

Bail-Ins, Optimal Regulation, and Crisis Resolution

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Abstract

We develop a tractable dynamic contracting framework to study bank bail-in regimes. In the presence of a repeated monitoring problem, the optimal bank capital structure combines standard debt, which induces liquidation and provides strong incentives, and bail-in debt, which restores solvency but provides weaker incentives. When there are fire sales, optimal policy entails joint regulation: a bail-in regime reduces standard debt while leverage regulation reduces total debt. Bail-ins replace bailouts as a recapitalization tool.

1 Introduction

In the aftermath of the 2008 financial crisis, the question of orderly bank resolution has received significant attention on both sides of the Atlantic. In many advanced economies, governments employed bailouts to stem financial turbulence in late 2008 and early 2009.¹ Bailouts were arguably very effective at stabilizing financial markets, but have been criticized for leading to moral hazard and perverse redistribution.² As a result, the US (Title II of the Dodd-Frank Act) and the EU (Bank Recovery and Resolution Directive) have introduced “bail-ins,” which allow resolution authorities to impose haircuts on (long-term) debt holders. The goals of bail-in regimes include ensuring that “creditors and shareholders will bear the losses of the financial company” and that “[n]o taxpayer funds shall be used to prevent the liquidation of any financial company under [Title II]” (Dodd-Frank Act Sections 204, 214), and bail-in regulation was coupled with statutory provisions against bailouts.³ Nevertheless, important concerns emerge with the introduction of bail-ins. If bank solvency can be improved by introducing state contingencies into debt contracts, then what prevents banks from efficiently doing so using private contracts?⁴ Moreover, why are bail-ins preferable from a regulatory perspective to other liability instruments—such as (outside) equity—or to bailouts as a recapitalization tool? Studying these issues requires a framework in which bail-in debt is part of an optimal liability structure.

The main contribution of this paper is to provide a simple and tractable dynamic contracting model in which long-term bail-in debt is part of an optimal liability structure. The optimal contract of our model can be implemented with a combination of short-term standard debt and long-term bail-in debt. We show that in the presence of fire sales from liquidation, a social planner’s optimal

¹Two examples in the US are the Troubled Asset Relief Program (TARP), which authorized the government to buy toxic bank assets, and the Temporary Liquidity Guarantee Program (TLGP), which provided guarantees of bank debt.

²The Dodd-Frank Wall Street Reform and Consumer Act (Dodd-Frank Act) lists “protect[ing] the American taxpayer by ending bailouts” as one of its main objectives, and lists “minimiz[ing] moral hazard” (Section 204) as one of the purposes of bail-ins.

³See [Geithner \(2016\)](#) and [Labonte \(2020\)](#) for descriptions on limitations placed, for example, on the Fed, Treasury, and FDIC.

⁴For example, banks could use contingent convertible (CoCo) securities that have gained traction in Europe, which are an internal recapitalization instrument with a trigger event (for example, the bank’s capital ratio falling below some threshold) for either a principal write-down or a conversion into equity.

regulatory intervention entails supplementing a maximum leverage requirement with a total loss-absorbing capacity (TLAC) requirement that can be also satisfied with long-term bail-in debt. The social planner's optimal regulation replaces bailouts with bail-ins. Statutory provisions that increase the costs of engaging in ex post bailouts improve welfare.

Our three period model centers on a repeated incentive problem in the tradition of [Innes \(1990\)](#). Banks raise funds from investors ex ante to finance lending. They must exert monitoring effort at both the initial date (0) and the middle date (1) in order to ensure the quality of their loans at the onset of the lending relationship and in its continuation. Monitoring effort is not contractible at either stage, so that incentive compatibility constraints define the bank's choice of effort. Banks write optimal liability contracts in a complete markets setting, and can choose to liquidate their project early at date 1 and sell it to arbitrageurs. Fire sales arise when the liquidation price declines in the number of projects arbitrageurs must purchase.

Our main result is that the privately optimal bank contract can be implemented with a combination of two debt instruments: (short-term) standard debt and (long-term) bail-in debt. Standard debt has a face value that does not depend on the bank's date 1 return, forcing liquidation when bank returns are low.⁵ Intuitively, a bank that does not liquidate at date 1 would have to be paid a minimum agency rent in order to be willing to exert effort in continuation. Standard debt provides strong incentives to the bank for initial monitoring effort by threatening a liquidation in which the bank no longer needs to be paid this minimum continuation agency rent. Thus while standard debt provides strong incentives by ensuring the bank receives no payoff in bad states, it requires costly liquidation that reduces investor repayment. Bail-in debt, on the other hand, avoids the resource costs of liquidation. It provides weaker incentives, however, because it transfers all cash flows to investors except for the minimum agency rent. It therefore is a softer incentive device that avoids resource losses. Both instruments retain the upside for the bank, which encourages effort. Other instruments such as outside equity transfer cash flows from the bank to investors when returns are

⁵Our model does not differentiate between standard short-term debt and (uninsured) deposits, and standard debt could be interpreted as a deposit. It could also be interpreted as a repurchase agreement, where insolvency arises when the value of collateral falls sufficiently far that it no longer covers the debt.

high, and so discourage effort. The bank finds it optimal not to use such instruments.⁶

The second part of our contribution is to leverage our framework to study optimal policy. Individual banks fail to internalize that more liquidations can reduce the liquidation price due to fire sales. We study the problem of a social planner that can choose any feasible bank contract, subject to the same constraints as private agents but internalizing the fire sale externality. In principle, the social planner can incentivize banks to write any feasible contract, for example incorporating outside equity. However, we show that the social planner finds it optimal for banks to use a combination of (short-term) standard and (long-term) bail-in debt. The planner does not require banks to introduce other liability instruments such as outside equity into their capital structures. Intuitively, as with the private bank, the social planner also perceives bail-in debt to be better than outside equity for incentive provision. We show that the planner can implement bail-in debt with a resolution authority that imposes write downs ex post, resembling the structure of existing regimes such as Title II.

The social planner optimally intervenes in both the level and composition of debt due to the presence of the fire sale. First, for a given total amount of debt, the social planner on the margin prefers less use of standard debt and more of bail-in debt. This is because greater use of standard debt increases liquidations, contributing to a lower liquidation price. Second, the social planner also on the margin prefers less overall debt. Intuitively, issuing more external debt allows the bank to scale up, but also pledges more of the bank's upside to investors, reducing effort at date 0. Lower effort increases the probability of low returns and liquidation, and therefore further exacerbates fire sales. We show that the social planner's optimum can be implemented using the combination of a simple maximum leverage requirement that restricts total debt, and a TLAC requirement that can also be satisfied with long-term bail-in debt. A strength of our framework is its ability to rationalize both of these requirements from a single underlying externality.

The post-crisis regime has emphasized replacing bailouts with bail-ins. We leverage our framework to study this important question. We introduce taxpayer-financed bailouts that can be

⁶Although we frame our model around banks, the core optimal contracting framework can also apply to non-financial corporates. We provide an interpretation of bail-ins in our model as a Chapter 11 bankruptcy reorganization process, with liquidations corresponding to Chapter 7.

undertaken by the planner by paying a fixed cost. We first provide a benchmark under commitment, and show that the social planner's optimum is to adopt the same socially optimal contract as before, and commit not to engage in any bailouts. That is, bail-ins replace bailouts. Intuitively, bail-ins and bailouts can achieve the same state contingencies in bank debt contracts, so that bailouts are at best resource transfers from taxpayers to banks. Our model thus substantiates a core principle of post-crisis regulatory reform, namely that the costs of bank resolution should be borne by bank investors and not by taxpayers. We contextualize this result by comparing it to optimal policy in the context of the "pre-crisis world;" the planner did not require bail-in debt to be issued, and banks did not privately issue bail-in debt. We show how bailouts provided a costly method of implementing the social optimum, but that banks optimally respond by only issuing standard debt to capitalize on bailouts. This highlights both how bail-ins can replace bailouts, and also how the introduction of bail-in regimes has seemingly "introduced" bail-in debt into banks' capital structures.

Even without commitment over bailouts, we show that the social planner still replaces bailouts with bail-ins. In particular, the social planner optimally sets a sufficiently strict restriction on use of standard and total debt that no banks are bailed out in equilibrium. Because the temptation to engage in bailouts ex post restricts the amount of (both types of) debt the bank can issue without triggering bailouts, increasing the fixed cost of bailouts increases welfare. This rationalizes the statutory provisions designed to make bailouts more difficult that have accompanied the introduction of bail-in authority.

Finally, we discuss our model in the context of too-big-to-fail institutions and demand-based theories of standard debt. We argue that partial liquidations through a good bank/bad bank approach can be preferable to an all-or-nothing resolution approach for large banks. We compare our model to demand-based (safety premia) theories of debt, and discuss how a combination of the two theories can provide a more complete view of bank capital structure.

Related Literature. We relate to a growing literature on bail-ins.⁷ [Keister and Mitkov \(2021\)](#)

⁷There are also related literatures on contingent debt instruments ([Flannery 2002](#), [Raviv 2004](#), [Sundaresan and Wang 2015](#), [Pennacchi and Tchisty 2019](#), with [Flannery 2014](#) providing a broader overview) and optimal derivatives

show that banks may not write down their (deposit) creditors if they anticipate government bailouts, motivating mandatory bail-ins. [Chari and Kehoe \(2016\)](#) use a costly state verification model to show that standard debt is the only renegotiation-proof contract, implying that bail-ins serve only to reduce the level of standard debt. [Pandolfi \(2021\)](#) studies a related [Holmstrom and Tirole \(1997\)](#) incentive problem but takes standard debt contracts as given. The paper argues that bail-in resolution can lead to a credit market collapse by weakening incentives, thus motivating liquidations or partial bailouts. [Mendicino et al. \(2018\)](#) study the optimal TLAC composition for protecting insured deposits under private benefit taking and risk shifting, taking contracts as given. [Walther and White \(2020\)](#) show that precautionary bail-ins can signal adverse information and cause a bank run, resulting in an overly weak bail-in regime and motivating bail-in rules based on public information. [Colliard and Gromb \(2018\)](#) study how bail-ins and bailouts affect the negotiation process of distressed bank restructuring. [Bolton and Oehmke \(2019\)](#) study the trade-offs between single- and multi-point-of-entry resolution of global banks. [Dewatripont and Tirole \(2018\)](#) study how bail-ins can complement liquidity regulation. [Berger et al. \(2020\)](#) provide a quantitative analysis of bailouts versus bail-ins. Our main contribution is to develop a tractable dynamic contracting framework based on an incentive problem, in which the privately optimal contract can be implemented with a combination of standard and bail-in debt. We leverage this framework to study the optimal design of bail-in regimes and the role of bailouts in the pre- and post-crisis crisis response toolkits.

A vast literature studies theories of debt.⁸ Our paper is closely related to the dynamic optimal contracting literature that studies repeated unobservable effort.⁹ [DeMarzo and Sannikov \(2006\)](#) and [DeMarzo and Fishman \(2007\)](#) implement optimal contracts using combinations of long-term debt, credit lines, and equity. Credit lines provide firms with financial flexibility following bad returns, but are costly to revolve. This costly financial flexibility serves a similar role to our bail-in debt.

protection ([Biais et al. 2016](#), [Biais et al. 2019](#)).

⁸Apart from incentive problems, theories of debt include costly state verification ([Townsend 1979](#)), liquidity provision ([Diamond and Dybvig 1983](#)), and asymmetric information ([Myers and Majluf 1984](#), [Nachman and Noe 1994](#)). We also connect in particular to the related literature that emphasizes the monitoring role of banks ([Diamond 1984](#), [Holmstrom and Tirole 1997](#)).

⁹Relatedly, debt contracts that become *more* expensive to service (higher interest rate or coupon payment) have been emphasized to promote liquidation in settings with repeated cash flow diversion ([Biais et al. 2007](#)) and screening ([Manso et al. 2010](#)).

Long-term debt requires coupon payments, forcing default and liquidation once the credit line is exhausted. The liquidation threat parallels the role of our standard debt. Our paper contributes to this literature by incorporating its ingredients (repeated unobservable effort) and insights into a simple framework that we use to rationalize the coexistence of standard and bail-in debt, which are especially important in the bank regulatory context. We use our framework to study normative policy implications for the design of bail-in regimes. A number of papers separately emphasize the cash flow transfer (Jensen and Meckling 1976, Innes 1990, Dewatripont and Tirole 1994, Hébert 2018) and liquidation threat (Calomiris and Kahn 1991, Diamond and Rajan 2001) values of debt.¹⁰ Our paper is also related to Bolton and Scharfstein (1996), who study strategic default with cash flow diversion. They emphasize the value of easy-to-renegotiate debt for preventing non-strategic default and hard-to-renegotiate debt for preventing strategic default.

A large literature studies macroprudential regulation in the presence of pecuniary externalities (Bianchi and Mendoza 2010, Bianchi and Mendoza 2018, Caballero and Krishnamurthy 2001, Dávila and Korinek 2018, Farhi et al. 2009, Lorenzoni 2008), aggregate demand externalities (Farhi and Werning 2016, Korinek and Simsek 2016, Schmitt-Grohé and Uribe 2016), and fiscal externalities (Chari and Kehoe 2016, Farhi and Tirole 2012), which motivate ex ante interventions such as leverage requirements. Our model rationalizes interventions that jointly regulate the level and composition of debt, providing a simultaneous role for both leverage regulation and a TLAC requirement that can be satisfied with bail-in debt.

2 Model

The three-period economy, $t = 0, 1, 2$, has a unit continuum of banks, investors, firms, and arbitrageurs. Banks are run by their owners. Banks invest in a firm of variable scale $Y_0 = A_0 + I_0 > 0$ by using their own funds (inside equity), $A_0 > 0$, and by signing contracts with investors to raise $I_0 \geq 0$.

¹⁰In similar spirits, Philippon and Wang (2022) studies use of bailout tournaments to provide equity-like incentives for lower risk taking while Zentefis (2021) studies disciplining effects of bailouts accompanied with managerial equity stake dilutions.

Investors are deep-pocketed at date 0 and can finance any investment scale. Firms are penniless and have an outside option of zero. We allocate the entire value of the bank-firm lending relationship to the bank. Arbitrageurs buy projects (firms) that are liquidated prior to maturity.

Banks and investors are risk-neutral and do not discount the future. The economy features idiosyncratic uncertainty, but for simplicity features no aggregate uncertainty.

2.1 Bank Projects

Banks extend financing to firms, thereby establishing a lending and monitoring relationship with those firms. When first extending funds to firms, banks monitor their borrowers, ensuring that the projects undertaken are of good quality. In doing so, banks develop specialized knowledge of that firm, and are uniquely able to monitor and collect from the firm in continuation. This relationship is the foundation of banking in our model. Because we allocate all value of the lending relationship to the bank, we omit firms going forward and refer to the relationship as bank projects.

Our model proceeds similarly to a multi-period version of [Innes \(1990\)](#). At each of dates 1 and 2, the bank experiences a stochastic quality shock $R_t \in [\underline{R}, \bar{R}]$, which adjusts the project scale to $Y_t = R_t Y_{t-1}$. This means the final project scale is $Y_2 = R_2 R_1 Y_0$. The project pays off one unit of the consumption good per unit of final scale if held to maturity at date 2, but yields no dividend at date 1. The shocks R_t are independent and idiosyncratic, with densities $f_t(R_t | e_{t-1}) = e_{t-1} f_{tH}(R_t) + (1 - e_{t-1}) f_{tL}(R_t)$. Both states R_t are contractible, but the distribution of R_t depends on the bank's non-contractible monitoring effort at the prior date, e_{t-1} . Date 0 effort is continuous, $e_0 \in [0, 1]$, while date 1 effort is binary, $e_1 \in \{0, 1\}$. Higher values e_{t-1} indicate greater effort (“working”) and lower values indicate lower effort (“shirking”). We think of e_0 as an initial monitoring/screening of borrowers and e_1 as continued due diligence and collections. For expositional convenience, we normalize $\mathbb{E}[R_2 | e_1 = 1] = 1$.

Higher effort is valuable because it improves returns, but shirking yields a private benefit to bankers. We assume that f_t satisfies the *monotone likelihood ratio property* (MLRP).

Assumption 1. Defining the likelihood ratio $\Lambda_t(R_t) \equiv \frac{f_{tH}(R_t)}{f_{tL}(R_t)}$, then $\Lambda_t' > 0$ on its support.

