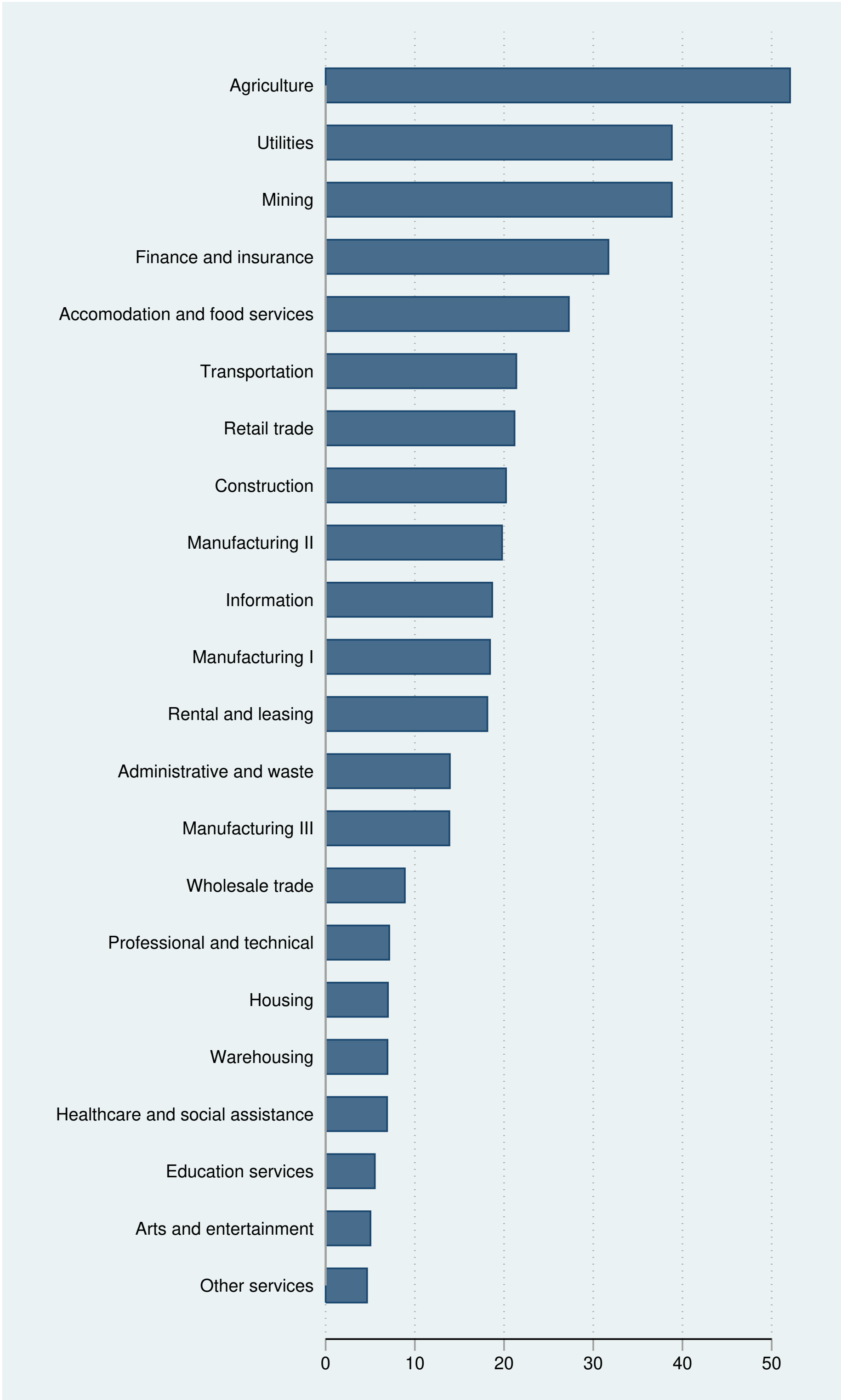

Figure 14. Monthly Price Adjustment Frequency $1 - \ominus$ From Pasten et al. (2017)

Note. We map the sector-specific monthly price adjustment frequency from Pasten et al. (2017) to the 22 sector categories.

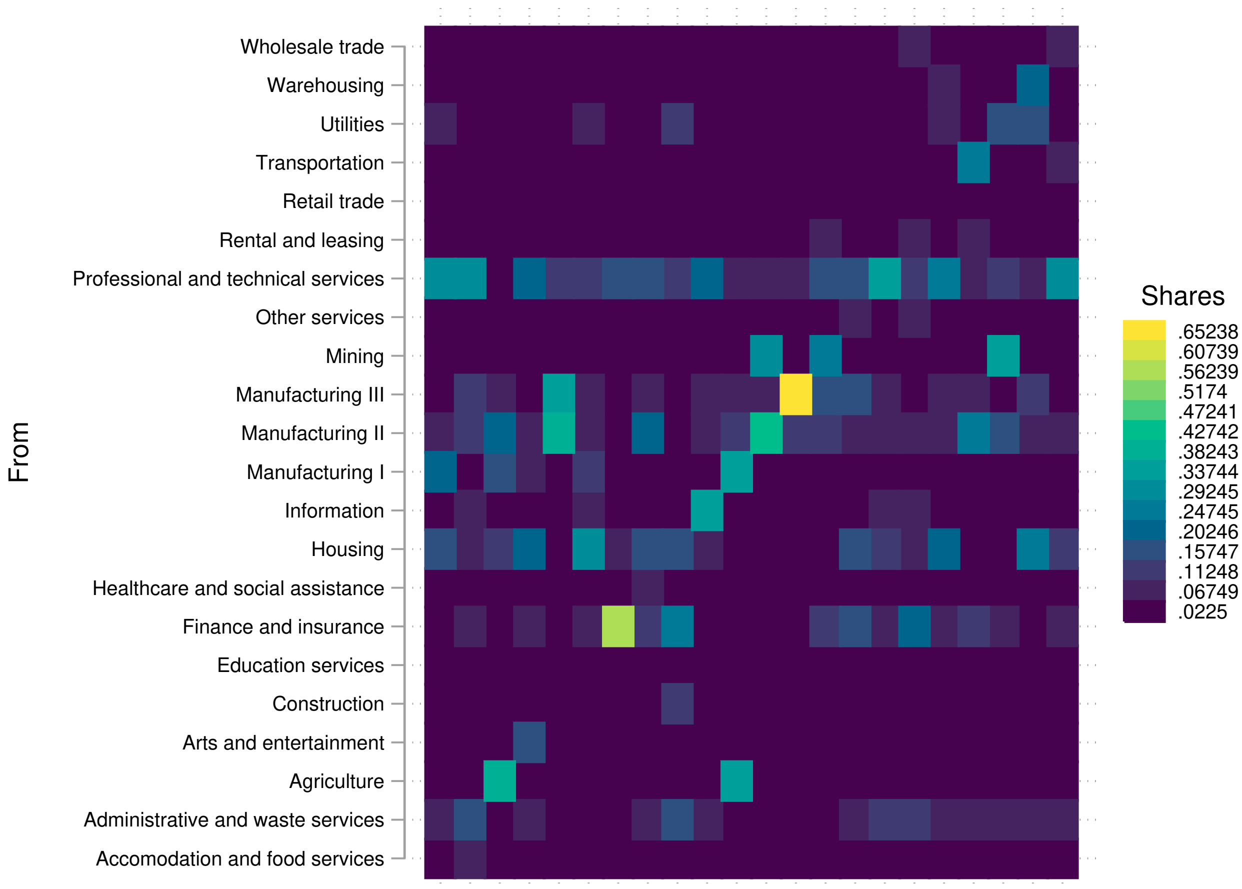

Figure 15. The Input-Output Network α^x_{jk}

Note. We compute the share of nominal expenditure on intermediate inputs from sector k (y-axis) for each sector j (x-axis) every year, then we average the ratio across 1997-2015.

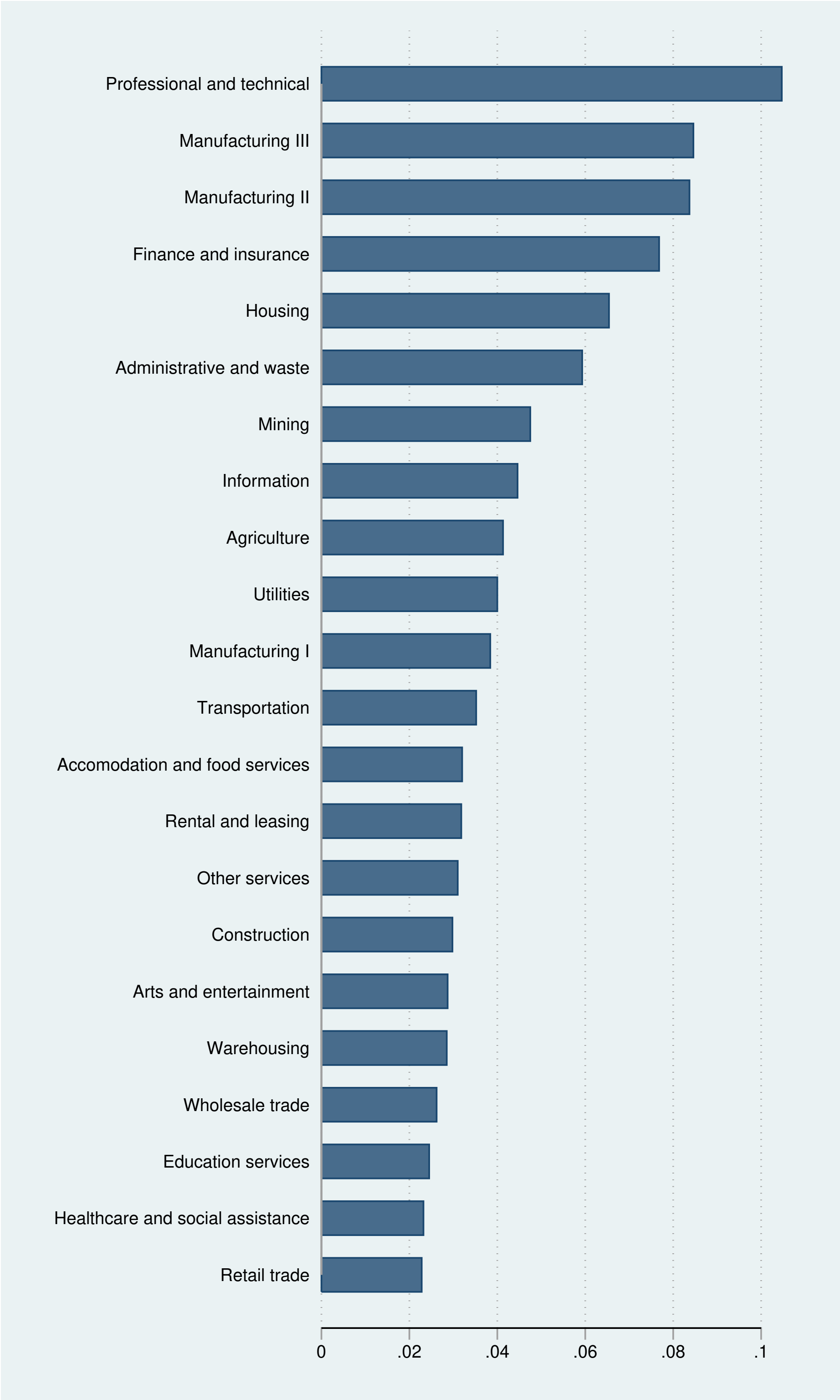

Figure 16. Sectoral Centrality α^x_{jk}

Note. We compute the centrality measure following Carvalho (2014) using the input-output matrix α^x_{jk}.

E Robustness Checks of Empirical Regularities

E.1 Expenditure Heterogeneity using CEX-IO Dataset

Expenditure-share weighted sectoral heterogeneity for different income percentiles. In Figure 17, we show the comparison of expenditure-share weighted sectoral heterogeneity: for pre-tax income percentiles in navy and after-tax income percentiles in red. The pre-tax income percentiles uses FINCBTAX in the CEX dataset, and the after-tax income percentiles uses FINCATAX instead. The results illustrates that our empirical regularities are robust to using either income measures.

Expenditure-share weighted sectoral heterogeneity for different levels of aggregation. In Figure 18, we show four sets of expenditure-share weighted sectoral heterogeneity at different levels of aggregation for after-tax income percentiles: at 66-sector level (the full BEA I-O) in red, 22-sector level (2-digit NAICS) in blue, 9-sector level (1-digit NAICS) in green, and 3-sector level (manufacturing, agriculture, and services) in gray. The empirical regularities are robust between the 66-sector level and the 22-sector level. But if we split the consumption categories into agriculture, manufacturing, and services like in Comin et al. (2021), the results are not consistent. Figure 19 shows the correlation of sectoral features between different levels of aggregation versus the 66 sectors. The results start to stabilize after 22-sector aggregation.

Factor shares. Figure 20 plots the expenditure-weighted three factor shares using with different levels of sectoral disaggregation: (1) labor shares, (2) capital shares, and (3) intermediates shares. The relationship between income percentile and factor shares is robust across measures and disaggregation levels.

Monthly frequency of price change. Figure 21 plots the expenditure-weighted monthly frequency of price change using three measures with different levels of sectoral disaggregation: (1) Pasten et al. (2017), (2) Nakamura and Steinsson (2008) with sales, and (3) Nakamura and Steinsson (2008) without sales. The hump-shaped relationship between income percentile and price rigidity is robust across measures and disaggregation levels, where the middle-income households have the least rigid consumption basket. Our results using Nakamura and Steinsson (2008) measures, either with or without sales are consistent with [Cravino 2020], where they use the same dataset for price rigidity.

Markups. Figure 22 plots the expenditure-weighted markups using three markups measures with different levels of sectoral disaggregation: (1) accounting-profits markups (AP), (2) user-cost markups (UC), and (3) production-function markups (PF).

Centrality. Figure 23 plots the expenditure-weighted markups using two measures with different levels of sectoral disaggregation: (1) Katz-Bonacich centrality and (2) outdegree. The relationship between income percentile and network centrality is robust across measures and disaggregation levels.

Investment and Government Spending Shares. Figure 24 plots the expenditure-weighted investment shares and government spending shares with different levels of sectoral disaggregation. The relationship between income percentile and investment or government spending shares is robust across measures and disaggregation levels.

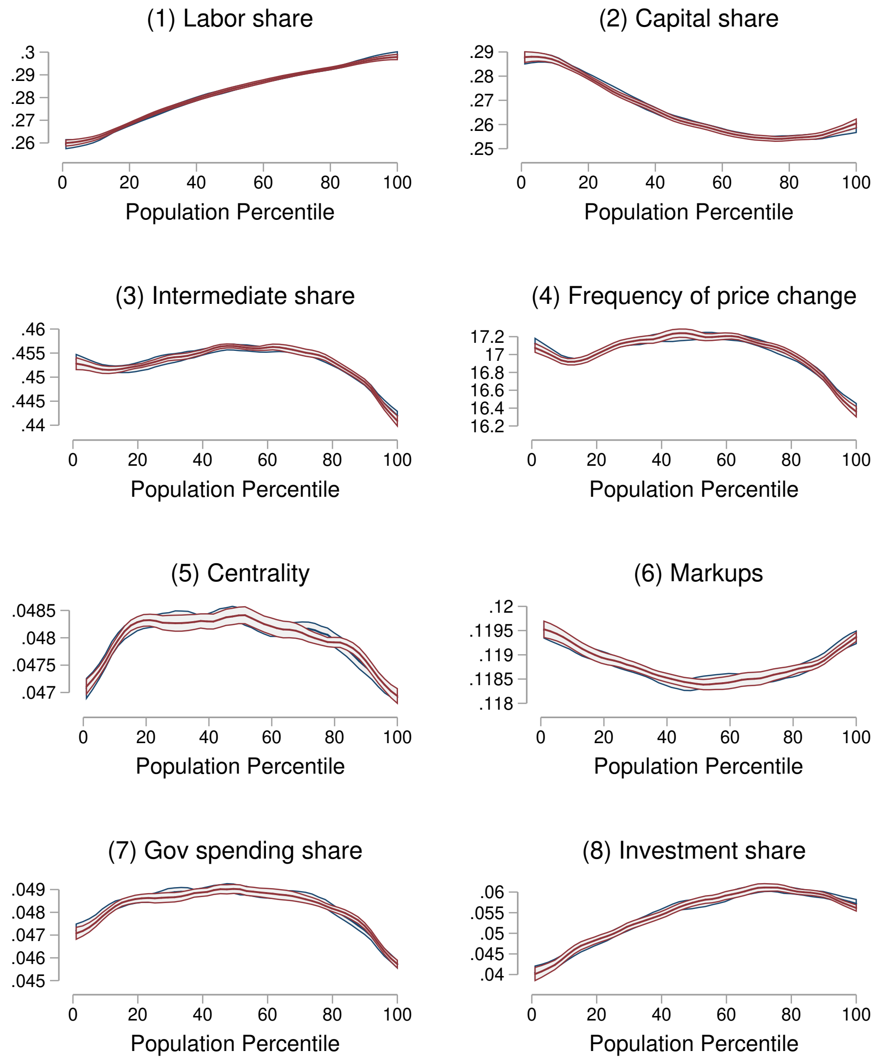

Figure 17. Expenditure-share Weighted Sectoral Heterogeneity for Pre-tax and After-tax Income Percentiles

Note. We plot the expenditure-weighted sectoral features as a function of households' pre-tax income percentile (in blue), and after-tax income percentile (in red), average over the sample period. The horizontal axis for each panel is household percentiles, each representing 1% of the population. The vertical axis reports the average sectoral features, such as labor share, capital share, intermediate share, frequency of price change, centrality, markups, government spending share, and investment share, weighted by the expenditure share across 22 sectors by households of the corresponding percentile.

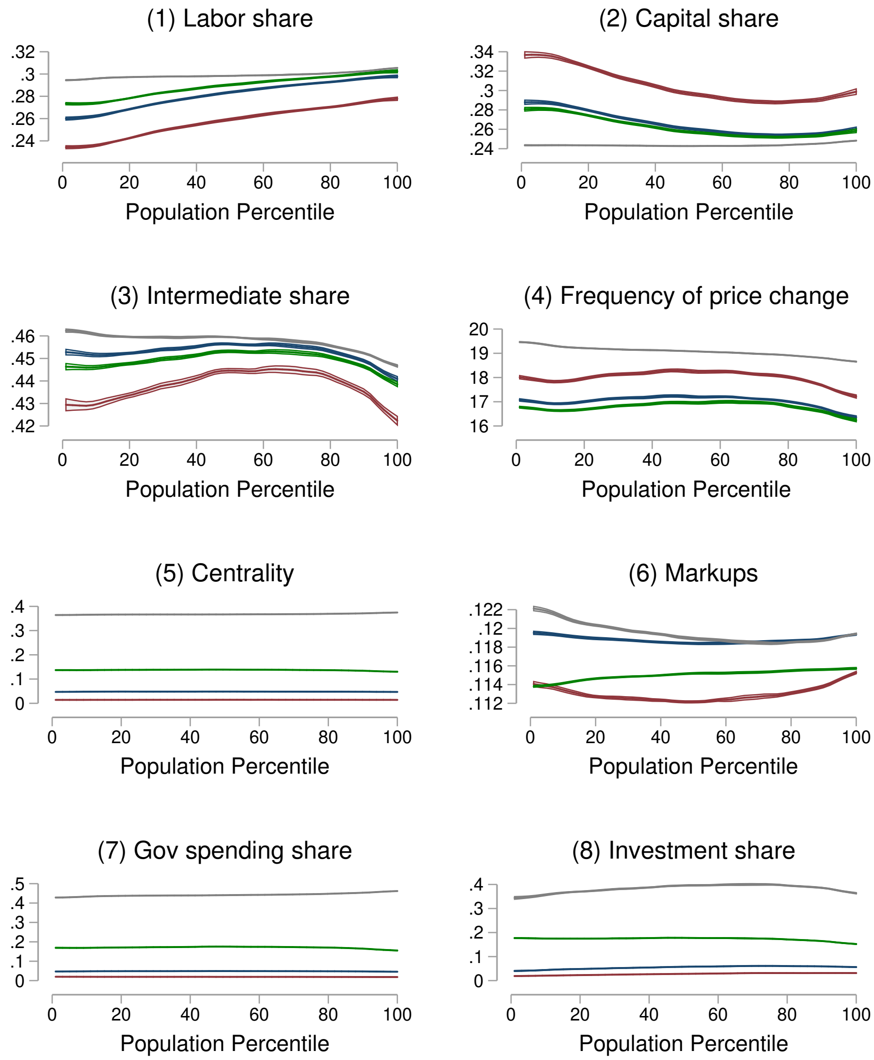

Figure 18. Expenditure-share Weighted Sectoral Heterogeneity for After-tax Income Percentiles with Different Levels of Aggregation

Note. We plot the expenditure-weighted sectoral features as a function of households' after-tax income percentile, average over the sample period. The horizontal axis for each panel is household percentiles, each representing 1% of the population. The vertical axis reports the average sectoral features, such as labor share, capital share, intermediate share, frequency of price change, centrality, markups, government spending share, and investment share, weighted by the expenditure share by households of the corresponding income percentile across 66 sectors in red, 22 sectors in blue, 9 sectors in green, and 3 sectors in gray.