All enumerated assumptions are maintained throughout the paper. MLRP is a standard assumption in generating debt contracts, and implies that high (low) returns are a signal that the bank exerted high (low) monitoring effort. The banker's date 0 private benefit is $B_0(e_0)Y_0$, where B_0 is decreasing and concave with $B_0(1) = 0$. The banker's date 1 private benefit is $(1 - e_1)B_1Y_1$ for $0 < B_1 < 1$.

Banks can liquidate their project prematurely at date 1, after observing R_1 but prior to exerting effort at date 1. If the project is liquidated, it yields γY_1 units of the consumption good at date 1 and nothing at date 2. Banks take the endogenous liquidation price $\gamma \in [\underline{\gamma}, \bar{\gamma}] \subset (0, 1)$ as given. We denote $\alpha(R_1) \in \{0, 1\}$ the liquidation choice, with $\alpha(R_1) = 1$ being liquidation. We will later introduce an assumption to ensure that liquidations are ex post inefficient in that they reduce payoffs available to both banks and investors, which will take the form of an upper bound on $\bar{\gamma}$ (see Assumption 4 below).

We make the following assumptions on bank projects.

Assumption 2. The bank projects satisfy:

- (a) $\frac{d}{de_0} [\mathbb{E}[R_1|e_0] + B_0(e_0)] > 0$ and $\mathbb{E}[R_2|e_1 = 0] + B_1 < 1$
- (b) $(1 - B_1)\mathbb{E}[R_1|e_0 = 1] < 1$
- (c) $F_{2H}(1 - B_1) = 0$
- (d) $-B_0'(1) \leq \bar{B}_0$, where \bar{B}_0 is defined in Appendix A.1.
- (e) $|B_0''| \geq \underline{B}_0'' > 0$, where \underline{B}_0'' is defined in Appendix A.1.
- (f) $\mathbb{E}[R_2|e_1 = 0] < \underline{\gamma}$

Part (a) of Assumption 2 ensures that higher effort levels increase total project value (including private benefits). Part (b) ensures finite project scale. Part (c) ensures that bail-in debt can be implemented as long-term debt. Part (d) rules out liquidating dividends being part of the optimal contract. Part (e) guarantees that a bank liquidation generates negative externalities by lowering the

equilibrium liquidation price (for example, under quadratic effort costs this is a lower bound on the magnitude of the coefficient on the quadratic term). Part (f) assumes that the expected date 2 return under low effort is lower than the lowest possible liquidation price, which means that funds available to repay investors are lower under date 1 low effort than under liquidation. This will ensure that the privately optimal contract induces high effort at date 1.

2.2 Bank and Investor Payoffs

In order to raise investment $I_0 \geq 0$ at date 0, the bank offers a contract to investors that specifies: (i) state-contingent investor repayment $x_1(R_1)$ at date 1 and $x_2(R_1, R_2)$ at date 2; (ii) a state-contingent liquidation rule $\alpha(R_1)$; (iii) state-contingent bank payoff $c_1(R_1), c_2(R_1, R_2)$.¹¹ It is important to highlight that the state-contingent liquidation rule $\alpha(R_1)$ contractually specifies after which return realizations R_1 the bank is liquidated, although in the implementation of our optimal contract liquidations will be forced by short-term standard debt that cannot be fully repaid. Without loss of generality, we define $c_2 = x_2 = 0$ in liquidation (i.e., when $\alpha(R_1) = 1$). Along a history R_1, R_2 , investor repayment and bank consumption must satisfy the resource constraints

$$c_1(R_1) + x_1(R_1) = \alpha(R_1)\gamma R_1 Y_0, \quad (1)$$

$$c_2(R_1, R_2) + x_2(R_1, R_2) = (1 - \alpha(R_1))R_1 R_2 Y_0. \quad (2)$$

Contracts are subject to limited liability constraints for banks, given by

$$c_1(R_1), c_2(R_1, R_2) \geq 0. \quad (3)$$

Limited liability is not required for investors, although in general the optimal contract can be implemented with non-negative payoffs $x_1(R_1), x_2(R_1, R_2) \geq 0$.

The bank's effort levels are chosen after contracts are signed (see Section 2.3), but both banks

¹¹ x_t is the actual amount received by investors, and is distinct from the face value of liabilities (that is, promised repayment). We set up the problem in terms of actual repayment, and later map it into promised liabilities.

and investors infer the effort levels the bank will choose from the contract signed. We denote these effort levels $e_0^*, e_1^*(R_1)$ and leave their dependency on the contract signed implicit.

Bank Payoffs The total payoff to the bank along history (R_1, R_2) , given a contract and effort choices $e_0^*, e_1^*(R_1)$, is

$$c_1(R_1) + c_2(R_1, R_2) + B_0(e_0^*)Y_0 + (1 - \alpha(R_1))(1 - e_1^*(R_1))B_1R_1Y_1. \quad (4)$$

Investor Payoffs and Participation Constraint. We define the date 1 expected payoff to investors as

$$x(R_1 | e_1^*, \alpha, x_1, x_2) = \begin{cases} x_1(R_1) + \mathbb{E} \left[x_2(R_1, R_2) \middle| e_1 = e_1^*(R_1) \right], & \alpha(R_1) = 0 \\ x_1(R_1), & \alpha(R_1) = 1 \end{cases}. \quad (5)$$

To streamline notation, we use the notational shorthand $x(R_1) = x(R_1 | e_1^*, \alpha, x_1, x_2)$, leaving implicit that the function x depends on $(e_1^*, \alpha, x_1, x_2)$. The lifetime expected payoff to investors from the contract is $-I_0 + \mathbb{E}[x(R_1) | e_0 = e_0^*]$. The voluntary investor participation constraint states that investors must at least break even in expectation on the contract they signed. Since $I_0 = Y_0 - A_0$, it is therefore given by

$$Y_0 - A_0 \leq \mathbb{E}[x(R_1) | e_0 = e_0^*]. \quad (6)$$

It is important to highlight that the participation constraint depends on investors' expectations of the effort levels $e_0^*, e_1^*(R_1)$ that the bank will choose (recall that $x(R_1)$ depends on $e_1^*(R_1)$).

Finally, we assume repayment monotonicity at both dates: $x(R_1)$ must be monotone in R_1 (i.e., expected investor repayment is monotone in R_1) and $x_2(R_1, R_2)$ must be monotone in R_2 (for given R_1). Formally, these assumptions are

$$R_1 \geq R_1' \Rightarrow x(R_1) \geq x(R_1'), \quad (7)$$

$$R_2 \geq R_2' \Rightarrow x_2(R_1, R_2) \geq x_2(R_1, R_2'). \quad (8)$$

Monotonicity is a common assumption in many settings of optimal contracts or security design. It generates the flat face value of liabilities in high-return states.¹²

2.3 Banker Effort and Incentive Compatibility

Because monitoring effort is non-contractible, the bank sequentially chooses effort to maximize its utility after contracts have been signed. This gives rise to incentive compatibility constraints. We proceed by backward induction.

Date 1 Effort Choice. At date 1, after R_1 is realized and assuming the bank is not liquidated, the bank optimally chooses date 1 effort $e_1(R_1) \in \{0, 1\}$ to maximize its payoff. We say that high effort is *incentive compatible* at date 1 if bank payoff from $e_1(R_1) = 1$ is higher than from $e_1(R_1) = 0$, that is

$$\mathbb{E} \left[c_2(R_1, R_2)(\Lambda_2(R_2) - 1) \middle| e_1 = 0 \right] \geq B_1 R_1 Y_0. \quad (9)$$

Note that the conditioning is on $e_1 = 0$ because we have defined the likelihood ratio as $\Lambda_2 = \frac{f_{2H}}{f_{2L}}$. Since Λ_2 is increasing (MLRP), higher payoffs $c_2(R_1, R_2)$ when R_2 is high increase incentives for high effort, whereas higher payoffs when R_2 is low reduce incentives for effort.

The bank's optimal effort choice at date 1 is $e_1^*(R_1) = 1$ if equation (9) is satisfied, and is $e_1^*(R_1) = 0$ if equation (9) is violated.

¹²For example, one justification offered is that banks would be incentivized to pad their returns, for example by secretly borrowing from a third party (Nachman and Noe 1990, Nachman and Noe 1994). Absent monotonicity, the optimal contract would have a live-or-die feature at the top. Note that monotonicity does not preclude a bank from issuing an individual instrument whose payoff profile is non-monotone, but rather states that the overall structure summed across instruments must be monotone.

Date 0 Effort Choice. Define the bank's date 1 expected total payoff (including the date 1 private benefit) as

$$c(R_1|e_1^*, \alpha, c_1, c_2, Y_0) = \begin{cases} c_1(R_1) + \mathbb{E} \left[c_2(R_1, R_2) + (1 - e_1^*(R_1))B_1 R_1 Y_0 \middle| e_1 = e_1^*(R_1) \right], & \alpha(R_1) = 0 \\ c_1(R_1), & \alpha(R_1) = 1 \end{cases} \quad (10)$$

We adopt the notational shorthand $c(R_1) = c(R_1|e_1^*, \alpha, c_1, c_2, Y_0)$ for convenience. Note that c does not depend on e_0^* . At date 0, the bank's optimal date 0 effort choice solves

$$\max_{e \in [0, 1]} \mathbb{E} \left[c(R_1) \middle| e_0 = e \right] + B_0(e)Y_0. \quad (11)$$

Therefore, date 0 optimal effort e_0^* is given by the equation¹³

$$-B_0'(e_0^*)Y_0 = \mathbb{E}_0 \left[c(R_1)(\Lambda_1(R_1) - 1) \middle| e_0 = 0 \right]. \quad (12)$$

From MLRP and since $-B_0$ is increasing and convex, optimal bank effort e_0^* is higher when the bank receives a higher expected payoff $c(R_1)$ following high returns R_1 , and lower when expected payoff is high following low returns.

2.4 Bank Optimal Contracting

The bank takes the liquidation price γ as given and signs a contract with investors, which we enumerate in full by $\mathcal{C} = \{\alpha(R_1), x_1(R_1), x_2(R_1, R_2), c_1(R_1), c_2(R_1, R_2), e_0^*, e_1^*(R_1), I_0, Y_0\}$. The contract must be feasible, defined below.

Definition 1 (Feasible Contracts). A bank contract \mathcal{C} is *feasible*, given a liquidation price γ , if: (i) bank limited liability is satisfied (equation 3); (ii) the resource constraints are satisfied (equations 1 and 2); (iii) the investor participation constraint is satisfied (equation 6); (iv) repayment monotonicity

¹³Even at the corner solution of $e_0^* = 1$, the equation below holds with equality. Intuitively, a bank that faced a nonbinding incentive constraint at $e_0^* = 1$ would pledge more repayment to external investors to increase project scale.

is satisfied (equations 7 and 8); (v) effort choices e_0^* and $e_1^*(R_1)$ are incentive compatible (equations 9 and 12); (vi) project scale is $Y_0 = I_0 + A_0$.

The bank chooses a feasible contract \mathcal{C} to maximize its own expected utility,

$$\mathbb{E} \left[c(R_1) \middle| e_0 = e_0^* \right] + B_0(e_0^*)Y_0. \quad (13)$$

It is helpful to note that given Assumption 2(f), the privately optimal contract in our model always induces $e_1^*(R_1) = 1$ when the bank is not liquidated.

Liquidations and Ex Post Renegotiation. Our model has ruled out renegotiation at date 1 when the contract has committed to liquidation, $\alpha(R_1) = 1$. Banks can implement the outcome of any feasible renegotiation using state-contingent contracts, meaning renegotiation cannot increase ex ante welfare. However, renegotiation creates a potential time consistency problem: liquidations in our model will be an *ex ante* efficient method of providing incentives even though they are *ex post* inefficient. Thus, although outside of our model, a bank could use liabilities that are difficult to renegotiate as a means of enforcing liquidation. For example, it could issue runnable demand deposits dispersed over many creditors (Calomiris and Kahn, 1991; Diamond and Rajan, 2001).¹⁴

2.5 Arbitrageurs

A representative arbitrageur purchases bank projects at date 1 and converts them into the consumption good using a technology $\mathcal{F}(\Omega)Y_0$, where Ω is the fraction of total bank projects purchased relative to initial scale.¹⁵ Arbitrageur surplus at date 1 from purchasing projects is $\mathcal{F}(\Omega)Y_0 - \gamma\Omega Y_0$,

¹⁴This idea is also consistent with the design of the Title II process, which focuses debt write-downs on long-term debt and not on short-term debt or deposits, due to a concern that “the threat of a restructuring may cause clients to flee and short-term creditors to withdraw their capital” (French et al. 2010). Moreover, Title II resolution includes a “clean holding company” requirement, which bars the top tier holding company (the target of resolution) from issuing any short-term debt to external investors (12 CFR §252.64).

¹⁵This technology is consistent with convex adjustment costs to capital stock that scale with the proportion of capital stock being adjusted (e.g. Bernanke et al. 1999). Assuming the technology is $\mathcal{F}(\Omega Y_0)$ would yield similar insights but add an additional size externality that motivates the planner to maintain lower project scale.

so that their demand satisfies

$$\frac{\partial \mathcal{F}}{\partial \Omega} = \gamma, \quad (14)$$

where the assumption $\gamma \in [\underline{\gamma}, \bar{\gamma}]$ is ensured by $\underline{\gamma} \leq \frac{\partial \mathcal{F}}{\partial \Omega} \leq \bar{\gamma}$.

Arbitrageurs have date 0 wealth of $\bar{A} - A_0$, but cannot borrow against future income. Their lifetime welfare is $u(\bar{A} - A_0) + (\mathcal{F}(\Omega) - \gamma\Omega)Y_0$, with $u'(\bar{A}) > 1$ so that the borrowing constraint binds. The intertemporal borrowing constraint gives rise to a distributive externality (Dávila and Korinek 2018) that makes fire sales Pareto inefficient (see Appendix B.1). The inefficient distributive externality arises because the borrowing constraint creates means arbitrageurs have a higher marginal value of wealth at date 0 than at date 1.¹⁶

2.6 Market Clearing and Equilibrium

We study a symmetric equilibrium in which all banks offer the same contract. The market for liquidated assets must clear at date 1, that is

$$\Omega(\alpha, e_0^*) = \int_{\underline{R}}^{\bar{R}} \alpha(R_1)R_1 f_1(R_1|e_0^*) dR_1. \quad (15)$$

We use the notational shorthand $\Omega = \Omega(\alpha, e_0^*)$. Using equations (14) and (15), we can write the equilibrium liquidation price as

$$\gamma(\Omega) = \frac{\partial \mathcal{F}(\Omega)}{\partial \Omega}, \quad \Omega = \int_{\underline{R}}^{\bar{R}} \alpha(R_1)R_1 f_1(R_1|e_0^*) dR_1. \quad (16)$$

If $\frac{\partial \mathcal{F}}{\partial \Omega}$ is constant and does not depend on Ω , then the liquidation price is constant and there is no fire sale. By contrast if $\frac{\partial \gamma}{\partial \Omega} = \frac{\partial^2 \mathcal{F}}{\partial \Omega^2} < 0$, there is a *fire sale*: more liquidations reduce the liquidation price. We define the liquidation price elasticity $\sigma = -\frac{\Omega}{\gamma} \frac{\partial \gamma}{\partial \Omega}$, which need not be constant. To ensure the social planner also does not find it optimal to induce low effort in continuation (even accounting for fire sale spillovers), we make the following assumption.

¹⁶This externality is similar to the case where there are multiple date 1 aggregate states, and incomplete markets prevent arbitrageurs from equating the marginal value of wealth across date 1 states.

Assumption 3. The liquidation price elasticity is bounded above, $\sigma \leq \bar{\sigma} = \frac{\gamma - \mathbb{E}[R_2 | e_1=0]}{\gamma}$.

In conjunction with Assumption 3, Assumption 2(f) ensures that low continuation effort reduces payoffs available to investors relative to liquidation, which will guarantee the social planner also finds it optimal to write contracts with high effort in continuation.