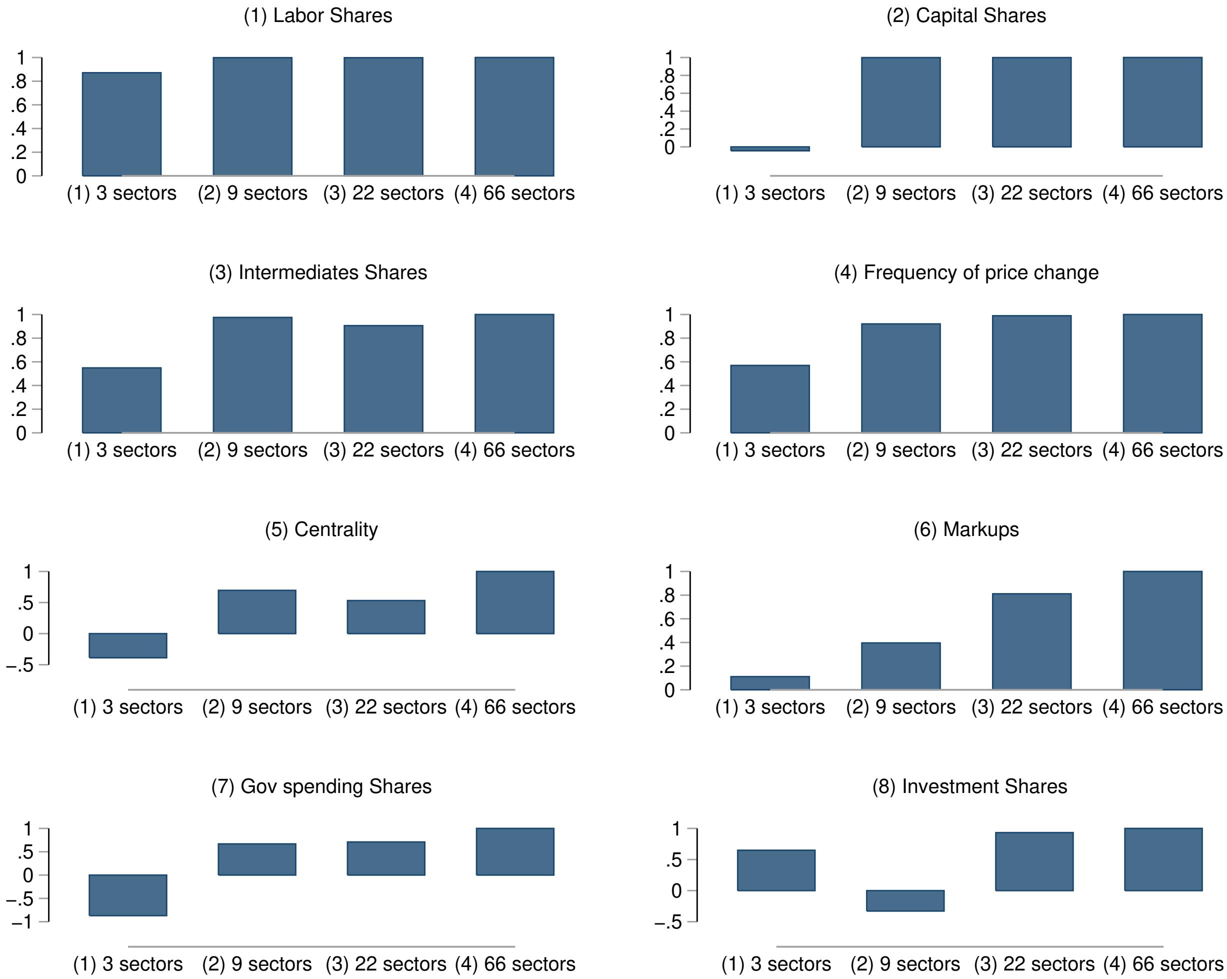

Figure 19. Correlation at Different Levels of Disaggregation Using CEX-IO

Note. We plot the correlation of corresponding expenditure-weighted sectoral features for each households' after-tax income percentiles, aggregating at full I-O sectors (66 sectors), 2-digit NAICS sectors (22 sectors), 1-digit NAICS sectors (9 sectors), and a manufacturing/agriculture/services split (3 sectors). We use the 66 sectors data points as the base.

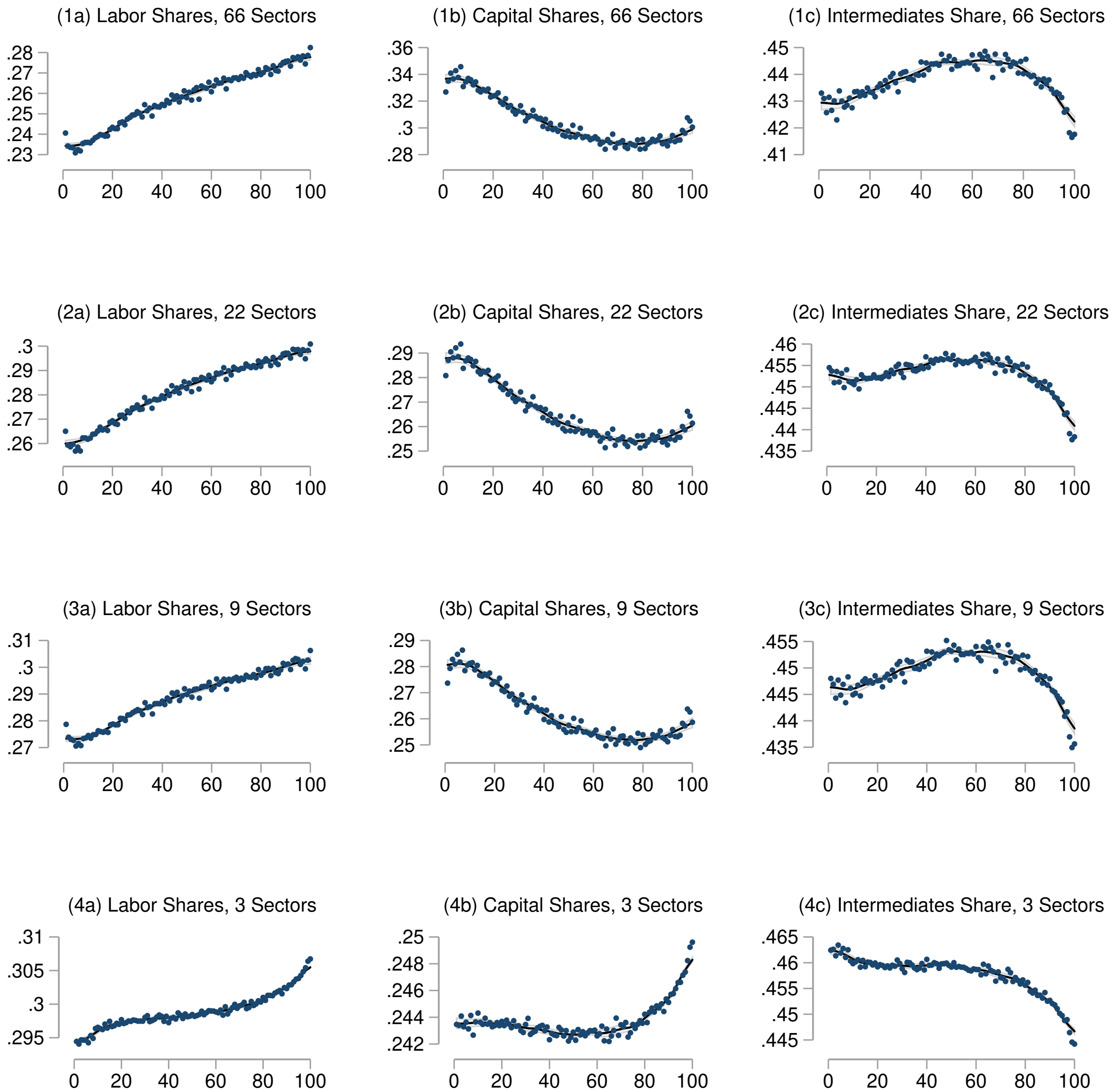

Figure 20. Robustness Checks for Expenditure-weighted Factor Shares Across Income

Note. We plot the expenditure-weighted factor shares (labor shares, capital shares, and intermediates shares) for 100 income percentiles across dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy. The line is the local polynominal fit, and the shaded area is the 95% confidence interval.

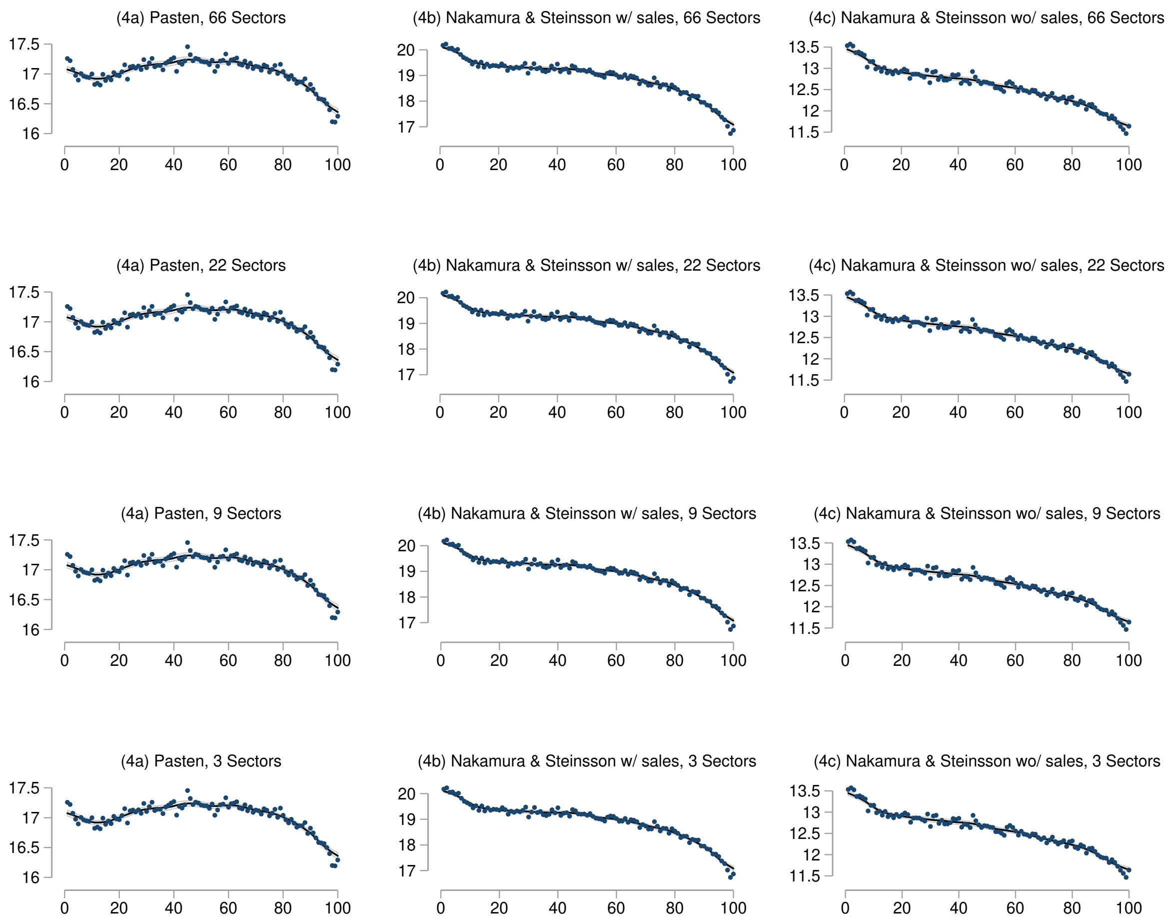

Figure 21. Robustness Checks for Expenditure-weighted Frequency of Price Change Across Income

Note. We plot the expenditure-weighted monthly frequency of price change for 100 income percentiles across two dimensions: different price rigidity measures and dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy. The line is the local polynominal fit, and the shaded area is the 95% confidence interval.