A symmetric competitive equilibrium of the model is a bank contract \mathcal{C} , arbitrageur purchases Ω , and a liquidation price γ such that: (i) the contract \mathcal{C} is optimal for banks, given price γ ; (ii) investors' participation constraint is satisfied (i.e., investors optimize); (iii) purchases Ω are optimal for arbitrageurs, given price γ ; (iv) the liquidation market clears.

3 Privately Optimal Contracts

In this section, we show that the privately optimal contract written by banks can be implemented by a combination of two debt instruments. The first, which we call *standard debt*, has a fixed face value that does not depend on R_1 , and liquidates the bank in low-return states. The second, which we call *bail-in debt*, has a face value that can be written down based on R_1 , and restores bank solvency when total debt exceeds the amount banks can pledge to investors while maintaining incentives.

3.1 Pledgeable Income and Liabilities

Our model gains considerable tractability because the date 1 binary effort problem can be reduced to a Holmstrom and Tirole (1997) style pledgeability constraint. This allows us to represent the date 0 contracting problem as choosing a liquidation rule $\alpha(R_1)$ and expected payoffs $c(R_1), x(R_1)$ subject to the pledgeability constraint, with the exact distribution of payoffs defined by following Lemma.¹⁷

Lemma 1. *Suppose that for state R_1 , the optimal contract sets $\alpha(R_1) = 0$ and $e_1^*(R_1) = 1$. Then:*

¹⁷In equilibrium, date 1 incentive compatibility does not always bind, in which case there can be multiple monotone continuation contracts that achieve the same bank and investor expected payoffs and maintain incentive compatibility at both dates. For consistency, we use a debt contract when (9) does not bind. Equation (9) does not bind when equation (18) does not bind, that is bank consumption exceeds its minimum agency rent.

1. An optimal repayment scheme is $x_1(R_1) = 0$ and

$$x_2(R_1, R_2) = \begin{cases} R_2 R_1 Y_0, & R_2 \leq R_2^u(R_1) \\ R_2^u(R_1) R_1 Y_0, & R_2 > R_2^u(R_1) \end{cases}, \quad (17)$$

for a threshold $R_2^u(R_1) \leq \bar{R}_2^u$, where \bar{R}_2^u is a constant defined implicitly by the equation $\int_{\bar{R}_2^u}^{\bar{R}} [R_2 - \bar{R}_2^u] (f_{2H}(R_2) - f_{2L}(R_2)) dR_2 = B_1$.

2. Incentive compatibility at date 1 holds if $R_2^u(R_1) \leq \bar{R}_2^u$, or equivalently if

$$c(R_1) \geq b R_1 Y_0, \quad (18)$$

where $b = \int_{\bar{R}_2^u}^{\bar{R}} [R_2 - \bar{R}_2^u] f_{2H}(R_2) dR_2$ is a constant.

All proofs are contained in Appendix A. Lemma 1 represents date 1 incentive compatibility (equation 9) as a minimum minimum agency rent required as a fraction of expected final project value, $R_1 Y_0$.¹⁸ That is, when the bank is not liquidated, bank expected consumption must be at least $c(R_1) \geq b R_1 Y_0$ while investor expected repayment must be at most $x(R_1) \leq (1 - b) R_1 Y_1$ in order to ensure date 1 high effort is incentive compatible.

Lemma 1 also shows that when the bank is not liquidated and high effort is induced at date 1, the monotone investor payoff profile associated with expected repayment $x(R_1)$ is, unsurprisingly, debt. This debt has a total date 2 face value of $R_2^u(R_1) R_1 Y_0$, which therefore depends on R_1 . Importantly, bank high date 1 effort $e_1^*(R_1) = 1$ combined with $F_{2H}(1 - B_1) = 0$ (Assumption 2(c)) guarantees that the date 2 return satisfies $R_2 \geq 1 - B_1 \geq 1 - b$. As a result, the bank will be able to repay the R_1 -contingent face value at date 2 with certainty, that is $x(R_1) = R_2^u(R_1) R_1 Y_0$. Hence, the debt $x(R_1)$ owed at date 2 is risk-free from a date 1 perspective. A consequence is that if $R_2^u(R_1) R_1$ is constant over a range of R_1 , then investor repayment both in expectation and at date 2 is constant for sure over that range.

¹⁸Recall that $\mathbb{E}[R_2 | e_1 = 1] = 1$.

The statement of Lemma 1 allows us to more clearly introduce the upper bound on $\bar{\gamma}$ that guarantees liquidations are ex post inefficient because pledgeable income to investors $1 - b$ is higher than the liquidation value γ .

Assumption 4. The liquidation value is bounded above by $\bar{\gamma} \leq 1 - b$.

Mapping to Promised Liabilities. We show how to represent optimal contracts in our model in the more natural setting of promised liabilities. A promised repayment is denoted $L(R_1)$. If $L(R_1) \leq (1 - b)R_1Y_0$, then the bank can make its promised repayment since it is no greater than its pledgeable income, $x(R_1) = L(R_1)$, so that the date 1 market value of external liabilities equals promised repayment. If instead $L(R_1) > (1 - b)R_1Y_0$, then promised repayment exceeds pledgeable income, the bank is liquidated ($\alpha(R_1) = 1$), and actual repayment to investors is $x(R_1) = \gamma R_1 Y_0$. This last statement relies on the fact that liquidating dividends to the bank are not optimal, as we verify in the proof of Proposition 1.

In the remainder of the main text of the paper, we represent contracts in terms of promised liabilities L rather than in terms of a liquidation rule and promised repayment (α, x) . We refer to L as the face value of liabilities.

3.2 Privately Optimal Contract Terms

We begin by characterizing the privately optimal bank contract in terms of two thresholds in the date 1 return, R_ℓ^p and R_u^p . We use the superscript notation p to indicate the privately optimal values of these thresholds, as distinct from generic thresholds R_ℓ, R_u . We then associate these two thresholds with the two debt instruments. These thresholds summarize the privately optimal liability structure of the bank.

Proposition 1. *A privately optimal bank contract has a liability structure*

$$L(R_1) = \begin{cases} (1-b)R_\ell^p Y_0, & R_1 \leq R_\ell^p \\ (1-b)R_1 Y_0, & R_\ell^p \leq R_1 \leq R_u^p \\ (1-b)R_u^p Y_0, & R_u^p \leq R_1 \end{cases}, \quad (19)$$

where $0 \leq R_\ell^p \leq R_u^p \leq \bar{R}$. The bank is liquidated if and only if $R_1 \leq R_\ell^p$. These thresholds, when interior and not equal,¹⁹ are given by

$$\underbrace{\frac{1 - \Lambda_1(R_\ell^p)}{(1 - e_0^*) + e_0^* \Lambda_1(R_\ell^p)}}_{\text{Incentive Provision}} \underbrace{\frac{1}{|B_0''(e_0^*)|} b \lambda G}_{\text{Liquidation Costs}} = b + \lambda (1 - b - \gamma), \quad (20)$$

$$\underbrace{\frac{F_{1L}(R_u^p) - F_{1H}(R_u^p)}{|B_0''(e_0^*)|}}_{\text{Incentive Provision}} \underbrace{\lambda G}_{\text{Investor Repayment}} = (\lambda - 1) \left(1 - F_1(R_u^p | e_0^*) \right), \quad (21)$$

where $G = \int_{R_\ell^p}^{R_u^p} \gamma R_1 (f_{1H}(R_1) - f_{1L}(R_1)) dR_1 + \int_{R_\ell^p}^{\bar{R}} (1-b) \min\{R_1, R_u^p\} (f_{1H}(R_1) - f_{1L}(R_1)) dR_1 \geq 0$ and where $\lambda > 1$ is the Lagrange multiplier on the investor participation constraint (equation 6).

Optimal date 0 effort satisfies

$$-B_0'(e_0^*) = \int_{R_\ell^p}^{\bar{R}} \max\{bR_1, R_1 - (1-b)R_u^p\} (f_{1H}(R_1) - f_{1L}(R_1)) dR_1. \quad (22)$$

An optimal bank contract is defined by three regions, illustrated in Figure 1 (which is illustrated for any R_ℓ, R_u , not just the privately optimal levels).²⁰ In the first region, $R_1 \leq R_\ell^p$, the face value of liabilities exceeds pledgeable income and the bank is liquidated following a low date-1 return.

¹⁹For the remainder of the paper, we assume that the thresholds are interior and not equal, except when explicitly stated otherwise. Generally speaking, R_ℓ^p will be interior when the likelihood ratio $\Lambda_1(\bar{R})$ is sufficiently small, that is when \bar{R} is a sufficiently good signal of low effort. R_u^p will generally be interior when $\Lambda_1(\bar{R})$ is sufficiently large, that is when \bar{R} is a sufficiently good signal of high effort.

²⁰In the proof of Proposition 1, see Appendix A.3.1 for a comment on non-uniqueness of total promised repayment $L(R_1)$ below R_ℓ^p . Non-uniqueness arises in this region because any face value of liabilities above $(1-b)R_1 Y_0$ results in bank liquidation. We have chosen the face value of liabilities that correspond to standard debt, which seems most natural in the context of banks and bail-ins. Moreover, uniqueness is restored if there is an $\varepsilon \rightarrow 0$ premium for standard debt, for example due to tax benefits of debt. The face value of liabilities is unique above R_ℓ^p .

In the second region, $R_\ell^p \leq R_1 \leq R_u^p$, all pledgeable income of the bank is transferred to investors, leaving the bank with only the minimum agency rent needed to induce high effort at date 1. In the third region, $R_1 \geq R_u^p$, all additional income generated by higher returns accrues to the bank and investors receive the same amount $(1 - b)R_u^p Y_0$ regardless of the date 1 return realization.

Equation (20) describes the marginal trade-off the bank faces in choosing the liquidation threshold R_ℓ^p . On the one hand, liquidating the bank results in a resource loss to banks and investors, valued on the margin at $b + \lambda(1 - b - \gamma)$. On the other hand, pledging to liquidate the bank provides higher-powered monitoring incentives at date 0, reflected in the term $\frac{1 - \Lambda_1(R_\ell^p)}{(1 - e_0^*) + e_0^* \Lambda_1(R_\ell^p)} \frac{1}{|B_0''(e_0^*)|} b$, by depriving the bank of its minimum agency rent $bR_\ell^p Y_0$ necessary to ensure date 1 incentive compatibility. Higher powered incentives increase effort at date 0 and so increase repayment to investors. This increase in effort is valued at the marginal value of relaxing the participation constraint times how much investor repayment is increased by higher effort, that is λG . The optimal choice of R_ℓ^p trades off these two effects. Since effort is optimally chosen by the bank to maximize its own utility, by Envelope Theorem the effect of the change in effort on bank utility is second order and so does not appear directly in the equation. Observe that the liquidation threshold satisfies $\Lambda_1(R_\ell^p) < 1$, that is at R_ℓ^p the likelihood ratio is less than 1 and is more in line with low effort having been exerted.

Equations (20) highlights two (direct) dependencies of R_ℓ^p on the liquidation value γ . First, all else equal (including Lagrange multipliers and the effort level), an increase in γ leads to an increase in the private cost of liquidation, which reduces the bank's desired level R_ℓ^p of standard debt. Second, all else equal (including Lagrange multipliers and the effort level), an increase in γ leads to a *fall* in G (since $f_{1H}(R_1) < f_{1L}(R_1)$ for $R_1 \leq R_u^p$). This intuitively reflects that higher effort levels increase investor repayment by less because liquidation values are high, making the cost of low-effort-induced liquidations low. The lower return to effort (in the form of investor repayment) pushes for a lower liquidation threshold, all else equal (including Lagrange multipliers and the effort level). In Appendix B.3 we consider a concrete case in which the private benefit of date 0 effort is linear, and we show that both R_ℓ^p and R_u^p are increasing in γ .

Equation (21) summarizes the marginal trade-off in choice of R_u^p . On the one hand, the binding

investor participation constraint implies that transferring pledgeable income to investors is valuable because it allows the bank to borrow more and increase project scale (i.e., $\lambda - 1 > 0$). On the other hand, increasing the total debt level reduces bank consumption in high-return states, where the likelihood ratio $\Lambda_1(R_1)$ is high and the signal of high effort is stronger. This lowers bank effort at date 0 (equation 12), which reduces investor repayment, again valued at how much repayment is valued times how much repayment is increased by effort, λG . The optimal choice of R_u^p equalizes these two effects on the margin.

We next associate the privately optimal contract with two liability instruments: short-term standard debt and long-term bail-in debt.

Corollary 1. *The privately optimal contract can be implemented with a combination of short-term standard debt with face value $(1 - b)R_\ell^p Y_0$ due at date 1, which cannot be written down contingent on the idiosyncratic state R_1 , and long-term bail-in debt with face value $(1 - b)(R_u^p - R_\ell^p)Y_0$ due at date 2, which can be written down contingent on the idiosyncratic state R_1 .*

Corollary 1 provides an implementation of the optimal contract in this setting that is associated with bail-ins. Short-term standard debt is due at date 1, and so must be rolled over. In the region $R_1 \leq R_\ell^p$, the face value of standard debt exceeds pledgeable income, so that debt cannot be rolled over and liquidation is forced. In the region $R_\ell^p \leq R_1 \leq R_u^p$, the face value of bail-in debt is written down to $(1 - b)(R_1 - R_\ell^p)Y_0$, which allows the bank to roll over its short-term debt and repay investors while maintaining its minimum agency rent. In the region $R_1 \geq R_u^p$, bail-in debt is not written down, all investors are repaid the face value of debt, and the bank exceeds its minimum agency rent. As highlighted in Section 3.1, Assumption 2(c) guarantees that after the required long-term debt write downs are imposed (if any), both the full face value of short-term debt and the remaining face value of long-term debt are risk free in continuation.

For the remainder of the paper, we associate (short-term) standard and (long-term) bail-in debt directly with the thresholds R_ℓ and R_u , rather than writing out their associated (face value) liabilities. That is, we refer to R_ℓ as the level of standard debt, $R_u - R_\ell$ as the level of bail-in debt, and R_u as

the level of total debt.²¹

Bail-in Debt or (Outside) Equity? Proposition 1 and Corollary 1 highlight why bail-in debt can be a valuable loss-absorbing instrument for banks, relative to equity. Bail-in debt combines the incentive properties of standard debt with the loss-absorbing properties of equity. It generates a maximal cash flow transfer below R_u and a flat investor payoff above R_u , similar to standard debt, but does so without liquidating the bank (as standard debt does). By contrast, equity transfers the upside of the bank to investors. Transferring more of the upside of the bank to investors worsens incentives due to MLRP, since higher returns signal that the bank likely exerted high effort. Bail-in debt recapitalizes the bank in the same manner as equity on the downside, but generates better incentives on the upside. This leads banks to prefer bail-in debt to equity as a loss-absorbing instrument even in the private optimum without any intervention by a social planner. In Section 5, we study why banks did not issue bail-in debt prior to the advent of bail-in regimes in the context of bailouts.

3.3 The Role of Agency Problems and Costly Liquidation

Our model features three key ingredients: an initial incentive problem ($B_0(0) > 0$), a continuation incentive problem ($B_1 > 0$), and costly liquidations ($\gamma < 1$). Absent one of the three ingredients, the optimal contract in our model can be implemented without combining standard and bail-in debt.²²

Proposition 2. *The privately optimal contract can be implemented with a single liability instrument if $B_0(e_0) = 0$, $B_1 = 0$, or $\gamma = 1$. In particular,*

(a) *If $B_0(e_0) = 0$, then the privately optimal contract can be implemented with bail-in debt.*

²¹Bail-in debt can also be interpreted as a *contingent convertible* (CoCo) debt instrument (see Avdjiev et al. 2020 and Flannery 2014 for more background). Bail-in debt in our model is a principal write-down CoCo debt security that applies at the point of non-viability. Other implementations of the contract in our model include, for example: (i) standard debt and outside equity, with a managerial compensation scheme to pay the bank $c(R_1)$; (ii) partial-bail-in debt, which can only be written down to $(1 - b)R_l Y_0$.

²²For case (c) with $\gamma = 1$, we naturally relax the bounding assumption \bar{B}_0 that had ruled out liquidating dividends as it had assumed $\gamma < 1$.

(b) If $B_1 = 0$, then the privately optimal contract can be implemented with long-term debt.²³

(c) If $\gamma = 1$, then the privately optimal contract can be implemented with standard debt.