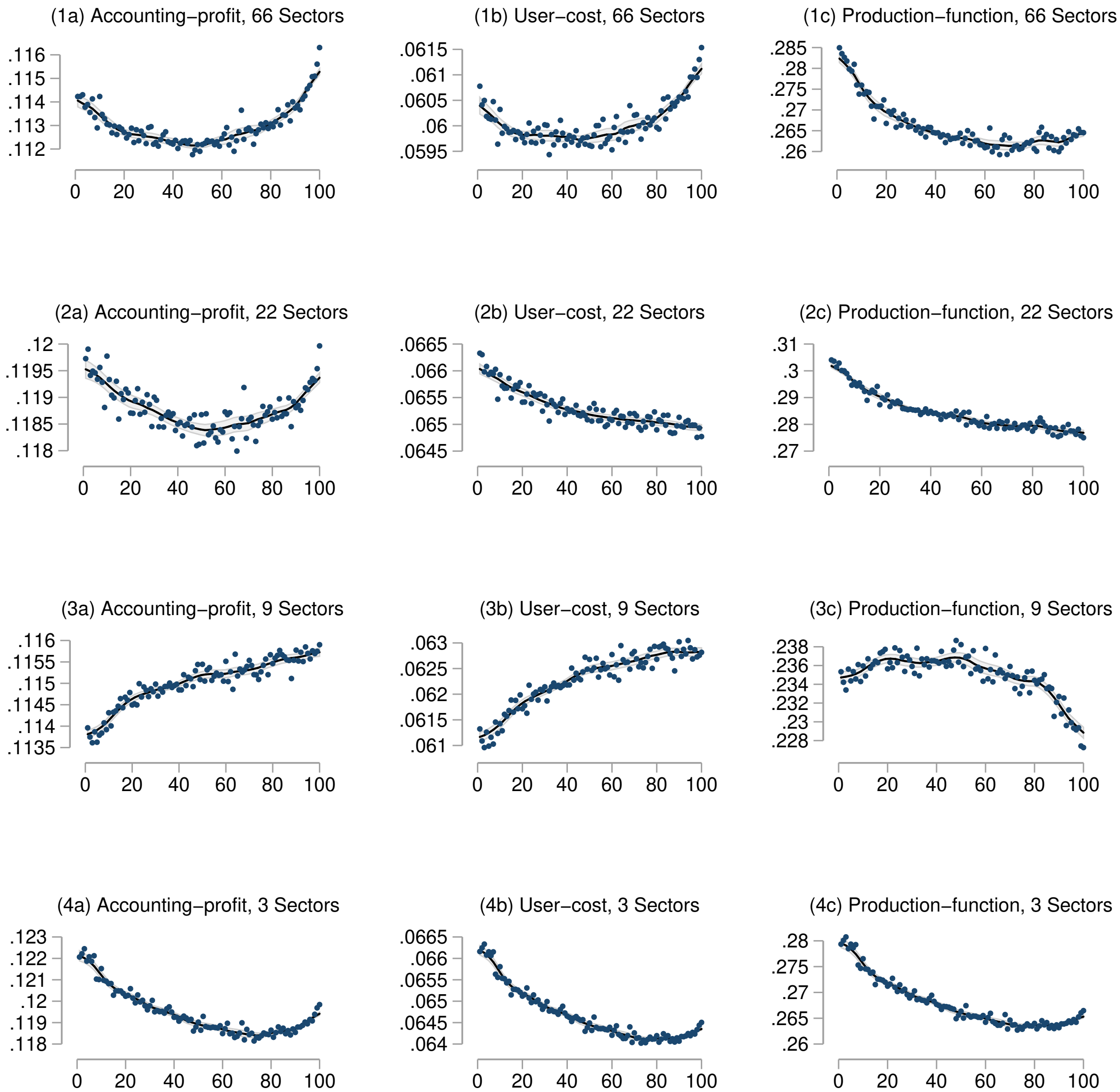

Figure 22. Robustness Checks for Expenditure-weighted Markups Across Income

Note. We plot the expenditure-weighted markups for 100 income percentiles across two dimensions: different markups measures and dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy. The line is the local polynominal fit, and the shaded area is the 95% confidence interval.

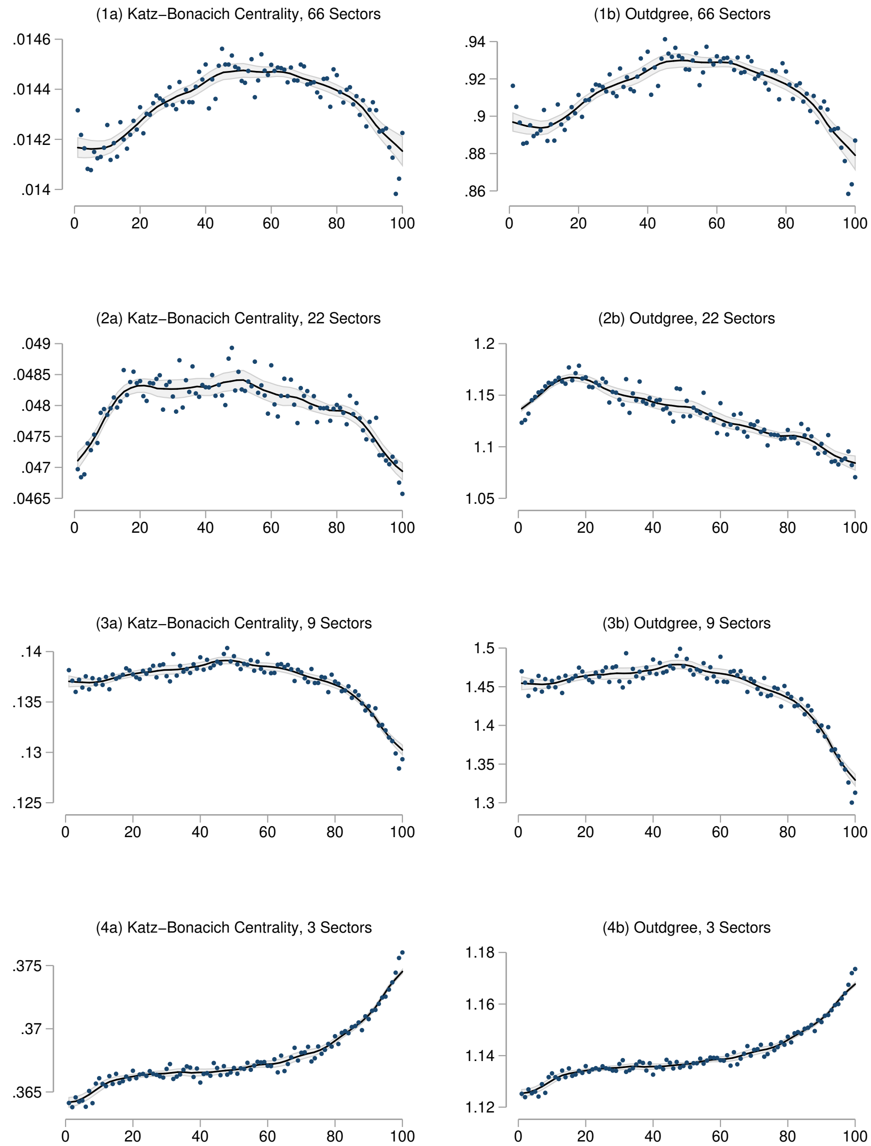

Figure 23. Robustness Checks for Expenditure-weighted Centrality Across Income

Note. We plot the expenditure-weighted centrality for 100 income percentiles across two dimensions: different production network centrality measures and dis-aggregation levels of the economy. Panel (1a) - (2a) are for 66-sector economy with two measures of centrality; panel (1b) - (2b) are for a 22-sector economy; panel (1c) - (2c) are for a 3-sector economy. The line is the local polynominal fit, and the shaded area is the 95% confidence interval.

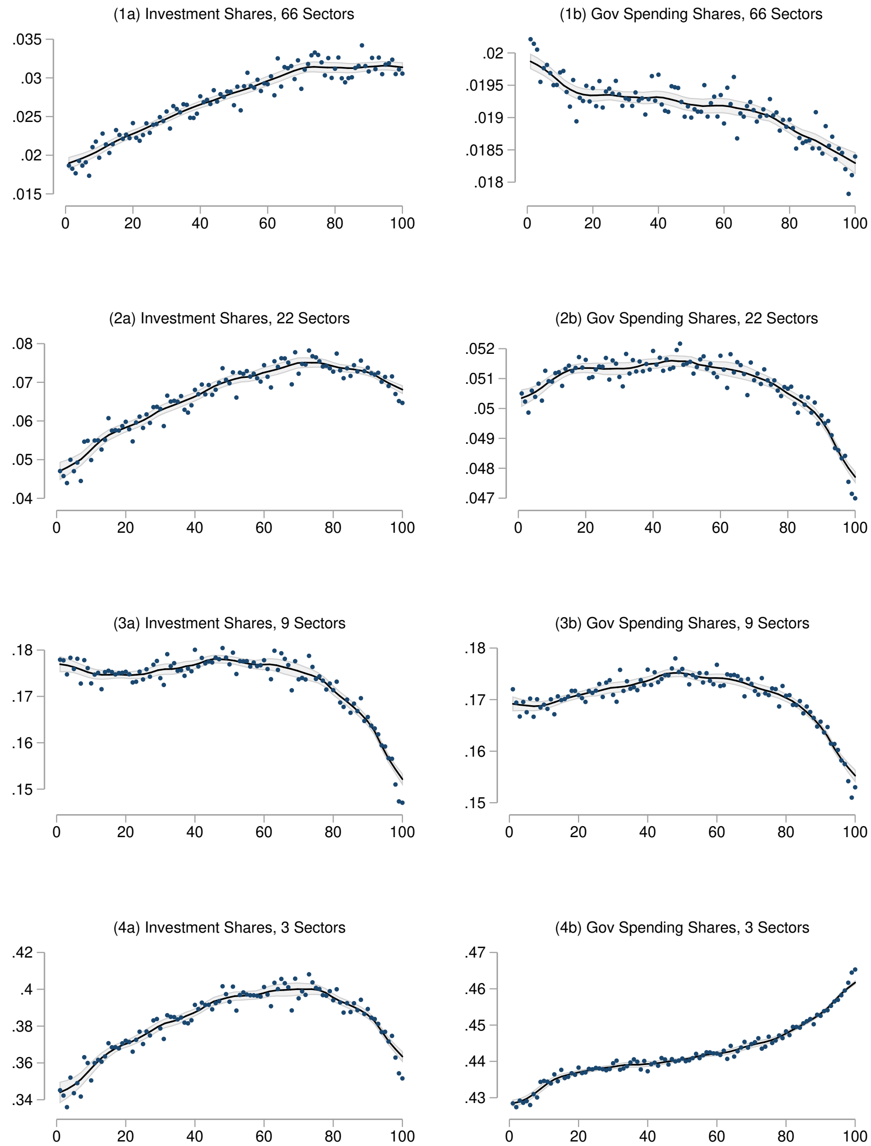

Figure 24. Robustness Checks for Expenditure-weighted Investment and Gov Spending Shares Across Income

Note. We plot the expenditure-weighted investment shares and government spending shares for 100 income percentiles across three dis-aggregation levels of the economy. Panel (1a) - (2a) are for 66-sector economy with investment and government spending shares respectively; panel (1b) - (2b) are for a 22-sector economy; panel (1c) - (2c) are for a 3-sector economy. The line is the local polynominal fit, and the shaded area is the 95% confidence interval.

E.2 Earning Heterogeneity using ACS-IO Dataset

Earnings-share weighted sectoral heterogeneity across different sample periods . In Figure 25 we show three sets of earnings-share weighted sectoral heterogeneity using three sample periods in a 22-sector economy: (1) from 2000-2004 in blue, (2) from 2005-2009 in red, and (3) from 2010-2015 in green. The relationship has been very robust across the time periods.

Earnings-share weighted sectoral heterogeneity for different levels of aggregation . Similar to the robustness check we have done for the CEX-IO Dataset, we show four sets of earnings-share weighted sectoral heterogeneity at different levels of aggregation for total income percentiles in Figure 26: at 66-sector level (the full BEA I-O) in red, 22-sector level (2-digit NAICS) in blue, 9-sector level (1-digit NAICS) in green, and 3-sector level (manufacturing, agriculture, and services) in gray. Figure 27 shows the correlation of sectoral features between different levels of aggregation versus the 66 sectors. The empirical regularities start to converge after a fair amount of disaggregation at 22 sectors.

Factor shares. Figure 28 plots the earnings-weighted three factor shares using with different levels of sectoral disaggregation: (1) labor shares, (2) capital shares, and (3) intermediates shares.

Monthly frequency of price change. Figure 29 plots the earnings-weighted monthly frequency of price change using three measures with different levels of sectoral disaggregation: (1) Pasten et al. (2017), (2) Nakamura and Steinsson (2008) with sales, and (3) Nakamura and Steinsson (2008) without sales.

Markups. Figure 30 plots the earnings-weighted markups using three markups measures with different levels of sectoral disaggregation: (1) accounting-profits markups (AP), (2) user-cost markups (UC), and (3) production-function markups (PF).

Centrality. Figure 31 plots the earnings-weighted markups using two measures with different levels of sectoral disaggregation: (1) Katz-Bonacich centrality and (2) outdegree.

Investment and Government Spending Shares. Figure 32 plots the earnings-weighted investment shares and government spending shares with different levels of sectoral disaggregation.