When $B_0(e_0) = 0$, there is no required agency rent at date 0 (incentive compatibility is maintained with any contract with monotone bank payoff), but there is a required agency rent at date 1. Therefore, the bank can ensure incentive compatibility at both dates by using a debt contract set according to Lemma 1. However, R_1 still requires the contract to adjust the level of debt in continuation to maintain date 1 incentive compatibility. As a result, a bail-in debt contract suffices. As a result, a date 0 incentive problem is necessary in our model to generate an optimal contract that combines standard and bail-in debt.

The second and third cases of Proposition 2 show that the date 0 incentive problem alone is not sufficient to generate a privately optimal contract that combines standard and bail-in debt. In the second case with $B_1 = 0$, Lemma 1 tells us that $b = 0$ and so all income is pledgeable to investors. Therefore, the bank can guarantee zero consumption, $c(R_1) = 0$, without having to liquidate the project prior to maturity. This case is analogous to Innes (1990), and means that the bank finds it optimal to only use long-term debt to transfer cashflows to investors while avoiding costly liquidation. In contrast in the third case, with $\gamma = 1$ but $B_1 > 0$, there is a limit to pledgeable income but no bankruptcy costs from liquidation. Banks can repay any amount $x(R_1) \leq R_1 Y_0$ by liquidating their project, and the pledgeability constraint ceases to be relevant. Banks use only standard debt.

In all cases of Proposition 2, the key property of debt is the cash flow transfer from the bank to investors in low-return states and fixed repayment in high-return states (as in e.g., Innes 1990, Hébert 2018). In absence of ex ante incentive problems, cash flow transfer is achieved with bail-in debt. In the absence of continuation agency rents, cash flow transfer is achieved with long-term debt. In the absence of bankruptcy costs, cash flow transfer is achieved with standard debt. However, if there are ex ante incentive problems, continuation agency rents, and bankruptcy costs, then bail-in

²³Note that it could also be implemented with bail-in debt.

debt cannot enact a full cash flow transfer, while standard debt enacts a full cash flow transfer at a resource cost. A role emerges for both forms of debt in the optimal contract.

4 Optimal Policy

In this section, we study optimal policy of a social planner that internalizes the fire sale externality of Section 2.6. In particular, we study the optimal contract chosen by the planner, who must respect the private incentive constraints of the bank but internalizes the fire sale externality. In principle, the planner could implement any feasible capital structure, for example requiring banks to issue outside equity as a loss absorbing instrument. Nevertheless, we show that the social planner finds it optimal for banks to write contracts that combine standard and bail-in debt, that is bail-in debt is a socially optimal loss absorbing instrument. Moreover, the planner intervenes in both the level and composition of debt: the planner on the margin prefers not only less use of standard debt, but also less overall debt. Our model rationalizes joint regulation of not only the total leverage of the bank, but also the composition of bank TLAC, from the single underlying fire sale externality.

4.1 Social Optimum

The social planner's problem is to choose a feasible contract (satisfying Definition 1) in order to maximize bank's utility (equation 13), internalizing that the equilibrium liquidation price γ is determined by equation (16).²⁴ Because the planner can choose any feasible contract, the planner is not required to choose a contract combining standard and bail-in debt, and could in principle employ other instruments such as outside equity. Nevertheless, we show in the following result that the planner's optimal contract combines standard and bail-in debt. For clarity, we use superscript s notation to differentiate objects under the social planner's contract from those of the private optimum.

²⁴For simplicity we have assigned a welfare weight of 0 to arbitrageurs, but in Appendix B.1 we show that similar results hold with positive welfare weights.

Proposition 3. *A socially optimal bank contract has a liability structure*

$$L(R_1) = \begin{cases} (1-b)R_\ell Y_0^s, & R_1 \leq R_\ell \\ (1-b)R_1 Y_0^s, & R_\ell \leq R_1 \leq R_u^s \\ (1-b)R_u^s Y_0^s, & R_u^s \leq R_1 \end{cases}, \quad (23)$$

where $0 \leq R_\ell^s \leq R_u^s \leq \bar{R}$. The bank is liquidated if and only if $R_1 \leq R_\ell^s$. These thresholds, when interior and not equal, are given by

$$\frac{1 - \Lambda_1(R_\ell^s)}{(1 - e_0^*) + e_0^* \Lambda_1(R_\ell^s)} \frac{1}{|B_0''(e_0^*)|} b \lambda^s G^s = b + \lambda^s (1 - b - \gamma^s) + \tau_\ell^s \quad (24)$$

$$\frac{F_{1L}(R_u^s) - F_{1H}(R_u^s)}{|B_0''(e_0^*)|} \lambda^s G^s = (\lambda^s - 1)(1 - F_1(R_u^s | e_0^*)) - \tau_u^s \quad (25)$$

where $G^s = \int_{\underline{R}}^{R_\ell^s} \gamma^s R_1 (f_{1H}(R_1) - f_{1L}(R_1)) dR_1 + \int_{R_\ell^s}^{\bar{R}} (1-b) \min\{R_1, R_u^s\} (f_{1H}(R_1) - f_{1L}(R_1)) dR_1 \geq 0$, where $\lambda^s > 1$ is the Lagrange multiplier on investor participation, and where the “wedges” τ_ℓ^s, τ_u^s are given by

$$\tau_\ell^s = \left(1 - \frac{1 - \Lambda_1(R_\ell^s)}{(1 - e_0^*) + e_0^* \Lambda_1(R_\ell^s)} \frac{1}{|B_0''(e_0^*)|} b L^s \right) \lambda^s \sigma \gamma^s \geq 0, \quad (26)$$

$$\tau_u^s = \frac{F_{1L}(R_u^s) - F_{1H}(R_u^s)}{|B_0''(e_0^*)|} L^s \lambda^s \sigma \gamma^s \geq 0, \quad (27)$$

where $L^s = \int_{\underline{R}}^{R_\ell^s} R_1 (f_{1L}(R_1) - f_{1H}(R_1)) dR_1 \geq 0$. Optimal date 0 effort satisfies

$$-B_0'(e_0^*) = \int_{R_\ell^s}^{\bar{R}} \max\{bR_1, R_1 - (1-b)R_u^s\} (f_{1H}(R_1) - f_{1L}(R_1)) dR_1. \quad (28)$$

Even though the social planner has the ability to write any feasible contract \mathcal{C} , for example using outside equity, Proposition 3 shows that the social planner finds it optimal to write a contract of the same structural form as private banks chose. That is, the socially optimal contract can also be implemented with a combination of standard and bail-in debt. The planner thus agrees with the bank that the optimal capital structure should make use of these two debt instruments, and not other

instruments such as outside equity. We have denoted R_ℓ^s and R_u^s to be the planner's choices of the two instruments.

However, the fire sale spillover results in an additional *social cost of liquidation* from the planner's perspective that is not internalized by banks: increases in total liquidations reduce the liquidation price, which reduces investor repayment and tightens the investor participation constraint. To compare the marginal incentives of the social planner relative to the bank in its capital structure choice, it is instructive to think of the planner's first order conditions in terms of the "wedge" approach. Equations (24) and (25) are the counterparts of equations (20) and (21), respectively, in the private optimum, excepting for the respective additions of the extra terms $+\tau_\ell^s$ and $-\tau_u^s$ on the right hand sides. That is, the terms τ_ℓ^s and τ_u^s represent the "wedges" that would need to be inserted into the bank's first order conditions to make them hold at the planner's optimal contract for the planner's Lagrange multiplier λ^s . These wedges are defined by equations (26) and (27), and reflect a *marginal* difference in incentives of the planner and private agents, evaluated at the social optimum. After adopting the marginal approach, we revisit the question of whether the planner uses more or less of each instrument from a global perspective.

The wedge τ_ℓ^s appears as an additional social cost of liquidations that arises when standard debt is increased. That is, a positive wedge reflects an incentive for the planner to reduce standard debt on the margin, relative to private banks. Intuitively, the size of the wedge is given by the total increase in liquidations, the term in parentheses, times the social cost of liquidations, $\lambda^s \sigma \gamma^s$. The total increase in liquidations combines both a direct and indirect effect. The direct effect is that more standard debt directly prompts more liquidations, which directly lowers liquidation prices. The indirect effect is that a higher liquidation threshold increases date 0 effort, which reduces the probability of insolvency and so reduces liquidations. Given Assumption 2(e), the direct effect dominates, that is an increase in standard debt increases total liquidations in the economy, which has a social cost $\lambda^s \sigma \gamma^s$. The social cost, all else equal, increases with the value of bank net worth, λ^s , and with the fire sale elasticity, σ . The planner's positive wedge on standard debt thus indicates that the planner on the margin prefers less use of standard debt than private banks in order to reduce

the severity of fire sales.

The wedge τ_u^s on use of total debt appears in the planner's first order condition akin to a decrease in the revenue received from issuing more total debt to investors. That is, a positive wedge reflects a lower marginal incentive for the planner to increase total debt. The wedge on use of total debt is positive, $\tau_u^s \geq 0$, which arises due to an indirect incentive effect. Intuitively, as total debt increases, the bank's optimal effort level declines, which raises the probability of the bank's date 1 return being below R_ℓ^s and hence a liquidation occurring. This increase in total liquidations is again evaluated at the social cost of liquidations, $\lambda^s \sigma \gamma^s$. As a result, the planner also prefers less use of total debt by the bank as another method of mitigating liquidations and fire sales. The planner thus intervenes in not only the composition of debt, but also in the total level of debt.

The above discussion is based on marginal analysis conducted at the social optimum. In Appendix B.3, we study a special case of linear private benefits, $B_0(e_0) = b_0(1 - e_0)$, and show that the planner's contract indeed involves less use of both instruments relative to the private optimum, that is $R_\ell^s \leq R_\ell^p$ and $R_u^s \leq R_u^p$.

Implementation with Joint Regulation. In the proof of Proposition 3, we show that the social planner can induce the bank to choose the socially optimal debt levels by setting two requirements: (i) banks must set total debt to be at most $R_u \leq R_u^s$; (ii) banks must set standard debt to be at most $R_\ell \leq R_\ell^s$. The first constraint on total debt (as a fraction of scale) can be interpreted as a minimum equity capital requirement or a maximum leverage requirement. Such a requirement is a common bank regulatory tool in both the pre- and post-financial crisis toolkit. The second constraint can be interpreted as a portion of the total loss-absorbing capacity (TLAC) requirement that can be satisfied by long-term bail-in debt. These two requirements are consistent with US requirements for both minimum equity and long-term debt that can be subject to bail-ins (82 FR 8266). More broadly, the Basel III requirements incorporate both a minimum equity component of the Tier 1 capital requirement, and additionally allow CoCos to satisfy AT1 requirements and long-term debt to satisfy Tier 2 requirements (BCBS (2011)). A strength of our model is that the single fire sale

externality rationalizes both of these requirements.

4.2 Ex Post (Bail-in) Resolution as Optimal Policy

Although in our model bail-in debt involves pre-specified (ex ante) contractual write-downs, in practice bail-ins are also implemented via a resolution authority that imposes write downs ex post. We show that the social optimum can also be implemented using an ex-post resolution authority. Our resolution authority has discretion at date 1 to impose write downs on liability contracts that are designated “bail-inable,” but is prohibited from imposing write downs on contracts that are designated “non-bail-inable.” The objective of the resolution authority is, at the level of the individual bank (i.e., taking equilibrium prices as given), to maximize total recovery value to creditors, subject to write downs being Pareto efficient for creditors and the banker. In the implementation that follows, it will be necessary to allow for debt seniority.

Corollary 2. *The social optimum can be implemented using macroprudential policy and a resolution authority. The social planner imposes an ex ante requirement for the bank to issue R_ℓ^s in non-bail-inable senior debt and $R_u^s - R_\ell^s$ in bail-inable junior debt, where R_ℓ^s and R_u^s are given as in Proposition 3. The resolution authority implements the write downs of Proposition 3 ex post.*

Corollary 2 provides an implementation of the social optimum using a resolution authority. If $R_1 < R_\ell^s$, the resolution authority lacks capability to impose sufficient write downs to recapitalize the bank, and so cannot intervene. If $R_\ell^s \leq R_1 \leq R_u^s$, the resolution authority writes down bail-inable debt to $R_1 - R_\ell^s$. This maximizes creditor recovery and is Pareto efficient, since bail-inable debt is junior. If $R_1 > R_u^s$, the resolution authority’s objective is achieved without write downs. Thus the same outcome as in Proposition 3 is achieved. Finally, the planner mandates the level and composition of debt ex ante for the same reason as under the contractual implementation.²⁵

The implementation of optimal policy in Corollary 2 is consistent with several properties in

²⁵In our model, bank fundamentals R_1 are common knowledge, so there is no informational time consistency problem as in Walther and White (2020).

the design of bail-in regimes in practice, for example Title II. Bail-in regimes subordinate bail-in(able) long-term debt to standard short-term debt, that is standard debt enjoys absolute priority in bankruptcy, liquidation, and resolution.²⁶ Moreover, the objective of Pareto efficiency is consistent with the No Creditor Worse Off principle of bank resolution (BRRD Article 73).

In practice, bail-in long-term debt and contingent convertible debt (CoCos) are subject to differential regulatory treatment. In addition to a common equity Tier 1 capital requirement, Basel III includes an AT1 capital requirement towards which certain CoCos qualify, whereas bail-in debt serves as Tier 2 capital (BCBS (2011)). This provides a regulatory advantage to CoCos in Europe and other jurisdictions that allow CoCos as AT1.²⁷ In contrast, the US does not include CoCos in Tier 1 capital and further requires that to satisfy the minimum long-term debt requirement, “eligible external LTD [is] prohibited from including contractual triggers for conversion into or exchange for equity” (82 FR 8266).²⁸ Although our model does not clearly differentiate between bail-in debt and CoCos, from a regulatory perspective it highlights the importance of distinguishing a (Tier 1) *equity* capital requirement that cannot be satisfied by CoCos, from a (Tier 2) bail-in debt requirement that could potentially be satisfied by CoCos. In particular, it highlights that both bail-in debt and CoCos should be restricted by a maximum leverage requirement. Although outside of our model, a regulator could require a minimum fraction of Tier 2 capital be comprised of bail-in debt or CoCos based on which of the two instruments was preferred.

²⁶In practice, short-term debt priority has three implementations. The first is contractual: bail-in debt is junior to short-term debt. The second is organizational: short-term debt is issued at the operating subsidiary, whereas long-term debt is issued at the top-tier holding company. The third is legal: national bankruptcy law confers priority to short-term debt in the case of banks. The US induces seniority through organizational form, and we could implement the US approach under Corollary 2 by assuming that bail-inable debt is held at a resolvable holding company, whereas non-bail-inable is held at a non-resolvable operating subsidiary.

²⁷Avdjiev et al. (2020) and Fiordelisi et al. (2020) provide more details on CoCos and regulatory treatment.

²⁸In addition, debt used to satisfy TLAC requirements must be plain-vanilla, implying a fixed face value (82 FR 8266)

5 Bail-ins and Bailouts

One of the stated goals of bail-in regimes is to replace bailouts. A strength of our framework is that we can leverage it to examine important policy questions such as this one. In this section, we extend our analysis of Section 4 by allowing the planner to engage in bailouts at date 1 by paying a fixed cost. If the planner has commitment and can jointly regulate the level and composition of debt, we show that the planner optimally commits to no bailouts. Absent commitment, we show that a social planner with joint regulation sets standard and bail-in debt levels so that no bailouts occur in equilibrium. An increase in the fixed cost of bailouts improves welfare, rationalizing why bail-in regulation has been paired with statutory provisions to make it harder to engage in bailouts.

5.1 Introducing Bailouts

We extend our setup to allow the planner to engage in bailouts. At date 1, the planner's bailout authority can pay a fixed cost $F \geq 0$ to be able to undertake bailouts (e.g., [Farhi and Tirole \(2012\)](#)). We define $T(R_1) \geq 0$ to be a bailout given at date 1 to a bank with return R_1 . The smallest bailout that recapitalizes a bank with return R_1 that cannot both repay its liabilities and maintain its minimum agency rent is $T(R_1) = L(R_1) - (1 - b)R_1Y_0$. Bailout funds are raised from taxpayers at date 1. As with investors, taxpayers have deep pockets and can finance any bailout. Taxpayer utility is $c_0^T + c_1^T$, where $c_0^T = A_0^T + T_0$ and $c_1^T = A_1^T - \mathbb{E}[T(R_1)|e_0 = e_0^*]$. We interpret T_0 as a "bailout fund," i.e., a transfer from banks to households at date 0 to compensate for ex post bailouts (so that the bank endowment at date 0 is $A_0 - T_0$). A bailout fund is necessary to achieve a Pareto improvement if bailouts are used in equilibrium, but is not needed if bailouts do not occur.

5.2 A Commitment Benchmark

We take as our starting point the planner's problem in Section 4.1 and allow the planner to also choose its bailout authority's bailout policies with commitment. We start with the assumption of commitment to provide conceptual benchmarks. Formally, we characterize Pareto efficient contracts

\mathcal{C} and bailout rules (T_0, T_1) . That means we solve the (constrained) Pareto problem, which is to maximize the sum of bank and taxpayer welfare with a Pareto weight of 1 on banks and ω^T placed on taxpayers.²⁹ Recall that ex-post bailouts T_1 can always in principle be offset by a transfer T_0 to ensure taxpayers are no worse off, so requiring Pareto efficiency does not immediately rule out use of bailouts. The problem is otherwise the same as in Section 4. We obtain the following result.

Proposition 4. *In the model with committed bailouts, the socially optimal contract of Proposition 3, with no bailouts ($T_0 = T_1 = 0$), is Pareto efficient.*

Proposition 4 shows that a planner with commitment achieves Pareto efficiency in the model with committed bailouts by exclusively relying on bail-ins to recapitalize banks. This means bailouts are not used at any point on the Pareto frontier. To understand why, suppose the planner bailed out the marginal liquidated bank $R_1 \uparrow R_\ell^s$. There are two effects of a bailout. First, the liquidation threshold is lowered, that is R_ℓ^s effectively falls. But under the socially optimal contract of Proposition 3, the marginal cost of liquidations is perfectly balanced against the marginal benefit of incentive provision, meaning a reduction in R_ℓ^s has zero net welfare effect. Second, a bailout also transfers resources from taxpayers to banks. The marginal social benefit of the resource transfer is the value of relaxing the participation constraint, λ^s . The marginal social cost of the resource transfer is the burden to taxpayers, ω^T . To select the point on the Pareto frontier that is consistent with the ex-ante distribution of resources between banks and taxpayers, we select the Pareto weight $\omega^T = \lambda^s$. This implies that the net welfare gain from ex-ante resource transfers is zero, and therefore there is also no welfare gain from bailing out the marginal otherwise-liquidated bank.³⁰ Therefore, the optimal contract of Proposition 3, with no bailouts or date 0 transfers, maximizes the planner's welfare criterion, and so we have found a Pareto efficient allocation without bailouts.³¹

It is well known that when debt contracts are non-contingent by assumption, bailouts can be

²⁹As in Section 4, we maintain a welfare weight of 0 on arbitrageurs.

³⁰Other points on the Pareto frontier imply an initial redistribution of wealth via T_0 , but still have no bailouts.

³¹Dewatripont and Tirole (2018), Farhi and Tirole (2021), and Keister and Mitkov (2021) emphasize that bailouts can be used as an insurance mechanism, independent of the value of preventing liquidations per se. In our model, this can happen in reduced form by assuming taxpayers discount the future while investors do not.

Pareto efficient because they insert contingencies into otherwise non-contingent contracts (Bianchi 2016, Jeanne and Korinek 2020). Our model endogenously generates not only non-contingent standard debt, but also contingent bail-in debt. Thus the planner can achieve the same state-contingencies with bail-in debt as it could with bailouts. As a result under full commitment, the planner can achieve an efficient outcome by using more bail-in debt, rather than by bailing out standard debt, provided that the planner also has the ability to jointly regulate both the level and composition of debt.

Bailouts without Joint Regulation: Although pre-crisis regulation included leverage requirements to regulate the total debt level of banks, the ability to regulate its composition was only introduced post-crisis by bail-in regimes (e.g., Title II of Dodd-Frank). Having only a leverage requirement would inhibit the ability of the planner to achieve the socially optimal contract with regulation. Maintaining the assumption of commitment, we show that the planner can use the combination of leverage regulation and bailouts to implement the outcome of the socially optimal contract in a Pareto efficient manner, while incurring only the fixed cost F of bailouts. The following result formalizes how bailouts implement the social optimum in a Pareto efficient manner.

Proposition 5. *With bailout commitment, the social planner can implement the social optimum of Proposition 3 in a Pareto efficient manner by setting: (i) leverage regulation $R_u = R_u^s$; (ii) bailouts for banks with $R_1 \in [R_\ell^s, R_u^s]$; (iii) a bailout fund $T_0 = \mathbb{E}[T(R_1)|e_0 = e_0^*(R_\ell^s, R_u^s)]$. Banks only issue standard debt, that is $R_\ell^p = R_u^s$.*

In absence of a bail-in regime, the planner can engineer the socially optimal outcome by making use of bailouts. Leverage regulation $R_u = R_u^s$ ensures that banks have to set total debt equal to its socially optimal level. Bailouts ensure that banks $R_\ell^s \leq R_1 \leq R_u^s$ are not liquidated. Anticipating bailouts, banks respond by setting $R_\ell^p = R_u^s$, that is not issuing any bail-in debt. This outcome mimics the payoff profile of the socially optimal contract, but in the process transfers resources of amount $\mathbb{E}[T(R_1)|e_0 = e_0^*]$ from taxpayers to the bank.³² The planner unwinds this ex

³²Bailouts go to repay investors at date 1, but the bank extracts expected bailout payments from investors by holding

post transfer by using an ex ante transfer (“bailout fund”), forcing banks to cover the expected costs of bailouts ex ante in a lump sum payment to taxpayers. This results in Pareto efficiency, since taxpayers receive back the amount they have to pay in bailouts (i.e., it is as-if banks are paying for their own bailouts).

As with Proposition 4, Proposition 5 provides an idealized result in the world of bailout commitment. It is instructive, however, because of the key contrast of this environment with that of Proposition 4: the only change was that the planner was not able to jointly regulate the level and composition of debt. In this sense, it is the advent of bail-in regulation that allows the planner to regulate both the level and composition of debt that allows for bail-ins to fully replace bailouts.

Why Didn’t Banks Issue Bail-in Debt before 2008? The combination of Propositions 4 and 5 provides an explanation for why bail-in debt was “introduced” into the capital structure of banks in the post-crisis world. Absent joint regulation of the level and composition of debt, bailouts provided a potentially costly method of implementing the social optimum by preventing costly liquidations of intermediate-return banks. In anticipation of bailouts, banks responded pre-crisis by only issuing standard debt and not issuing bail-in debt. However, the introduction of bail-in regimes has allowed interventions in both the level and composition of debt, enabling the planner to achieve efficient outcomes without bailouts. Thus the advent of bail-in regimes has “introduced” bail-in debt into banks’ capital structures.³³

5.3 Bail-ins replace bailouts without commitment

We now study the case in which the planner cannot commit to the bailout rules adopted by its bailout authority at date 1. Instead, in keeping with much of the literature, we assume the bailout authority

them to their participation constraint ex ante.

³³Another variant of this idea comes from part (c) of Proposition 2: if banks were protected from the costs of liquidation, for example by the anticipation of bailouts, they would not be incentivized to issue bail-in debt. Expectation of bailouts would increase recovery values both directly by protecting creditors (debt guarantees such as TLGP) and indirectly by stabilizing resale markets (asset purchases such as TARP). Resale market stabilization in particular can help explain why even smaller banks, which may not have expected direct bailouts, might nevertheless not use bail-in debt.

chooses bailout policies at date 1 to maximize a utilitarian objective with equal weights on banks, investors, and taxpayers, accounting for the fixed cost F of bailouts.³⁴ Even though the bailout authority chooses bailout policies ex post without commitment, our main result for this section is that with joint regulation, the planner's still achieves Pareto efficiency by eliminating bailouts. Moreover, increasing the fixed cost F increases welfare, rationalizing statutory provisions against bailouts. To streamline exposition, we move directly to considering contracts combining standard and bail-in debt, (R_ℓ, R_u) .

At date 1, the bailout authority makes bailout decisions (taking the contracts signed and date zero effort choice as given). The total resource loss to banks and investors from liquidations across all banks is given by $(1 - \gamma(\Omega))\Omega Y_0$. The bailout authority therefore bails out no banks if the fixed cost F of bailouts is higher than the total losses from liquidations,

$$(1 - \gamma(\Omega))\Omega Y_0 \leq F, \tag{29}$$

and otherwise bails out every bank that would otherwise be liquidated. For example, in the limiting case of $F = 0$, any nontrivial level of standard debt (i.e., any $R_\ell > \underline{R}$) would result in bailouts of every bank with $R_1 \leq R_\ell$ ex post. Anticipating bailouts, banks would only issue standard debt (as in Proposition 5). This means a planner lacking joint regulation (i.e., only a leverage requirement) would face a choice between forcing banks to minimal leverage or to engage in full bailouts of any distressed banks.³⁵ We focus our analysis on the case with joint regulation.

We now turn to the ex ante optimal debt levels of the social planner. Akin to Section 5.2, we solve the (constrained) Pareto problem, that is characterizing Pareto efficient debt levels (R_ℓ, R_u) and bailout rules (T_0, T_1) , but subject to the constraint that bailouts T_1 are chosen ex post by the bailout authority. Recall that ex-post bailouts T_1 can always in principle be offset by a transfer T_0

³⁴Recall that even though investors receive no surplus ex ante under the contract, ex post they lose value from liquidations. Results extend to allowing the bailout authority to also put the same welfare weight on arbitrageurs, but for consistency in main text we simplify and maintain the zero weight on arbitrageurs.

³⁵See for example Farhi and Tirole (2012) and Chari and Kehoe (2016) for models in which full bailouts can arise due to fixed bailout costs or to purely utilitarian bailouts.

to ensure taxpayers are no worse off, so requiring Pareto efficiency does not immediately rule out debt levels that result in bailouts. We now show that for *any* F , Pareto efficient debt levels eliminate bailouts entirely.³⁶

Proposition 6. *Absent bailout commitment:*

1. For any $F \geq 0$, Pareto efficient debt levels (R_ℓ^s, R_u^s) are such that no banks are bailed out in equilibrium.
2. Welfare is increasing in the cost F of bailouts.

The first part of Proposition 6 shows that, as before, Pareto efficiency is associated with debt levels that ensure that no bailouts occur ex post. In fact, the intuition comes from Proposition 4. As in Proposition 4, bailouts are redundant as a recapitalization instrument when bail-ins are available. In particular, any debt levels under which bailouts occurred ex post could be mimicked by instead reducing the level of standard debt to zero while maintaining the same overall leverage, without incurring the fixed cost of bailouts. Thus the planner finds it optimal to rule out bailouts through choice of debt levels. The above logic applies for any F , even $F = 0$.

The second part shows that welfare increases in the fixed cost F at every point on the Pareto frontier. This is because the “no bailouts” constraint of equation 29 limits the set of no-bailout debt levels. Increasing the cost of bailouts F relaxes that constraint and enables the planner to choose debt levels that result in more costly liquidations. While welfare is maximized by taking $F \rightarrow \infty$, that is ensuring bailouts are never tempting, the planner benefits from increasing the cost of bailouts F even when F remains finite.

Our model helps to understand that attempts to move the post-crisis regime towards a no-bailout world has two complementary components. The first important element is the use of regulatory limits – leverage and TLAC requirements – to reduce the temptation to engage in bailouts ex post.

³⁶If we were instead to assume the date 0 planner also put equal weights on banks and taxpayers, as does the bailout authority, the planner would still choose debt levels that eliminated bailouts as in Proposition 6, but would also use date 0 transfers T_0 to shift wealth to banks, since the marginal value of inside equity, which is $\lambda^s > 1$, is higher than the marginal value of wealth to taxpayers, which is 1. That is, the planner would simply move to a different point on the Pareto frontier that still involved no bailouts.

The second important element is to increase the cost of engaging in bailouts, which complements the first by enabling the planner to adopt looser regulatory requirements in the sense of allowing more costly liquidations to occur. The post-crisis introduction of bail-in regulation as a complement to equity capital requirements (leverage limits) has been complemented by pushes to make engaging in bailouts more difficult, which through the lens of our model makes F larger. Contrasted with Propositions 4 and 6, our results in Proposition 5 suggest that governments pre-2008 may have had more flexibility to engage in bailouts precisely because they lacked the bail-in instrument. In the context of Proposition 5, targeted bailouts towards intermediate-return banks allowed them to repay their debtholders while also maintaining their own minimum agency rent. Importantly, this implies that bailouts in our model not only protect debt holders, but also provide some protection to the (inside) equity stakeholders in the bank. However, in the absence of commitment, the conditional support of Proposition 5 is not incentive compatible for the government: the bailout authority optimally chooses to bail out all banks, or none.

Although post-crisis legislation has not made bailouts impossible (which would correspond to $F = \infty$), it arguably has increased the difficulty and costs of engaging in bailouts on several dimensions. First, the Dodd-Frank Act put in place restrictions on lender of last resort activities, including eliminating the ability to target loans to individual distressed institutions (Labonte, 2020; Geithner, 2016).³⁷ Geithner (2016) notes that “[t]his was designed to make it hard if not impossible for the Fed to undertake the types of programs it did to facilitate the acquisition of Bear Stearns by JPMorgan Chase and to prevent AIG’s failure.” Consistent with bail-in regimes making it possible and desirable to eliminate bailouts, Former Chairman Ben S. Bernanke notes that “[w]ith the creation of the liquidation authority, the ability of the Fed to make loans to individual troubled firms like Bear and AIG was no longer needed and, appropriately, was eliminated.”³⁸ Beyond restrictions on the Fed, post-crisis regulation has also put limits on the ability for other agencies to engage

³⁷Labonte (2020) notes additional restrictions, including that in implementing the Dodd-Frank Act, “the [Fed’s] final rule requires lending to be at a “penalty rate,” which it defines as a premium to the market rate prevailing in normal circumstances.”

³⁸Ben S. Bernanke, “Warren-Vitter and the Lender of Last Resort,” Brookings Institution, May 15, 2015. <http://www.brookings.edu/blogs/ben-bernanke/posts/2015/05/15-warren-vitter-proposal>

in the debt guarantee and capital infusion programs that were used in the crisis (Geithner 2016). For example, Geithner (2016) highlights that it “took away the FDIC’s discretion to guarantee the broader liabilities of banks and bank holding companies.” At the same time, Congress maintains the ability to enact support programs with legislation. This is consistent with the idea that the cost of engaging in bailouts may have increased through limitations on discretionary authority, but that cost remains finite. Through the lens of our model and Proposition 6, the post-crisis leverage and TLAC requirements play a dual role of managing fire sales and preventing bailouts, while statutory commitments raise bailout costs and alleviate the need for even more stringent regulatory requirements.

6 Discussion and Extensions

In this section, we connect our model to nonfinancial firms and bankruptcy, to too-big-to-fail institutions, and to demand-based explanations for standard debt.

6.1 Nonfinancial Firms and Bankruptcy

Although our model is framed in terms of banks, our optimal contracting framework could also be applied to nonfinancial corporates. An interpretation of our model as corresponding to corporate capital structure and bankruptcy can be made as follows. Chapter 7 of the US Bankruptcy Code provides for liquidation, while Chapter 11 provides for reorganization and debt restructuring process.³⁹ Chapter 11 reorganization requires that creditors in impaired classes should either have voted to accept the plan or be no worse off than in Chapter 7 liquidation (11 U.S.C. §1129).⁴⁰ Impaired classes can push for liquidation under Chapter 7, or can accept concessions such as the bail-in haircuts of our model in a reorganization plan. It is well known that different creditors

³⁹One important concern is that Chapter 11 may be imperfectly designed for banks (French et al. 2010), leading the US Treasury Department to adopt a proposal for a Chapter 14 bankruptcy process tailored to banks (Scott and Taylor, 2012; US Department of Treasury, 2018). Our model suggests that difficulties of resolving banks results from banks’ deliberate capital structure decisions, and our normative results in Section 4 suggest a role for the government to require greater use of bail-in debt even under a Chapter 14 process.