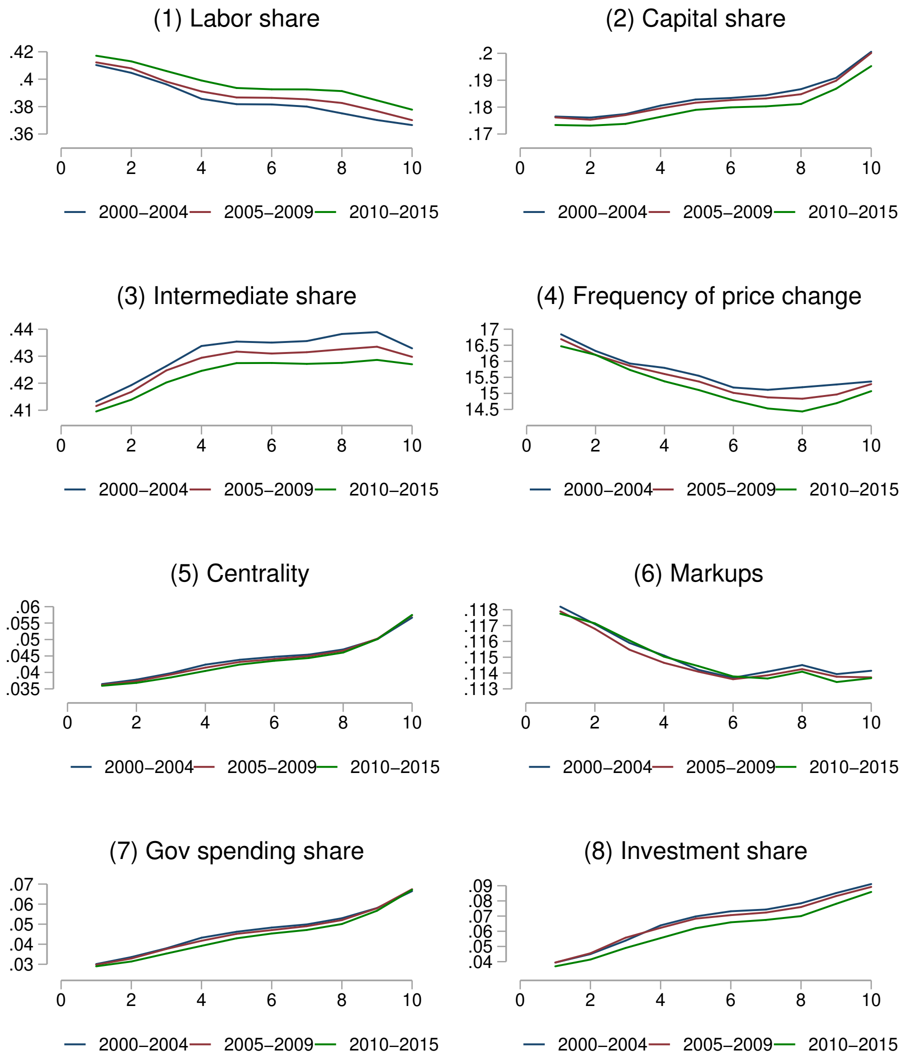

Figure 25. Earnings-share Weighted Sectoral Heterogeneity for Income Percentiles with Different Sample Periods in the 22-sector Economy

Note. We plot the earnings-weighted sectoral features as a function of households' total income percentile, average over the sample period (1) from 2000-2004 in blue, (2) from 2005-2009 in red, and (3) from 2010-2015 in green. The horizontal axis for each panel is household percentiles, each representing 10% of the population. The vertical axis reports the average sectoral features, such as labor share, capital share, intermediate share, frequency of price change, centrality, markups, government spending share, and investment share, weighted by the earnings share from 22 sectors by households of the corresponding income percentile.

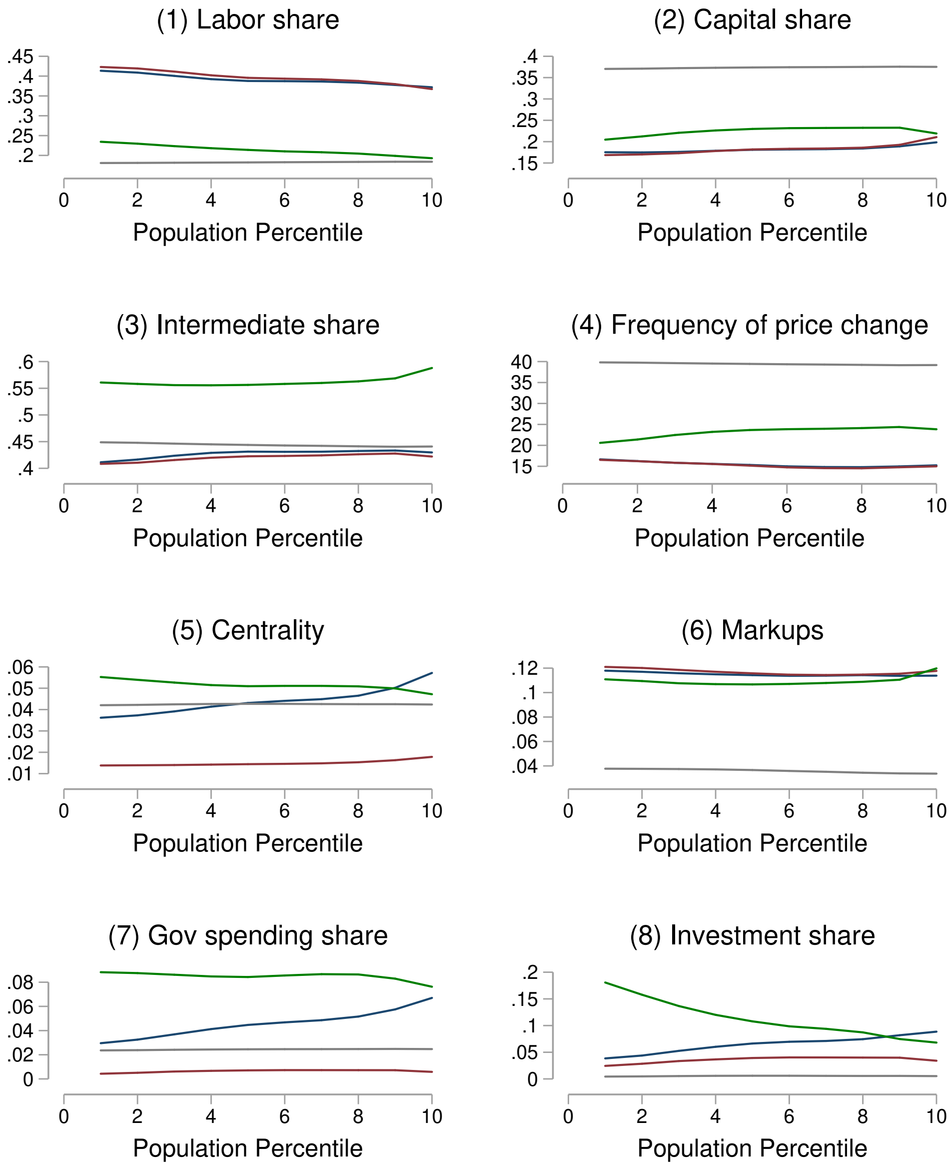

Figure 26. Earnings-share Weighted Sectoral Heterogeneity for Total Income Percentiles with Different Levels of Aggregation

Note. We plot the expenditure-weighted sectoral features as a function of households' after-tax income percentile, average over the sample period. The horizontal axis for each panel is household percentiles, each representing 1% of the population. The vertical axis reports the average sectoral features, such as labor share, capital share, intermediate share, frequency of price change, centrality, markups, government spending share, and investment share, weighted by the expenditure share by households of the corresponding income percentile across 66 sectors in red, 22 sectors in blue, 9 sectors in green, and 3 sectors in gray.

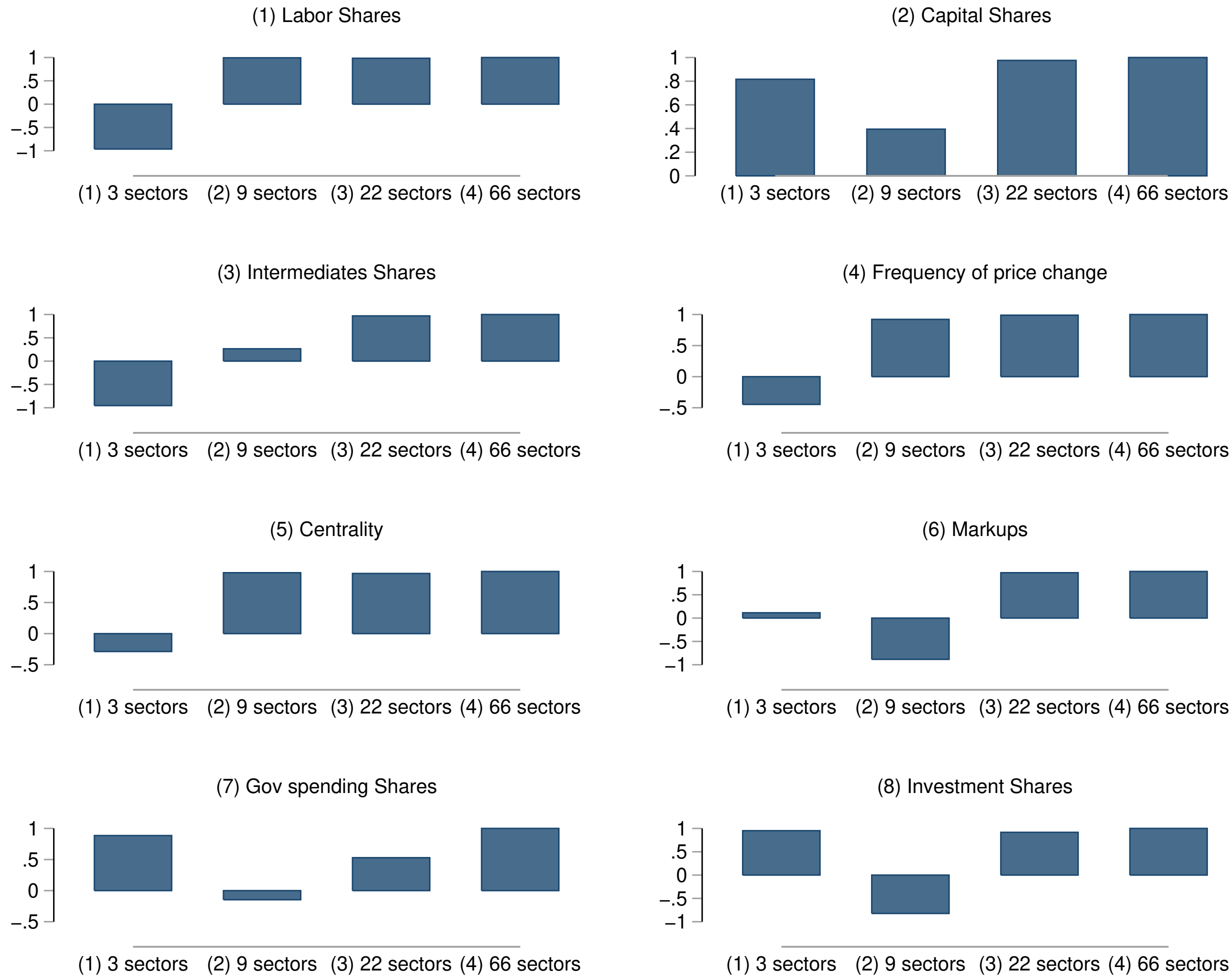

Figure 27. Correlation at Different Levels of Disaggregation Using ACS-IO

Note. We plot the correlation of corresponding earnings-weighted sectoral features for each households' total income percentiles, aggregating at full I-O sectors (66 sectors), 2-digit NAICS sectors (22 sectors), 1-digit NAICS sectors (9 sectors), and a manufacturing/agriculture/services split (3 sectors). We use the 66 sectors data points as the base.