⁴⁰This mirrors the no-creditor-worse-off condition of bail-in regimes, see Section 4.2.

have different incentives in the renegotiation process (e.g., [Bolton and Scharfstein 1996](#)): senior secured creditors often favor liquidation to avoid further impairment, whereas junior unsecured creditors often prefer reorganization to capitalize on convexity. Dispersing (secured senior) claims over many creditors can lead to disorderly collateral seizures and hold out problems that inhibit reorganization, whereas concentrating claims can mitigate hold out problems. We interpret standard debt as corresponding to dispersed (short-term) senior secured claims that promote Chapter 7, and bail-in debt as concentrated (long-term) junior unsecured claims that promote Chapter 11.⁴¹

In practice, Chapter 11 reorganization is common for distressed large nonfinancials and conversion to Chapter 7 liquidation is relatively uncommon ([Wruck 1990](#), [Bernstein et al. 2019](#), [Antill 2022](#)). In addition, nonfinancials often have lower leverage (lower R_u^p) and make less use of harder-to-resolve contracts (lower R_ℓ^p) such as short-maturity repurchase agreements that are exempt from the automatic stay.⁴² One explanation for why nonfinancials may make themselves fairly resolvable and have lower leverage is that they may have fairly high average liquidation discounts ([Antill, 2022](#)).⁴³ Consistent with these observations, in Appendix B.3, we study the special case of linear private benefits, $B_0(e_0) = b_0(1 - e_0)$, and show that both standard debt R_ℓ^p and total debt R_u^p are decreasing in the liquidation discount $1 - \gamma$.

6.2 Too-Big-To-Fail

Our model assumes all banks are small, but in practice bail-in regulation is often tailored to “too-big-to-fail” banks. Failure of a single large bank could generate a large enough fire sale that the

⁴¹Our normative results in Section 4 could potentially be viewed as rationalizing intervention in the non-financial corporate bankruptcy process if fire sale externalities are a concern, for example requiring greater issuance of easier to resolve junior unsecured debt. See [Antill and Clayton \(2024\)](#) for a related analysis of optimal intervention in the insolvency process for nonfinancials.

⁴²For example, the Flow of Funds (B.103) suggests that the debt-to-equity of nonfinancial corporates has been in the 20-40% range over the past decade, whereas capital requirements for systemically important financial institutions are in the range of 20% (i.e. a debt-equity ratio well above one). See [Gorton and Metrick \(2012\)](#) for a discussion of repurchase agreements.

⁴³[Antill \(2022\)](#) shows in a sample excluding acquisitions that switching from reorganization to liquidation in a given bankruptcy reduces expected creditor recovery across all debt claims by 42 cents on the dollar. Relatedly, [Bernstein et al. \(2019\)](#) provides evidence that firm liquidation persistently reduces the utilization of the firm’s real estate assets.

planner would prefer to avoid liquidation. In our model, this would mean $R_\ell^s \leq \underline{R}$.⁴⁴ Such regulation is privately costly because it reduces effort incentives, forcing lower debt and smaller project scale. However, this cost arises because we have modeled liquidation as an all-or-nothing decision. We now argue that partial liquidation of large banks can be desirable to mitigate externalities while providing incentives. We connect this idea to proposals for a good bank/bad bank approach to resolution. We then verbally discuss how our results might be affected by a large bank with an all-or-nothing liquidation decision.

Partial Liquidations of Large Banks. We model $N \geq 1$ ex ante identical large banks (in place of small banks). A large bank is an investment family (“holding company”) consisting of $\frac{1}{N}$ managers (“subsidiaries”), each of whom can undertake a project of the form in Section 2. Each holding company divides inside equity, $\frac{1}{N}A_0$, equally among its subsidiaries, and then coordinates external capital structure decisions for the family (i.e., the contract of each subsidiary).⁴⁵ We treat x as subsidiary cash flows pledged to external investors and c as cash flows pledged to the family.⁴⁶ Each manager independently operates her subsidiary to maximize the payoff of her project to her family. Incentive compatibility, monotonicity, and limited liability are specified at the subsidiary level. For simplicity, suppose there are no fire sales. It is easy to see in this case that every large bank chooses the same privately optimal contract as Proposition 1.⁴⁷ The privately optimal contract therefore results in a partial liquidation of each large bank, where subsidiaries with $R_1 \leq R_\ell^p$ are liquidated.

In this simple extension, subsidiary-level effort choice provides a role for partial liquidations of

⁴⁴This is a more extreme version of current regulations that impose surcharges in capital requirements for systemically important financial institutions.

⁴⁵For simplicity, our model abstracts away from the possibility that contracts are interconnected (that is, \mathcal{C} is only adapted to the individual subsidiary’s R_1) – for example a threat to liquidate other subsidiaries as well if one subsidiary performs badly. This is consistent with the idea that the manager must act to maximize payoff to her subsidiary’s equity holders, and not equity holders of other subsidiaries.

⁴⁶In our simple framework, there are two equivalent methods this could be achieved. One is that the subsidiary directly holds external bail-in debt. The other is that the holding company holds bail-in debt in the subsidiary, and then issues the same instrument externally. Thus a loss at the subsidiary is indirectly passed on to external investors. We treat these two methods as equivalent in the sense that both generate the same division of final payoffs x_t and c_t among outside investors and the family.

⁴⁷If there were fire sales, each large bank would internalize only a fraction $\frac{1}{N}$ of the fire sale cost, generating a role for the planner to intervene and impose the contract of Proposition 3.

poorly performing subsidiaries, while the holding company coordinates capital structure decisions. In addition to motivating a holding company structure that allows individual subsidiaries to fail, this extension also provides a novel rationale for clean holding company requirements: standard debt at the subsidiary level forces liquidation of that specific subsidiary. Partial liquidation could also be seen as a good bank/bad bank resolution approach, under which the best performing assets or subsidiaries (with $R_1 \geq R_\ell^p$) of the large bank are separated into the “good” bank and reorganized, while the worst performing assets or subsidiaries (with $R_1 < R_\ell^p$) are placed into the “bad” bank and liquidated. The size of the bad bank increases in R_ℓ^p , resulting in larger partial liquidations.

Large Banks without Partial Liquidations. What might happen if there were a large (positive mass) bank whose problem otherwise paralleled that of small (infinitesimal) banks, including the all-or-nothing liquidation choice? We verbally sketch and discuss this possibility when there is one large bank and a continuum small banks. Suppose that arbitrageurs have enough capacity to absorb small bank failures at a high fixed liquidation price, but failure by the large bank is enough to exhaust capacity and lead to a discretely lower liquidation price. The large bank would potentially want to use standard debt, internalizing that its failure leads to a fire sale but not caring about the negative externality on small banks. On the other hand, the social planner would have an incentive to reduce the probability of large bank failure to mitigate externalities on small banks. As in the baseline model, the planner faces a trade-off: failure of the large bank has social costs via the fire sale, but restricting its standard debt lowers its investment capabilities ex ante. In particular, if the likelihood ratio is sufficiently small at low returns, we conjecture that the planner would find it optimal for the large bank to use standard debt, even at the risk of large bank failure and accompanying fire sales. Intuitively as the likelihood ratio approaches zero, standard debt more closely resembles an *off-path* liquidation threat, since a bank that exerts high effort is unlikely to fail but a bank that exerts low effort is likely to fail.⁴⁸ As a result, the planner would likely be willing to endure small failure risks in exchange for larger ex ante scale, but might still sharply limit large bank standard debt relative to the private optimum. We further conjecture that if instead the

⁴⁸In fact, if $e_0^* = 1$ and $\Lambda_1(R_1') = 0$, then the liquidation threat is entirely off-path at $R_\ell \leq R_1'$.

severity of the fire sale were large and the likelihood ratio not too low, the planner would respond by requiring the large bank to set $R_l^s \leq \underline{R}$, eliminating large bank failures entirely. While in this environment the planner would impose a bail-in regime for the large bank, the planner would not impose the regime on small banks if small bank failures weren't contributing to fire sales.

6.3 Relation to Demand-Based Explanations for Standard Debt

Our model provides an incentive-based explanation for standard debt. Another important explanation is demand-based: investors assign a special preference to safe debt, which makes safe debt cheaper to issue (Bolton and Oehmke, 2019; Walther and White, 2020). Demand-based explanations are by no means mutually exclusive with ours, but it is important to highlight some of the differences.

Our model uses a single contracting friction, repeated unobservable effort, to rationalize the joint existence of standard and bail-in debt. A pure demand-based (safety premium) story has two important points to reconcile. First, a safety premium means that costly liquidations are also inefficient ex ante. Banks thus have strong incentives to write hedging contracts to protect standard debt and prevent liquidation. Yet, the costly insolvencies we see in practice are a basis for introducing bail-in regimes. Both Bolton and Oehmke (2019) and Walther and White (2020) reconcile this in part by assuming that banks are unable to write such hedging contracts (incomplete markets).⁴⁹ Our model predicts that banks do not hedge standard debt, that is banks use noncontingent contracts even though markets are complete. Our model thus provides a unified explanation for why we observe costly liquidations in practice. Second, a pure safety premium story implies Modigliani-Miller holds for the residual capital structure of the bank, giving no clear role for bail-in debt per se. Both Bolton and Oehmke (2019) and Walther and White (2020) incorporate one period of unobservable effort to give a clear role for bail-in debt over equity. Our model unifies the explanation for the desirability of both standard debt and bail-in debt over equity from repeated unobservable effort. Finally, our model uses the single externality of fire sales to rationalize joint regulation that pairs a

⁴⁹Bolton and Oehmke (2019) also offers an insightful observation that resolution authorities themselves may prevent hedging across countries “by ring-fencing assets” (p. 2390).

total leverage requirement (minimum equity requirement) with an additional limit on standard debt.

Incorporating a special preference for safe debt into our model can offer a more complete perspective on bank capital structure. For example, our model provides no role for deposit insurance, which impairs the incentive benefits of standard debt. Appendix B.2 provides a simple extension in which a planner protects a class of insured deposits to preserve their safety premium.⁵⁰ If liquidations are not too costly, the social planner allows banks to issue both insured standard debt (e.g., retail deposits) and uninsured standard debt (e.g., wholesale funding). Liquidation occurs when total standard debt exceeds pledgeable income, consistent with the common FDIC practice of resolving small insolvent banks by using either liquidation or merger, both of which could be seen as a possibly costly reallocation of the bank to the next best user.⁵¹

7 Conclusion

We develop a simple and tractable dynamic contracting model in which the privately optimal bank contract can be implemented with a combination of standard and bail-in debt. In the presence of fire sale externalities from bank failures, a social planner intervenes by altering both the level and composition of debt, rationalizing joint regulation of both elements of the bank's capital structure through a combination of a maximum leverage requirement and a TLAC requirement that can be satisfied by bail-in debt. Bail-ins are a desirable addition to the regulatory toolkit because bail-in debt is better suited than (outside) equity to address the incentive problem that motivated banks to issue standard debt in the first place. Our model suggests that bail-ins have the potential to replace bailouts in the post-crisis response kit, and that pairing bail-ins with statutory commitments against bailouts can be welfare-improving.

There are a few interesting possible directions in which our framework could be extended.

⁵⁰See for example Dewatripont and Tirole (2018) and Farhi and Tirole (2021) for safety premia as a motivation for deposit insurance.

⁵¹There is little meaningful difference between public and private insurance in this extension, so we could also view this as an explanation of secured standard debt, possibly also protected by hedges, and unsecured standard debt, not protected by hedges. It can also explain the exemption of repurchase agreements from the automatic stay: disorderly collateral seizure both protects secured creditors and promotes liquidation.

First, our model had two periods of effort choice and made assumptions so that high effort choice was optimal in the second stage. We conjecture that the key insights of our model would arise in a longer horizon model: liquidations would be costly but strong incentive devices, while maximal cash flow transfers (“bail-ins”) would be less costly but weaker incentive devices. It is possible that pledging contracts that induce future shirking (low effort) after low returns could be an optimal form of money burning, similar to liquidation, by reducing continuation agency rents. The policy implications of such a model would be an interesting avenue for future research. Second, our model assumes that the banker is not severable from the bank, that is the banker cannot be fired without also liquidating the bank. Standard debt thus transfers control rights to investors, whose best option is liquidation. If the banker were severable but firings were costly, we conjecture the government might prefer to use costly firings and bail-ins to provide high-powered incentives, rather than liquidations. Another interesting avenue for future research would be to consider socially optimal punishment schemes in this context.

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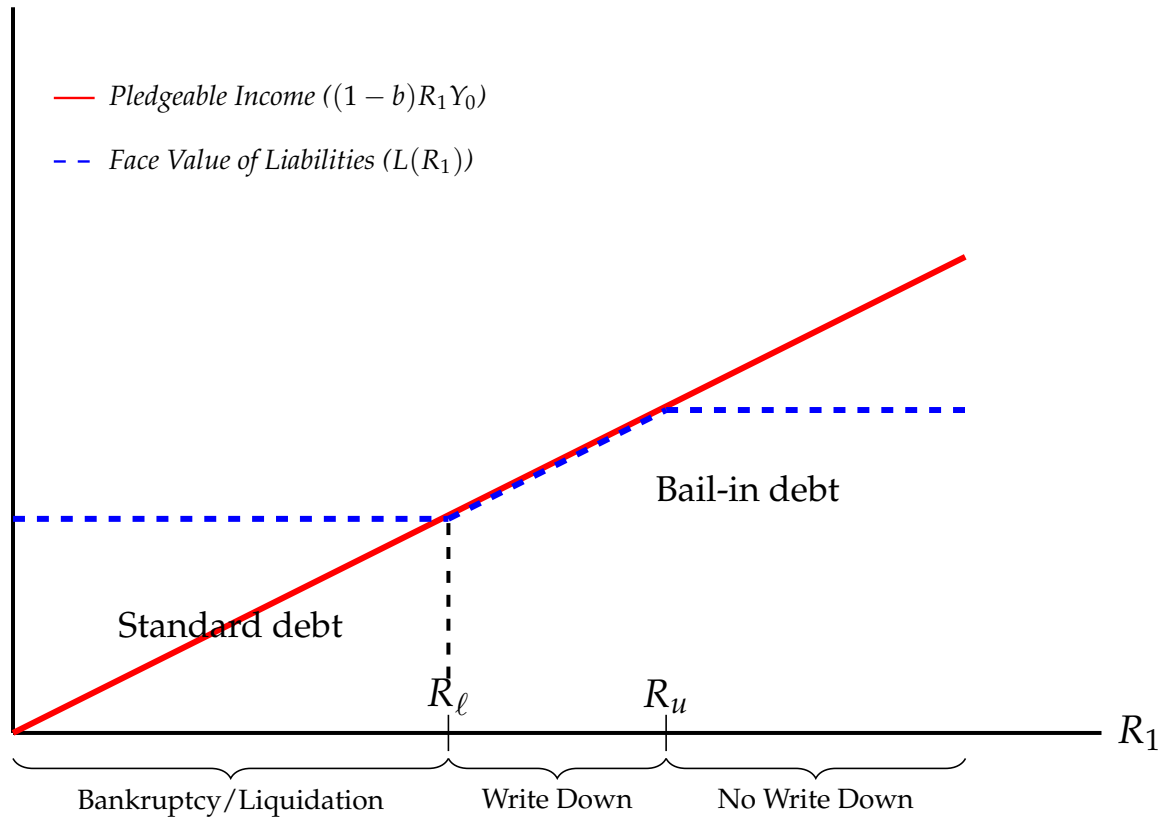


Figure 1: This figure provides an illustration for the form of the privately optimal contract. Up to a threshold R_ℓ , bank liabilities are constant and exceed pledgeable income, leading to liquidations. Between R_ℓ and R_u , the face value of liabilities is written down to coincide with pledgeable income (“bail-ins” or “write downs”). Above R_u , the face value of liabilities is constant. We refer to R_ℓ as the amount of “standard debt,” $R_u - R_\ell$ is the amount of “bail-in debt,” and R_u is the total amount of debt.

A Proofs

A.1 Definitions of Objects in Assumptions

We begin by defining the objects in Assumptions 2(d) and 2(e).

Assumption 2(d): Definition of \bar{B}_0 . We define

$$\bar{B}_0 = \mathbb{E}[R_1 - \tilde{x}(R_1)|e_0 = 1] - \mathbb{E}[R_1 - \tilde{x}(R_1)|e_0 = 0], \quad (30)$$

where $\tilde{x}(R_1) = \min\{\bar{\gamma}R^*, (1-b)R_1\}$, where R^* is defined implicitly by $\Lambda_1(R^*) = 1$, and where b is defined in Lemma 1. Intuitively, this bound means that the bank can issue $\bar{\gamma}R^*$ in bail-in debt while maintaining incentives for highest effort at date 0, $e_0^* = 1$.⁵²

Assumption 2(e): Definition of \underline{B}'_0 . We define

$$\underline{B}'_0 = b \cdot \sup_{R' | \Lambda_1(R') < 1} (\Lambda_1(R')^{-1} - 1) \int_{\underline{R}}^{R'} R_1 (f_{1L}(R_1) - f_{1H}(R_1)) dR_1. \quad (31)$$

This assumption ensures the incentive effects of liquidations are not so strong that the probability of bank failure decreases when the bank commits to liquidate more often. The lower bound is not necessary if the private benefit of effort is linear, $B_0(e_0) = b_0 \cdot (1 - e_0)$, since in that case effort is at an upper corner solution under optimal contracts.

A.2 Proof of Lemma 1

Consider the problem of the bank. The bank chooses a contract \mathcal{C} in order to maximize utility,

$$\mathbb{E}[c(R_1)|e_0 = e_0^*] + B_0(e_0^*)Y_0$$

subject to (IC-0),

$$-B'_0(e_0^*) = \mathbb{E}_0 \left[c(R_1)(\Lambda_1(R_1) - 1) \middle| e_0 = 0 \right],$$

⁵²Absent this assumption, optimal contracts would still combine standard and bail-in debt. However, in the corner solution where banks only issue standard debt, banks might find it optimal to commit to liquidation even when debt could be fully repaid, and pay themselves a liquidating dividend. This would serve the purpose of holding down repayment to investors to the standard debt level to maintain effort incentives, hence we assume that the bank can always pledge the liquidation value as a bail-in debt contract and maintain strong incentives.

to (IC-1),

$$\mathbb{E} \left[c_2(R_1, R_2)(\Lambda_2(R_2) - 1) \middle| e_1 = 0 \right] \geq B_1 R_1 Y_0,$$

to (P),

$$Y_0 - A_0 \leq \mathbb{E} [x(R_1) | e_0 = e_0^*].$$

and to monotonicity (M1) and (M2),

$$R_1 \geq R'_1 \Rightarrow x(R_1) \geq x(R'_1)$$

$$R_2 \geq R'_2 \Rightarrow x_2(R_1, R_2) \geq x_2(R_1, R'_2)$$

and the resource constraints (BC-1) and (BC-2),

$$c_1(R_1) = \alpha(R_1) \gamma R_1 Y_0 - x_1(R_1)$$

$$c_2(R_1, R_2) = (1 - \alpha(R_1)) R_1 R_2 Y_0 - x_2(R_1, R_2)$$

Observe that bank utility, (IC-0), (M1), and (P) depend only on (α, c, x) and not on the distribution of claims over time (c_t, x_t) .

The strategy is as follows. Let (α, c, x) be a contract that satisfies (IC-0), (P), and (M2). Take a return R_1 , if any, with $\alpha(R_1) = 0$. We look to see whether we can find a division of payoffs (c_t, x_t) so that it also satisfies (IC-1), (M2), (BC-1), and (BC-2). Formally, we can represent this problem as choosing (c_t, x_t) to maximize slack in incentive compatibility,

$$\mathcal{S}(R_1) = \mathbb{E} \left[c_2(R_1, R_2)(\Lambda_2(R_2) - 1) \middle| e_1 = 0 \right] - B_1 R_1 Y_0$$

subject to (M2), (BC-1), (BC-2), and subject to the definitions of $c(R_1)$ and $x(R_1)$. Thus the contract is feasible iff $\mathcal{S}(R_1) \geq 0$ for all R_1 with $\alpha(R_1) = 0$. The problem of maximizing slack in incentive compatibility, while pledging an amount of repayment to investors, is the dual of a standard [Innes \(1990\)](#) problem with binary effort, and hence we know the (weakly) optimal monotone contract is a debt contract (where $x_1(R_1) = 0$ is weakly optimal)

$$x_1(R_1) = 0$$

$$x_2(R_1, R_2) = \begin{cases} R_2 R_1 Y_0, & R_2 \leq R_2^u(R_1) \\ R_2^u(R_1) R_1 Y_0, & R_2 > R_2^u(R_1) \end{cases}$$

Since this contract satisfies (M2), it remains only to evaluate incentive compatibility from the

implied bank consumption profile generated from (BC-1) and (BC-2). We have

$$S(R_1) = \int_{R_2^u(R_1)}^{\bar{R}} \left[R_2 - R_2^u(R_1) \right] R_1 Y_0 (\Lambda_2(R_2) - 1) f_{2L}(R_2) dR_2 - B_1 R_1 Y_0,$$

which is nonnegative if

$$\int_{R_2^u(R_1)}^{\bar{R}} \left[R_2 - R_2^u(R_1) \right] (f_{2H}(R_2) - f_{2L}(R_2)) dR_2 \geq B_1.$$

Thus, define \bar{R}_2^u as the highest value that satisfies the above equation,

$$\int_{\bar{R}_2^u}^{\bar{R}} \left[R_2 - \bar{R}_2^u \right] (f_{2H}(R_2) - f_{2L}(R_2)) dR_2 = B_1,$$

and observe that it does not depend on R_1 . Thus, (IC-1) is satisfied only if $R_2^u(R_1) \leq \bar{R}_2^u$. Equivalently, a debt contract that guarantees (IC-1) exists only if

$$c(R_1) \geq b R_1 Y_0$$

where $b \equiv \int_{\bar{R}_2^u}^{\bar{R}} [R_2 - \bar{R}_2^u] f_{2H}(R_2) dR_2$. This concludes the proof.

A.3 Proof of Proposition 1

Exploiting Lemma 1, we represent the problem over (α, x, c) , with Lemma 1 providing the split (c_t, x_t) . After deriving the optimal contract, we represent it in terms of a liability structure L .

The bank's optimization problem is:

$$\max_{\alpha, x, c, e_0^*, Y_0} \mathbb{E}[c(R_1) | e_0 = e_0^*] + B_0(e_0^*) Y_0$$

subject to

$$-B_0'(e_0^*) Y_0 = \mathbb{E}_0 \left[c(R_1) (1 - \Lambda_1(R_1)^{-1}) \middle| e_0 = 1 \right].$$

$$Y_0 - A = \mathbb{E}[x(R_1) | e_0 = e_0^*]$$

$$R_1 \geq R_1' \Rightarrow x(R_1) \geq x(R_1')$$

$$c(R_1) \geq (1 - \alpha(R_1)) b R_1 Y_0$$

$$x(R_1) = \alpha(R_1) \gamma R_1 Y_0 + (1 - \alpha(R_1)) R_1 Y_0 - c(R_1)$$

where the first constraint is IC-0, the second is (P), the third is (M1), the fourth is the pledgeability constraint (equation 18 from Lemma 1), and the last is consolidated budget constraint from (BC-1) and (BC-2). It slightly eases exposition to write in terms of Λ_1^{-1} above. Bank limited liability is implied by the pledgeability constraint and so is dropped.⁵³

The proof strategy is to first show that if we conjectured the monotonicity constraint (M1), the contract would be non-monotone at the top. This will lead to an upper pooling region in investor repayment. We then derive the optimal contract, conjecturing a pooling threshold, and show monotonicity does not bind below that pooling threshold. Finally, we solve for the optimal thresholds.

Conjecture first that the monotonicity constraint never binds. For purely technical reasons, we introduce an investor limited liability constraint $x(R_1) \geq -\bar{x}$ on only the relaxed problem. We show that this relaxed problem generates a contract that violates monotonicity at the top, giving rise to an upper pooling region. The Lagrangian of this relaxed problem, substituting out x via the resource constraint, is

$$\begin{aligned} \mathcal{L} = & \mathbb{E}[c|e_0 = e_0^*] + B_0(e_0^*)Y_0 \\ & + \mu [\mathbb{E}[c(1 - \Lambda_1^{-1})|e_0 = 1] + B'_0(e_0^*)Y_0] \\ & + \lambda [\mathbb{E}[\alpha\gamma R_1 Y_0 + (1 - \alpha)R_1 Y_0 - c|e_0 = e_0^*] + A_0 - Y_0] \\ & + \mathbb{E}[\chi(c - (1 - \alpha)bR_1 Y_0)|e_0 = 1] \\ & + \mathbb{E}[\zeta((\alpha\gamma R_1 Y_0 + (1 - \alpha)R_1 Y_0 - c + \bar{x}))|e_0 = 1] \end{aligned}$$

where the last line is the investor limited liability constraint introduced for technical reasons $x \geq -\bar{x}$. The conditionings of the last two lines on $e_0 = 1$ are without loss and correspond to different selections of Lagrange multipliers. From here, first order condition for bank consumption $c(R_1)$ is

$$\begin{aligned} 0 &= f_1(R_1|e_0^*) + \mu(1 - \Lambda_1(R_1)^{-1})f_{1H}(R_1) - \lambda f_1(R_1|e_0^*) + \chi(R_1)f_{1H}(R_1) - \zeta f_{1H}(R_1) \\ &= \frac{f_1(R_1|e_0^*)}{f_{1H}(R_1)}(1 - \lambda) + \mu(1 - \Lambda_1(R_1)) + \chi(R_1) - \zeta(R_1) \\ &= (1 - \lambda)e_0^* + \mu - \left[(\lambda - 1)(1 - e_0^*) + \mu \right] \Lambda_1(R_1)^{-1} + \chi(R_1) - \zeta(R_1) \end{aligned}$$

By MLRP, there is a threshold R^* such that $\chi(R_1) > 0$ for $R_1 \leq R^*$ and $\zeta(R_1) > 0$ for $R_1 \geq R^*$. This threshold is given by

$$(1 - \lambda)e_0^* + \mu - \left[(\lambda - 1)(1 - e_0^*) + \mu \right] \Lambda_1(R^*)^{-1} = 0 \quad (32)$$

⁵³The problem is set up assuming $e_1^*(R_1) = 1$, which is then verified to be optimal.

However, this contract violates monotonicity unless $x(R_1)$ is constant for all R_1 or $R^* = \bar{R}$. Therefore, we have an upper pooling region in the optimal contract, where investor repayment is constant.⁵⁴

We now characterize the optimal contract using the following strategy. First, we conjecture a pooling threshold R_u with corresponding $x_u \equiv x(R_u)$, so that $x(R_1) = x_u$ for all $R_1 \geq R_u$. Note that this is without loss, since the pooling threshold could be $R_u = \bar{R}$. We then solve for the optimal contract below R_u , taking as given R_u and x_u , subject to monotonicity and subject to $x(R_1) \leq x_u$, and verify that the resulting contracting is monotone. In doing so, we characterize the space of implementable contracts (that satisfy monotonicity). Finally, we optimize over the choice of R_u and x_u .

Conjecture a pooling threshold R_u with corresponding liabilities x_u . Observe that in the analysis that follows if the constraint $x(R_1) \leq x_u$ binds for some $R'_u < R_u$, then we can without loss of generality discard (R_u, x_u) as a candidate and instead consider the contract (R'_u, x_u) . What remains therefore is monotonicity within $R_1 < R_u$ for candidate thresholds. We solve the relaxed problem not subject to monotonicity and verify it generates a monotone liability structure. The associated Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & \mathbb{E}[c|e_0 = e_0^*] + B_0(e_0^*)Y_0 \\ & + \mu [\mathbb{E}[c(1 - \Lambda_1^{-1})|e_0 = 1] + B'_0(e_0^*)Y_0] \\ & + \lambda [\mathbb{E}[\alpha\gamma R_1 Y_0 + (1 - \alpha)R_1 Y_0 - c|e_0 = e_0^*] + A_0 - Y_0] \\ & + \mathbb{E}[\chi(c - (1 - \alpha)bR_1 Y_0)|e_0 = 1] \end{aligned}$$

where note that we have now dropped the ad hoc investor limited liability constraint used above. Taking the derivative in consumption $c(R_1)$ for $R_1 \leq R_u$, we obtain

$$\begin{aligned} 0 = & (1 - \lambda) \frac{f_1(R_1|e_0^*)}{f_{1H}(R_1)} + \mu(1 - \Lambda_1(R_1)^{-1}) + \chi(R_1). \\ 0 = & (1 - \lambda)e_0^* + \mu - \left[(\lambda - 1)(1 - e_0^*) + \mu \right] \Lambda_1(R_1)^{-1} + \chi(R_1). \end{aligned}$$

Since the resulting contract is non-monotone if $R_u > R^*$ (MLRP), we discard candidate contracts with $R_u > R^*$. From the definition of R^* and from MLRP, we therefore have $\chi(R_1) > 0$ for all $R_1 < R_u$.

⁵⁴If $L(R_1)$ is constant, then the entire contract is pooled. If $R_1^* = \bar{R}$, then the results that follow apply setting $R_u = \bar{R}$ to be the pooling threshold.

Now, consider the derivative in liquidations $\alpha(R_1)$, given by⁵⁵

$$\frac{\partial \mathcal{L}}{\partial \alpha(R_1)} = \left[-\lambda(1-\gamma) + \frac{f_{1H}(R_1)}{f_1(R_1|e_0^*)} \chi(R_1)b \right] R_1 Y_0 f_1(R_1|e_0^*)$$

Substituting in for $\chi(R_1)$, observe that we have

$$\frac{f_{1H}(R_1)}{f_1(R_1|e_0^*)} \chi(R_1) = (\lambda - 1) + \mu \frac{\Lambda_1(R_1)^{-1} - 1}{e_0^* + (1 - e_0^*)\Lambda_1(R_1)^{-1}}$$

Therefore, by MLRP liquidations are only potentially valuable in the region where $R_1 \leq \hat{R}$, where \hat{R} is defined by $\Lambda_1(\hat{R})^{-1} = 1$. Since $\frac{\Lambda_1(R_1)^{-1} - 1}{e_0^* + (1 - e_0^*)\Lambda_1(R_1)^{-1}}$ is an increasing function of $\Lambda_1(R_1)^{-1}$ in the region $R_1 < \hat{R}$, then by MLRP there is a threshold R_ℓ such that $\alpha(R_1) = 1$ if and only if $R_1 < R_\ell$ (note it is possible for $R_\ell = \underline{R}$). Observe that since $\chi(R_1) > 0$ in this region, then $c(R_1) = 0$ in liquidation.

Finally, in the region (if non-empty) between R_ℓ and R_u , we know that $\chi(R_1) > 0$ and therefore $x(R_1) = (1 - b)R_1 Y_0$ for all $R_\ell \leq R_1 \leq R_u$.

As a result, the optimal contract is a three-part structure. First, there is a lower region $R_1 < R_\ell$ in which we have $\alpha(R_1) = 1$ and $x(R_1) = \gamma R_1 Y_0$ (and hence $c(R_1) = 0$). Second, there is a middle region $R_\ell \leq R_1 \leq R_u$ such that $\alpha(R_1) = 0$ and $x(R_1) = (1 - b)R_1 Y_0$, and hence $c(R_1) = bR_1 Y_0$. Finally, there is an upper region $R_1 \geq R_u$ in which $\alpha(R_1) = 0$ and $x(R_1) = x_u$, and hence $c(R_1) = R_1 Y_0 - x_u$.⁵⁶ To complete this part of the characterization, we show that there cannot be an upward discontinuity in x at R_u . If there were an upward discontinuity, we would have

$$x_u > \lim_{R_1 \uparrow R_u} x(R_1) = (1 - b)R_u Y_0$$

and hence the pledgeability constraint is violated at R_u , a contradiction. The capital structure is therefore continuous at R_u .⁵⁷

Finally, the above capital structure can be implemented by a liabilities contract $L(R_1) = (1 - b)R_\ell Y_0$ for $R_1 \leq R_\ell$ and $L(R_1) = x(R_1)$ for $R_1 > R_\ell$, as stated in the proposition. We have now

⁵⁵Implicitly, we are treating $\alpha(R_1)$ as a continuous variable in performing the differentiation. To do so, we implicitly incorporate the constraint $\alpha(R_1)(1 - \alpha(R_1)) = 0$, which ensures that implementable contracts must set $\alpha(R_1) \in \{0, 1\}$. The logic below is unaffected.