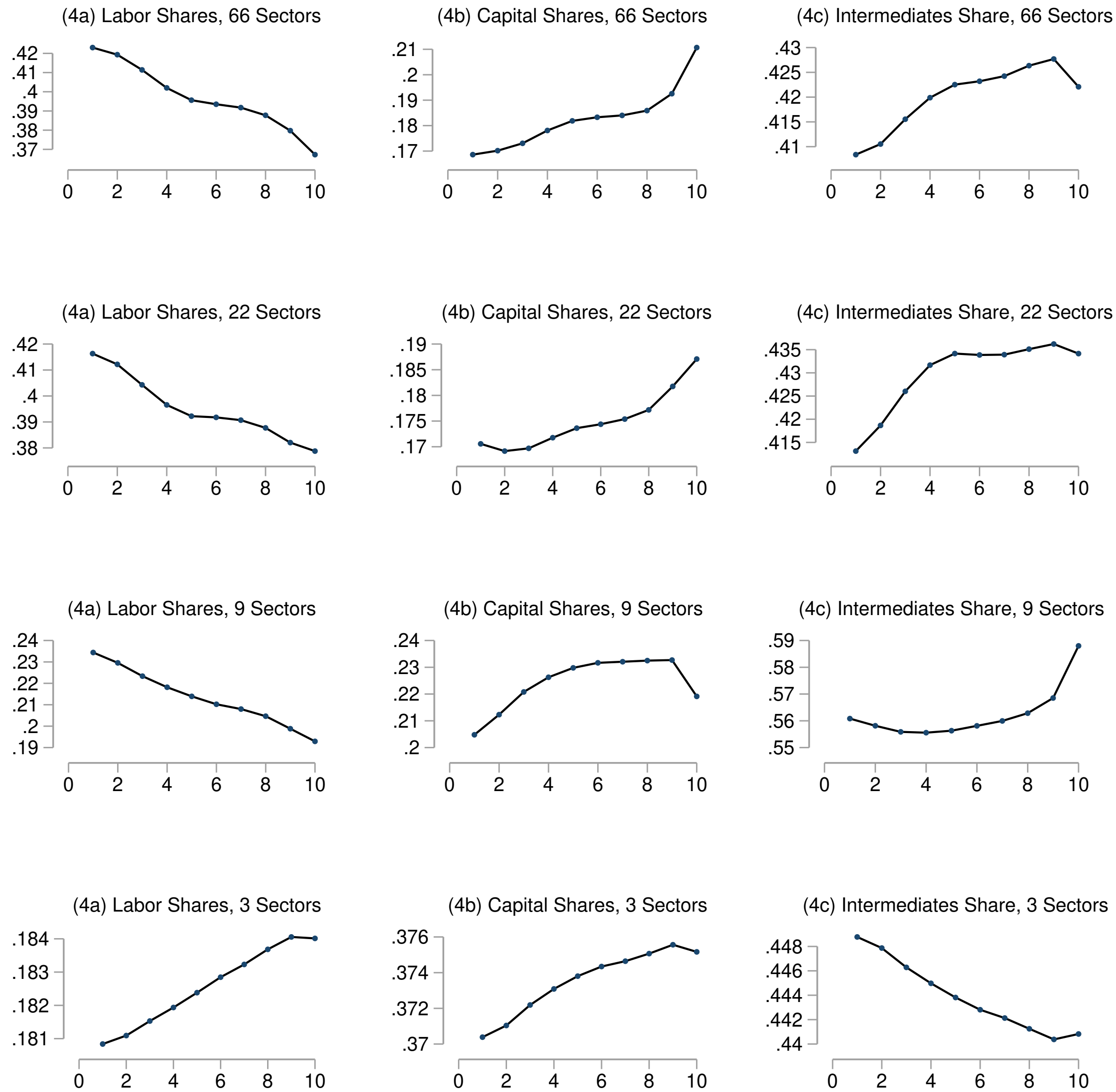

Figure 28. Robustness Checks for Earnings-weighted Factor Shares Across Income

Note. We plot the earnings-weighted factor shares (labor shares, capital shares, and intermediates shares) for 10 income percentiles across dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy.

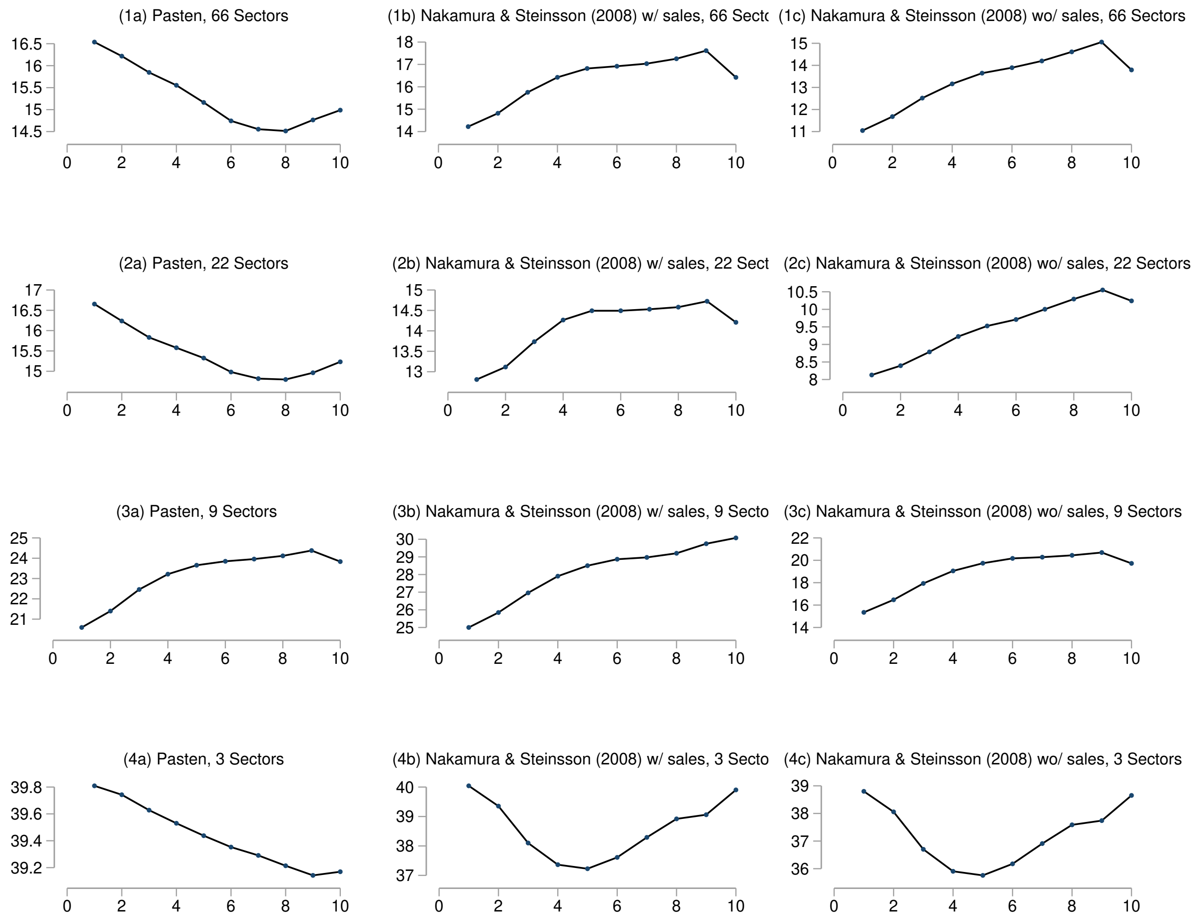

Figure 29. Robustness Checks for Earnings-weighted Frequency of Price Change Across Income

Note. We plot the earnings-weighted monthly frequency of price change for 10 income percentiles across two dimensions: different price rigidity measures and dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy.

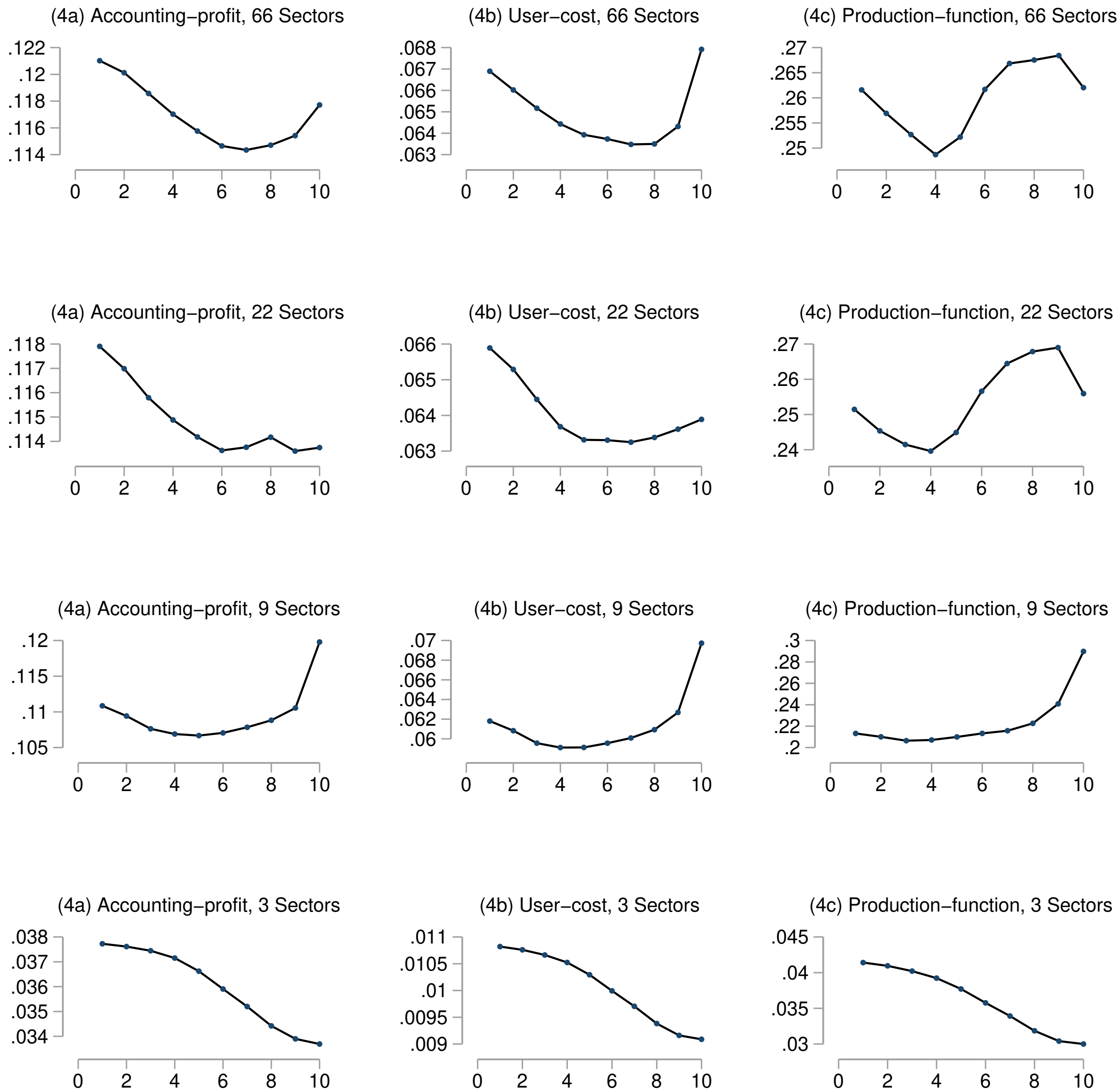

Figure 30. Robustness Checks for Earnings-weighted Markups Across Income

Note. We plot the earnings-weighted markups for 10 income percentiles across two dimensions: different markups measures and dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy.

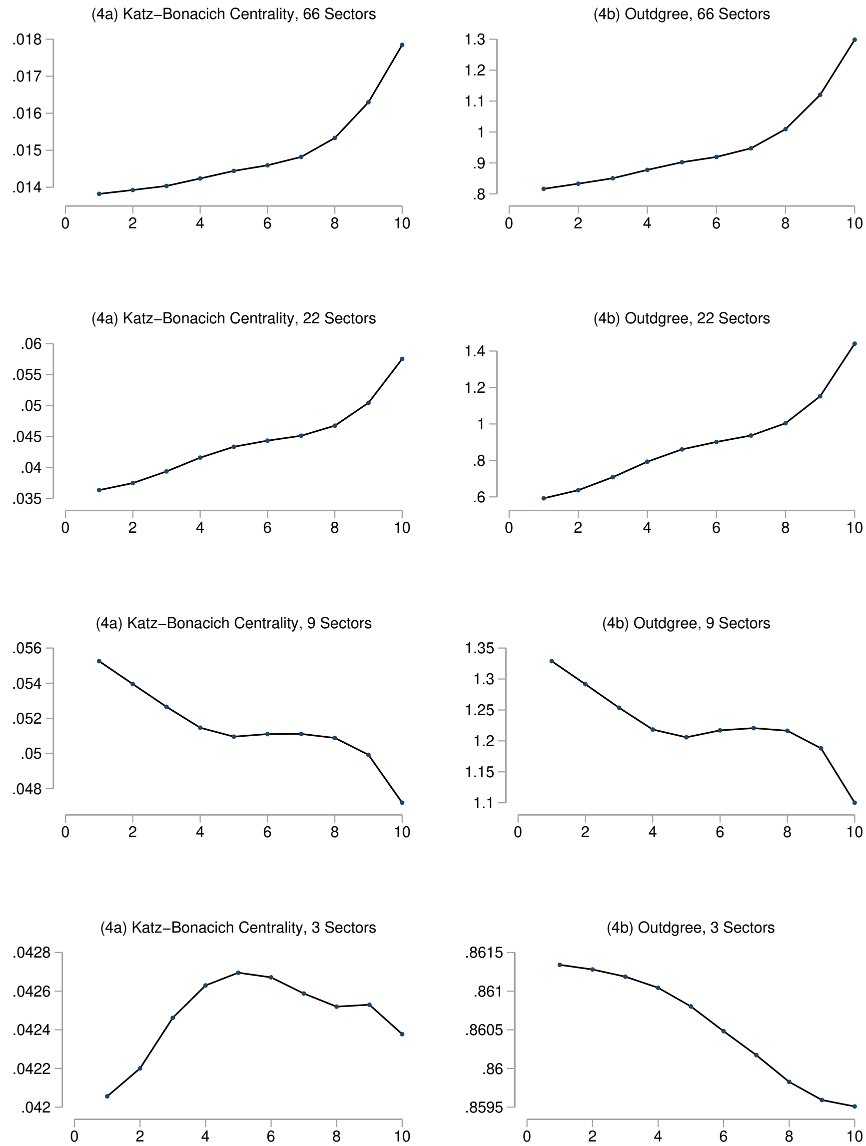

Figure 31. Robustness Checks for Earnings-weighted Centrality Across Income

Note. We plot the earnings-weighted centrality for 10 income percentiles across two dimensions: different production network centrality measures and dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy.

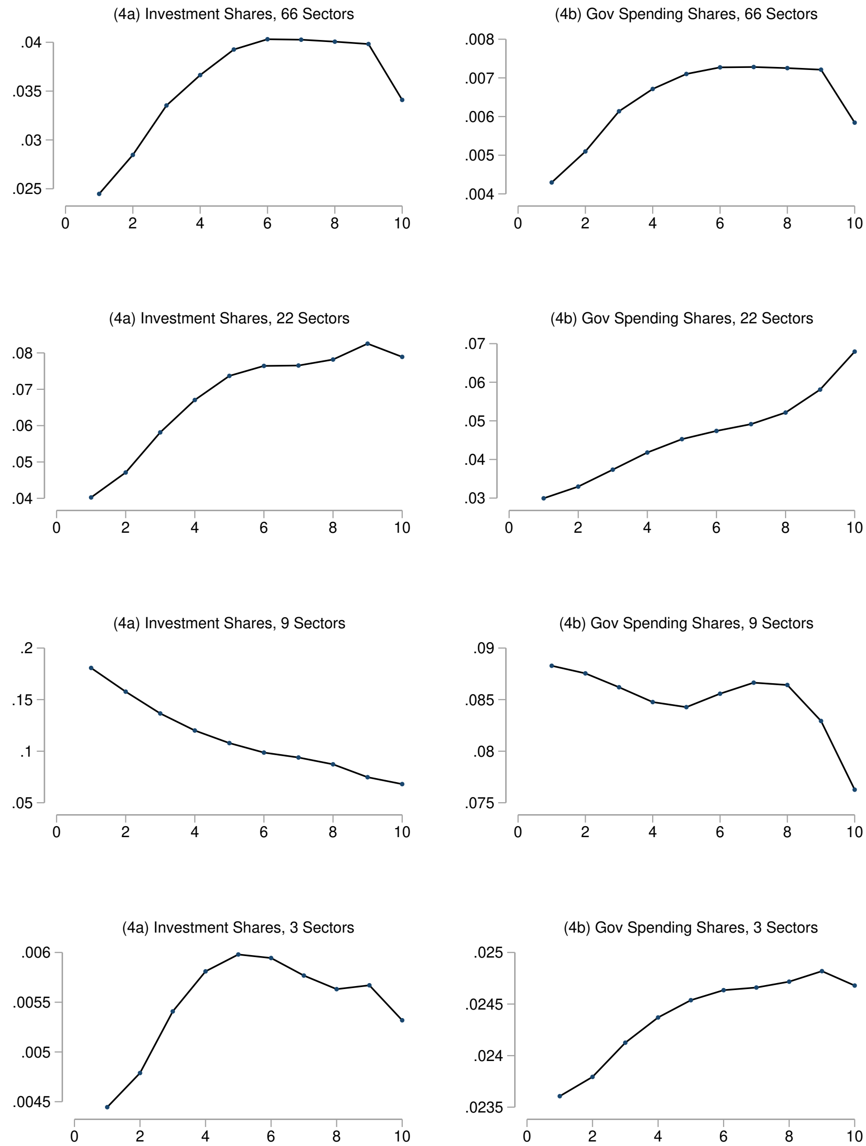

Figure 32. Robustness Checks for Earnings-weighted Investment and Gov Spending Shares Across Income

Note. We plot the earnings-weighted investment shares and government spending shares for 10 income percentiles across three dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy.

F Additional Details About nhCES

F.1 Theory and Notations

Preference. The preferences of a household of type i are ordered according to

$$\mathbb{E}_0 \int_0^\infty e^{-\int_0^t (\rho_i+\xi)ds} u(c_{i,t}) - \Phi\left(\left\{n_{ik,t}, \pi^w_{ik,t}\right\}_{k\in[0,1]}\right) dt,$$

where

$$c_{i,t} = \mathcal{D}_i^{NH}\left\{c_{i1,t},\, c_{i2,t},\, \ldots,\, c_{iN,t}\right\}$$

is a generalized nonhomothetic CES aggregator and $c_{ij,t}$ denotes the household i's consumption of goods from sector j at time t. $c_{i,t}$ depends on household's liquid asset holdings a, illiquid asset holdings b, and labor productivity z.

$n_{ik,t}$ is the labor hour supplied to union k. ρ is the discount rate. $\pi^w_{ik,t}$ is union k's wage inflation. Households die at rate ξ. The expectation operator is over future realizations of idiosyncratic earnings risk. We abstract from aggregate risk in this paper. Finally, we assume CRRA preferences, with

$$u(c_{i,t}) = \frac{c_{i,t}^{1-\gamma}}{1-\gamma}.$$

The cost function of labor hour and wage inflation, $\Phi(\cdot)$, will be discussed in more detail below.

Heterogeneous consumption baskets. The type-specific consumption aggregator is implicitly defined via

$$1 = \sum_j \left(\Omega_{i,j} c_{i,t}^{\varepsilon_j}\right)^{\frac{1}{\eta_c}} c_{ij,t}^{\frac{\eta_c-1}{\eta_c}}.$$

in which $c_{ij,t}$ denotes consumption by type i household of good produced in sector j, and $\Omega_{i,j}$ is the taste parameters of household i for good in sector j.

Nonhomothetic CES preferences still admit an ideal price index $P_{i,t}$, but it is now governed by additional economic effects. To derive this price index, consider the intratemporal cost minimization problem of the household type i

$$\min_{\{c_{ij,t}\}_j} \sum_j p_{j,t} c_{ij,t} - \phi_i \sum_j \left(\Omega_{i,j} j c_{i,t}^{\varepsilon_j}\right)^{\frac{1}{\eta_c}} c_{ij,t}^{\frac{\eta_c-1}{\eta_c}},$$

taking as given a desired level of real consumption $c_{i,t}$. The first-order condition yields

$$0 = p_{j,t} - \phi \frac{\eta_c - 1}{\eta_c} \left(\Omega_{i,j} c_{i,t}^{\varepsilon_j}\right)^{\frac{1}{\eta_c}} c_{j,t}^{\frac{-1}{\eta_c}}$$

or simply

$$c_{ij,t} = \Omega_{i,j} c_{i,t}^{\varepsilon_j} \left(\frac{\eta_c}{\eta_c - 1} \frac{p_{j,t}}{\phi_i} \right)^{-\eta_c}.$$

Plugging into the definition of $\mathcal{D}^{NH}$, we obtain

$$\phi_i^{1-\eta_c} = \sum_j \Omega_{i,j} \left(\frac{\eta_c}{\eta_c - 1} \right)^{1-\eta_c} p_{j,t}^{1-\eta_c} c_{i,t}^{\varepsilon_j}.$$

Different line of attack. The first-order condition can be written as

$$\phi_i \frac{\eta_c - 1}{\eta_c} \left(\Omega_{i,j} c_{i,t}^{\varepsilon_j} \right)^{\frac{1}{\eta_c}} c_{ij,t}^{\frac{\eta_c - 1}{\eta_c}} = p_{j,t} c_{ij,t}$$

Now summing across j, and defining the *expenditure share* $\omega_{ij,t} = (\Omega_{i,j} c_{i,t}^{\varepsilon_j})^{\frac{1}{\eta_c}} c_{ij,t}^{\frac{\eta_c - 1}{\eta_c}}$, we have

$$\phi_i \frac{\eta_c - 1}{\eta_c} = \sum_j p_{j,t} c_{ij,t} \equiv E_{i,t},$$

noting that by definition $\sum_j \omega_{ij,t} = 1$. We will also write $P_{i,t} c_{i,t} = E_{i,t}$. Thus, we have

$$c_{ij,t} = \Omega_{i,j} c_{i,t}^{\varepsilon_j} \left(\frac{p_{j,t}}{E_{i,t}} \right)^{-\eta_c} = \Omega_{i,j} \left(\frac{p_{j,t}}{P_{i,t}} \right)^{-\eta_c} c_{i,t}^{\eta_c + \varepsilon_j}.$$

This is the key equation for the intratemporal problem with nonhomothetic CES. Given parameters, sectoral prices $p_{j,t}$, and desired real consumption $c_{i,t}$, this equation defines the spending on each good that minimizes total expenditures while attaining the real consumption level.

Price index. The price level $P_{i,t}$ itself changes as real consumption $c_{i,t}$ changes due to a switching effect.

Plugging in for the optimal demand $c_{ij,t}$, we have

$$\begin{aligned} E_{i,t} = P_{i,t} c_{i,t} &= \sum_j \left\{ p_{j,t} \Omega_{i,j} \left(\frac{p_{j,t}}{P_{i,t}} \right)^{-\eta_c} c_{i,t}^{\eta_c + \varepsilon_j} \right\} \\ &= \sum_j \left\{ p_{j,t}^{1-\eta_c} P_{i,t}^{\eta_c} \Omega_{i,j} c_{i,t}^{\eta_c + \varepsilon_j} \right\} \end{aligned}$$

or simply

$$P_{i,t} = \left(\sum_j \Omega_{i,j} p_{j,t}^{1-\eta_c} c_{i,t}^{\varepsilon_j - (1-\eta_c)} \right)^{\frac{1}{1-\eta_c}}$$

which of course also implies

$$P_{i,t}c_{i,t} = \left(\sum_j \Omega_{i,j} p_{j,t}^{1-\eta_c} c_{i,t}^{\varepsilon_j}\right)^{\frac{1}{1-\eta_c}}.$$

Therefore, a household's consumption bundle price really takes the form

$$P_{i,t} = \mathcal{P}\left(\left\{p_{j,t}\right\}_j, c_{i,t}\right).$$

In particular, once we solve for the consumption policy function $c_{i,t}(a,b,z)$, the consumption basket price will take the form $P_{i,t}(a,b,z)$. In other words, every single household faces a different consumption price index!

Expenditure Shares. Furthermore, the expenditure shares can be expressed as

$$\omega_{ij,t} = \Omega_{ij,t}\left(\frac{p_{j,t}}{P_{i,t}}\right)^{1-\eta_c}\left(\frac{E_{i,t}}{P_{i,t}}\right)^{\varepsilon_j-(1-\eta_c)} \tag{31}$$

$$\sum_j \omega_{ij,t} = 1. \tag{32}$$

Elasticity of substitution between goods of different sectors. The nhCES has a constant elasticity of substitution between goods,

$$\frac{\partial \log\left(c_{ij,t}/c_{ik,t}\right)}{\partial \log\left(p_{k,t}/p_{j,t}\right)} = \eta_c, \quad \forall j,k \in \mathcal{N}, \quad \forall i \in \mathcal{I}.$$

Relative demand for goods of different sectors. The elasticity of the relative demand for two different goods with respect to aggregate real consumption $c_{i,t}$ is constant,

$$\frac{\partial \log\left(c_{ij,t}/c_{ik,t}\right)}{\partial \log c_{i,t}} = \varepsilon_j - \varepsilon_k, \quad \forall j,k \in \mathcal{N}, \quad \forall i \in \mathcal{I}.$$

The Hicksian demand for any pair of expenditure shares $\omega_{ij,t}$ and $\omega_{ik,t}$ satisfies

$$\log\left(\frac{\omega_{ij,t}}{\omega_{ik,t}}\right) = (1-\eta_c)\log\left(\frac{p_{j,t}}{p_{k,t}}\right) + \left(\varepsilon_j - \varepsilon_k\right)\log c_{i,t} + \log\left(\frac{\Omega_{ij,t}}{\Omega_{ik,t}}\right)$$

The first term on the right hand side shows the price effects characterized by a constant elasticity of substitution η_c, and the second term on the right hand side shows the change in relative sectoral demand as consumers move across indifference curves.

Marshallian demand. The income (expenditure) elasticity of demand for sectoral good j is given by

$$\eta_j \equiv \frac{\partial \log c_{i,t}}{\partial \log E_{i,t}} = \sigma + (1-\sigma)\frac{\varepsilon_j}{\bar{\varepsilon}}$$

where $\bar{\varepsilon} \equiv \sum_{j=1}^{J} \omega_j \varepsilon_j$ is the expenditure-weighted average of nonhomotheticity parameters. As Engel aggregation requires, the income elasticities average to 1 when sectoral weights are given by expenditure shares, $\sum_{j=1}^{S} \omega_j \boldsymbol{\eta}_j = 1$

Getting back to standard CES. And so if we set $\varepsilon_j = 1 - \eta_c$, we are back to the homothetic CES. In this case, the expenditure function becomes linear in the index of real consumption C, and the average cost of real consumption corresponds to the CES price index, $P = \left[\sum_{j=1}^{S} \Omega_i p_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$

Without loss of generality, we can normalize all the income elasticity and taste parameters such that those corresponding to a specific base good b equal a given arbitrary value, e.g., $\varepsilon_b = \Omega_b = 1$. Equation 31 allows us to write the real consumption as

$$\omega_{ib,t} = \left(\frac{p_{b,t}}{P_{i,t}}\right)^{1-\eta_c} (c_{i,t})^{\eta_c}$$

Therefore

$$\log c_{i,t} = (1-\eta_c) \log \left(\frac{E_{i,t}}{p_{b,t}}\right) + \log \omega_{ib,t} \tag{33}$$

Plugging in the Hicksian demand function, we get the moment condition used in GMM estimation,

$$\log \omega_{ij,t} = \log \Omega_{ij,t} + (1-\eta_c) \log \left(\frac{p_{j,t}}{p_{b,t}}\right) + (1-\eta_c)\left(\varepsilon_j - 1\right) \log \left(\frac{E_{i,t}}{p_{b,t}}\right) + \varepsilon_j \log \omega_{ib,t} \tag{34}$$

F.2 Estimation

The estimation process has two steps. First, we estimate the income elasticities of each sector $\varepsilon_{j,t}$, using the expenditure shares across households in the CEX dataset. Second, after obtaining the income elasticities, we estimate the consumer substitution elasticity η_c by regressing residual demand over relative price.

F.2.1 Income elasticities

The regression equation 34 can be re-written as

$$\log \left(\frac{\omega_{ij,t}}{\omega_{ib,t}}\right) = (1-\eta_c) \log \left(\frac{p_{j,t}}{p_{b,t}}\right) + (1-\eta_c)\left(\varepsilon_{j,t} - 1\right) \log \left(\frac{E_{i,t}}{p_{b,t}}\right) + \left(\varepsilon_{j,t} - 1\right) \log \omega_{ib,t} + \zeta_{ij,t} + \nu_{ij,t}$$

in which $\zeta_{ij,t} = \log \Omega_{ij,t}$ denotes the relative taste parameters which is a a linear function of the household profile vector $X_{i,t}$ including age, household size, and education. $\nu_{ij,t}$ is the error term.

Closely following Comin et al. (2021) when treating Indian data and [Hubmer (2022)], we assume that consumers face the same prices, conditional on time and control variables, as prices are absorbed by a sector-year fixed effect $\zeta_{j,t}$. I estimate

$$\log\left(\frac{\omega_{ij,t}}{\omega_{ib,t}}\right) = \zeta_{j,t} + \left(\varepsilon_{j,t} - \varepsilon_{b,t}\right)\log E_{i,t} + \Gamma'_{j,t}X_{j,t} + \nu_{ij,t}$$

for all $j \in S\backslash\{b\}$. The sector-time specific measurement error will be absorbed by $\zeta_{j,t}$. To address measurement error and endogeneity issues, we use after-tax income, education and occupation as instruments for total expenditure $E_{i,t}$. Our estimation results give us relative income elasticities ($\varepsilon_{j,t}$ - $\varepsilon_{b,t}$). We then recover all $\varepsilon_{j,t}$ by restricting their expenditure-weighted average to equal a constant. This constant is calibrated so that $\varepsilon_{j,t} > 0$ for all $j \in S$.

F.2.2 Substitution elasticity

In the second step of our estimation, we substitute in our baseline specification of income elasticities $\varepsilon_{j,t}$, and estimate substitution elasticity η_c from taking the difference of equation 34 with respect to time:

$$\Delta \log \omega_{j,t} = (1-\eta_c)\Delta \log\left(\frac{p_{j,t}}{P_t}\right) + \left(\varepsilon_{j,t} - 1\right)\Delta \log\left(\frac{E_t}{P_t}\right) + \Gamma'_{j,t}\Delta X_t + \nu_{ij,t}$$

in which, $\log \Omega_{ij,t}$ is assumed to be constant over time and therefore subsumed in taking difference. Our baseline calibration uses price indexes $p_{j,t}$ from the BEA GDP by Industry Table from 1997-2015. The aggregate price $P_t = \sum_j \omega_{j,t} p_{j,t}$. E_t is the aggregate expenditure by all households in year t.

F.2.3 Robustness

We use three measures to show that income elasticies are robust. First, we derive $\varepsilon_{j,t}$ for each year t using relative income elasticity $\varepsilon_{j,t} - \varepsilon_{b,t}$ for each year t and expenditure share $\omega_{j,t}$ for each year t. We take an average of the yearly data $\varepsilon_{j,t}$ and call it ε_j^{yearly}. This is our baseline. Second, we use the average of yearly relative income elasticities $\varepsilon_j^{average} - \varepsilon_b^{average}$ and the average expenditure share $\omega_j^{average}$ to derive ε_j^{mean}. Third, we use the constant income elasticity $\varepsilon_j - \varepsilon_b$ by performing the first-step estimation on all households data across the sample period. Together with the average expenditure share $\omega_j^{average}$, we obtain $\varepsilon_j^{constant}$.

Figure 33 plots the relationship of $\varepsilon_j^{constant}$ (Panel (A)), ε_j^{mean} (Panel (B)), yearly average of income elasticities between 1997-2006 (Panel (C)), yearly average of income elasticities between 2007-2015 (Panel (D)) and our baseline ε_j^{yearly}. It shows that income elasticities are stable over time and across measures.

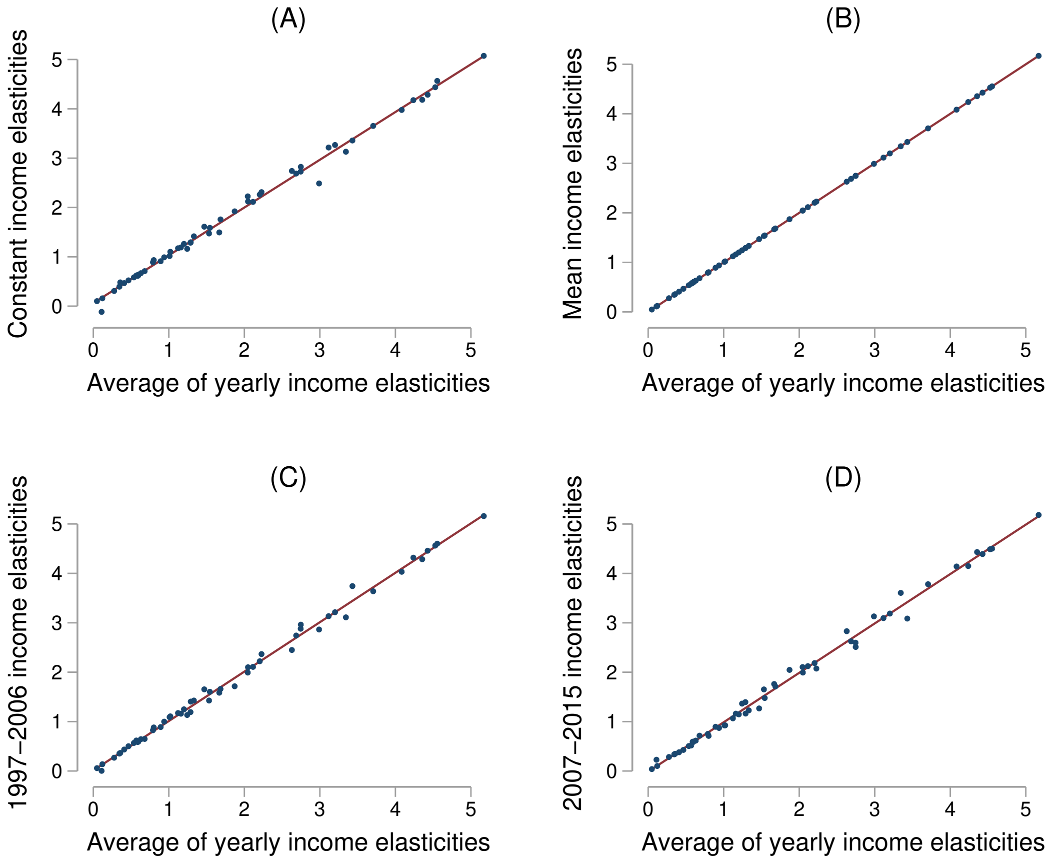

Figure 33. Robustness checks of income elasticities

Note. We plot the relationship of $\varepsilon_j^{constant}$ (Panel (A)), ε_j^{mean} (Panel (B)), yearly average of income elasticities between 1997-2006 (Panel (C)), yearly average of income elasticities between 2007-2015 (Panel (D)) and our baseline ε_j^{yearly}.