⁵⁶We have ruled out the possibility of $R_\ell = R_u$ but liquidations extend into the pooling region by the bound \bar{B}_0 (Assumption 2(d)). Formally, if this were to hold, then by continuity $x_u = \gamma R_\ell Y_0 \leq \bar{\gamma} R_\ell Y_0$. But since we have assumed that pledging $R_\ell = 0$ and $x_u = \bar{\gamma} R_\ell Y_0$ induces $e_0^* = 1$, then a contract with liquidations in the pooling region is trivially dominated by this contract.

⁵⁷Assumption 2(f) guarantees that low effort is never optimal. In the region $R_1 < R_u^*$, low effort is dominated by liquidation, since liquidation both increases investor payoff and reduces bank payoff. In the region $R_1 \geq R_u^*$, low effort is dominated by continuation, since continuation increases payoffs available to both investors and banks.

proven the first part of the proposition.

Optimal Thresholds. Finally, we characterize the optimal thresholds R_ℓ and R_u . To do so, we internalize the determination of effort, $e_0^*(R_\ell, R_u)$, into the problem, where e_0^* solves

$$-B'_0(e_0^*(R_\ell, R_u)) = \int_{R_\ell}^{R_u} bR(1 - \Lambda_1(R_1)^{-1})f_{1H}(R_1)dR_1 + \int_{R_u}^{\bar{R}} [R_1 - (1 - b)R_u](1 - \Lambda_1(R_1)^{-1})f_{1H}(R_1)dR_1$$

Focusing on the case where thresholds are interior and not equal, $\underline{R} < R_\ell < R_u < \bar{R}$, the bank Lagrangian is

$$\begin{aligned} \mathcal{L} = & \int_{R_\ell}^{R_u} bR_1Y_0f_1(R_1|e_0^*)dR_1 + \int_{R_u}^{\bar{R}} [R_1 - (1 - b)R_u]Y_0f_1(R_1|e_0^*)dR_1 + B_0(e_0^*)Y_0 \\ & + \lambda \left[A_0 - Y_0 + \int_{\underline{R}}^{R_\ell} \gamma R_1Y_0f_1(R_1|e_0^*)dR_1 + \int_{R_\ell}^{R_u} (1 - b)R_1Y_0f_1(R_1|e_0^*)dR_1 + \int_{R_u}^{\bar{R}} (1 - b)R_uY_0f_1(R_1|e_0^*)dR_1 \right] \end{aligned}$$

By Envelope Theorem, the optimality condition for R_ℓ is

$$0 = -bR_\ell Y_0 f_1(R_\ell | e_0^*) + \lambda \left(\gamma - (1 - b) \right) R_\ell Y_0 f_1(R_\ell | e_0^*) + \lambda \int_{\underline{R}}^{\bar{R}} x(R_1) (f_{1H}(R_1) - f_{1L}(R_1)) \frac{\partial e_0^*}{\partial R_\ell} dR_1$$

From above, we have

$$-B''_0(e_0^*(R_\ell, R_u)) \frac{\partial e_0^*}{\partial R_\ell} = bR_\ell (\Lambda_1(R_\ell)^{-1} - 1) f_{1H}(R_\ell)$$

and therefore substituting in and rearranging, we obtain

$$\frac{b(\Lambda_1(R_\ell)^{-1} - 1) \frac{f_{1H}(R_\ell)}{f_1(R_\ell | e_0^*)}}{|B''_0(e_0^*)|} \lambda \int_{\underline{R}}^{\bar{R}} \frac{x(R_1)}{Y_0} (f_{1H}(R_1) - f_{1L}(R_1)) dR_1 = b + \lambda (1 - b - \gamma).$$

From here, we define $G = \mathbb{E}[\frac{x(R_1)}{Y_0} | e_0 = 1] - \mathbb{E}[\frac{x(R_1)}{Y_0} | e_0 = 0]$ to obtain

$$\frac{1 - \Lambda_1(R_\ell)}{(1 - e_0^*) + e_0^* \Lambda_1(R_\ell)} \frac{1}{|B''_0(e_0^*)|} b \lambda G = b + \lambda (1 - b - \gamma)$$

giving the first result.

Analogously, the optimality condition for R_u is

$$0 = -(1 - b) \int_{R_u}^{\bar{R}} Y_0 f_1(R_1 | e_0^*) dR_1 + \lambda (1 - b) \int_{R_u}^{\bar{R}} Y_0 f_1(R_1 | e_0^*) dR_1 + \lambda \int_{\underline{R}}^{\bar{R}} x(R_1) (f_{1H}(R_1) - f_{1L}(R_1)) \frac{\partial e_0^*}{\partial R_u} dR_1$$

From above, we have

$$-B_0''(e_0^*(R_\ell, R_u)) \frac{\partial e_0^*}{\partial R_u} = -(1-b) \int_{R_u}^{\bar{R}} (1 - \Lambda_1(R_1)^{-1}) f_{1H}(R_1) dR_1$$

and therefore substituting and rearranging, we obtain

$$\lambda \frac{\int_{R_u}^{\bar{R}} (f_{1H}(R_1) - f_{1L}(R_1)) dR_1}{|B_0''(e_0^*)|} \int_{\underline{R}}^{\bar{R}} \frac{x(R_1)}{Y_0} (f_{1H}(R_1) - f_{1L}(R_1)) dR_1 = (\lambda - 1) \int_{R_u}^{\bar{R}} f_1(R_1 | e_0^*) dR_1$$

$$\frac{F_{1L}(R_1) - F_{1H}(R_u)}{|B_0''(e_0^*)|} \lambda G = (\lambda - 1)(1 - F_1(R_u | e_0^*))$$

which concludes the proof.

A.3.1 A Remark on Contract Uniqueness

The optimal contract, expressed in liabilities L , is not generally unique in the following sense. In the region $R \leq R_\ell$, the bank only needs a liability face value that is sufficient to liquidate the bank, and so any contract with monotone face value $L(R_1) > (1-b)R_1Y_0$ in this region is optimal. We selected the contract with a flat face value below R_ℓ due to its correspondence to standard debt.

A.4 Proof of Corollary 1

Consider the proposed liability structure. The amount $(1-b)R_\ell Y_0$ of short-term standard debt liquidates the bank when $R_1 \leq R_\ell$, generating the lower region. Long-term bail-in debt $(1-b)(R_u - R_\ell)$ is written down to $(1-b)(R_1 - R_\ell)$ in the region $R_\ell \leq R_1 \leq R_u$, so that the bank is always held to the agency rent over this region (while short-term standard debt is fully repaid). The full debt level $(1-b)R_u Y_0$ (both short-term standard debt and long-term bail-in debt) is repaid above R_u . Therefore, we replicate the contract in Proposition 1.

A.5 Proof of Proposition 2

We split the proof into the different cases.

Case 1: Suppose first that $B_0(e_0) = 0$. Optimal effort at date 0 satisfies the corner solution $e_0^* = 1$ if

$$E[c(R_1)(\Lambda(R_1) - 1) | e_0 = 0] \geq 0,$$

and satisfies $e_0^* = 0$ otherwise. For any monotone c , we have

$$E [c(R_1)(\Lambda_1(R_1) - 1) | e_0 = 0] = \text{cov}(c(R_1), \Lambda_1(R_1) - 1) \geq 0$$

where the inequality follows from MLRP. As a result, any monotone consumption rule c satisfies (IC-0) at $e_0^* = 1$. Thus the optimal contract structure is determined by date 2 payoffs of Lemma 1, with no liquidations at date 1, and corresponds to bail-in debt.

Case 2: Consider next $B_1 = 0$. From Lemma 1, $\bar{R}_2^u = \bar{R}$ and therefore $b = 0$. The RHS of (20) collapses to $\lambda(1 - \gamma) > 0$ while the LHS collapses to 0, and so banks choose a corner solution $R_\ell = \underline{R}$. Optimal contracts use only bail-in debt.

Case 3: Consider finally $\gamma = 1$. Any face value $L(R_1) \leq R_1 Y_0$ can then be repaid by liquidating assets, so that bank consumption is $c(R_1) = R_1 Y_0 - L(R_1)$ for any $L(R_1) \leq R_1 Y_0$. Therefore for any liability structure $L(R_1)$, we can define

$$(c(R_1), x(R_1)) = \begin{cases} (R_1 Y_0 - L(R_1), L(R_1)), & L(R_1) \leq R_1 Y_0 \\ (0, R_1 Y_0), & L(R_1) \geq R_1 Y_0 \end{cases}$$

where the relevant liquidation function $\alpha(R_1)$ is defined from the liability structure. Minimum pledgeability never binds. One interpretation is that if $(1 - b)R_1 Y_0 < L(R_1) < R_1 Y_0$, then we have liquidation with a liquidating dividend paid to equity.

Defining the problem in the repayment space, we then have

$$\max \int_{R_1} [R_1 Y_0 - x(R_1)] f_1(R_1 | e_0^*) dR_1 + B_0(e_0^*) Y_0$$

subject to

$$-B_0'(e_0^*) Y_0 = \int_{R_1} [R_1 Y_0 - x(R_1)] (1 - \Lambda_1(R_1)^{-1}) f_{1H}(R_1) dR_1$$

$$Y_0 - A = \int_{R_1} x(R_1) f_1(R_1 | e_0^*) dR_1$$

$$R_1 \geq R_1' \Rightarrow x(R_1) \geq x(R_1')$$

with $0 \leq x(R_1) \leq R_1 Y_0$. This problem is therefore identical to the baseline model except that liquidation is no longer an explicit choice variable but is instead implied by the repayment structure. MLRP implies the existence of an upper pooling region as in the proof of Proposition 1. Thus as in the proof of Proposition 1, we have an upper pooling region with $x(R_1) = x_u$ for $R_1 \geq R_u$ and $x(R_1) = R_1 Y_0$ for all $R \leq R_u$. The same continuity argument implies $x(R_u) = R_u Y_0$. Hence, we can

set $L(R_u) = R_u Y_0$, which is standard debt.

A.6 Proof of Proposition 3

The proof of Lemma 1 follows identically from before. The decision problem of the social planner is therefore

$$\max_{\alpha, x, c, e_0^*, \gamma} \mathbb{E}[c(R_1)|e_0 = e_0^*] + B_0(e_0^*)Y_0$$

subject to

$$-B'_0(e_0^*)Y_0 = \mathbb{E}_0 \left[c(R_1)(1 - \Lambda_1(R_1)^{-1}) \middle| e_0 = 1 \right].$$

$$Y_0 - A = \mathbb{E}[x(R_1)|e_0 = e_0^*]$$

$$R_1 \geq R'_1 \Rightarrow x(R_1) \geq x(R'_1)$$

$$c(R_1) \geq (1 - \alpha(R))bR_1Y_0$$

$$c(R_1) = \alpha\gamma R_1Y_0 + (1 - \alpha)\gamma R_1Y_0 - x(R_1)$$

$$\gamma = \gamma(\Omega), \quad \Omega = \int \alpha(R_1)R_1 f_1(R_1|e_0^*) dR_1$$

All aspects of this optimization problem are identical to the private program in the proof of Proposition 1, except that the planner internalizes the effect on the liquidation price. We proceed by internalizing the liquidation price determination γ and the liability structure x into the decision problem.

The proof now proceeds in parallel to that of Proposition 1. The Lagrangian of the relaxed problem not subject to monotonicity is analogous to Proposition 1, with the endogenous liquidation price,

$$\begin{aligned} \mathcal{L} = & \mathbb{E}[c|e_0 = e_0^*] + B_0(e_0^*)Y_0 \\ & + \mu^s [\mathbb{E}[c(1 - \Lambda_1^{-1})|e_0 = 1] + B'_0(e_0^*)Y_0] \\ & + \lambda^s [\mathbb{E}[\alpha\gamma(\Omega)R_1Y_0 + (1 - \alpha)R_1Y_0 - c|e_0 = e_0^*] + A_0 - Y_0] \\ & + \mathbb{E}[\chi^s (c - (1 - \alpha)bR_1Y_0)|e_0 = 1] \\ & + \mathbb{E}[\zeta^s ((\alpha\gamma(\Omega)R_1Y_0 + (1 - \alpha)R_1Y_0 - c + \bar{x}))|e_0 = 1] \end{aligned}$$

where we use the s notation to distinguish social planner's Lagrange multipliers from the bank's Lagrange multipliers. Since the liquidation price does not depend directly on $c(R_1)$, the first order

condition for consumption $c(R_1)$ thus takes the same form as in the proof of Proposition 1,

$$0 = (1 - \lambda^s)e_0^* + \mu^s - \left[(\lambda^s - 1)(1 - e_0^*) + \mu^s \right] \Lambda_1(R_1)^{-1} + \chi^s(R_1),$$

and so by MLRP there is a threshold R^s such that $\chi^s(R_1) > 0$ for $R_1 \leq R^s$ and $\chi^s(R_1) > 0$ for $R_1 \geq R^s$, where we have

$$(1 - \lambda^s)e_0^* + \mu^s - \left[(\lambda^s - 1)(1 - e_0^*) + \mu^s \right] \Lambda_1(R^s) = 0 \quad (33)$$

Therefore, as in the proof of Proposition 1, we have an upper pooling region.

Let R_u^s be the pooling threshold and x_u^s be the pooling level, so that $x(R_1) = x_u^s$ for $R_1 \geq R_u^s$. As before, take R_u^s and x_u^s as given, and solve for the optimal contract for $R_1 \leq R_u^s$. The planner's Lagrangian is

$$\begin{aligned} \mathcal{L} = & \mathbb{E}[c|e_0 = e_0^*] + B_0(e_0^*)Y_0 \\ & + \mu^s \left[\mathbb{E}[c(1 - \Lambda_1^{-1})|e_0 = 1] + B_0'(e_0^*)Y_0 \right] \\ & + \lambda^s \left[\mathbb{E}[\alpha\gamma(\Omega)R_1Y_0 + (1 - \alpha)R_1Y_0 - c|e_0 = e_0^*] + A_0 - Y_0 \right] \\ & + \mathbb{E}[\chi^s(c - (1 - \alpha)bR_1Y_0)|e_0 = 1] \end{aligned}$$

The same steps imply that implementable contracts must satisfy $R_u < R^s$ for the same definition of R^s , else the contract would be non-monotone. Taking the derivative in consumption $c(R_1)$ for $R_1 \leq R_u$, we obtain

$$0 = (1 - \lambda^s) \frac{f_1(R_1|e_0^*)}{f_{1H}(R_1)} + \mu^s(1 - \Lambda_1(R_1)) + \chi^s(R_1).$$

Therefore, as in the proof of Proposition 1, we have $\chi^s(R_1) > 0$. Next taking the FOC for liquidations $\alpha(R_1)$, we have

$$\frac{\partial \mathcal{L}}{\partial \alpha(R_1)} = \lambda^s(\gamma - 1)R_1Y_0f_1(R_1|e_0^*) + \chi^s bY_0f_{1H}(R_1) + \lambda^s \frac{d\gamma(\Omega)}{d\alpha(R_1)} \mathbb{E} \left[\alpha R_1 Y_0 \middle| e = e_0^* \right]$$

We have $\frac{d\gamma}{d\alpha(R_1)} = \frac{\partial \gamma}{\partial \Omega} R_1 f_1(R_1|e_0^*)$ and $\mathbb{E} \left[\alpha R_1 \middle| e = e_0^* \right] = \Omega$, so using the definition $\sigma = -\frac{\Omega}{\gamma} \frac{\partial \gamma}{\partial \Omega}$ we can write

$$\frac{\partial \mathcal{L}}{\partial \alpha(R_1)} = \left[\lambda^s(\gamma - 1 - \sigma\gamma) + \frac{f_{1H}(R_1)}{f_1(R_1|e_0^*)} \chi^s(R_1) b \right] f_1(R_1|e_0^*) Y_0.$$

Thus, as in the proof of Proposition 1, substituting in for $\chi^s(R_1)$ and applying MLRP shows that

there is a threshold R_ℓ^s for liquidations, that is $\alpha(R_1) = 1$ iff $R < R_\ell^s$. As in the proof of Proposition 1, since $\chi(R_1) > 0$ then $x(R_1) = \gamma R_1 Y_0$ when $R_1 < R_\ell^s$ and $x(R_1) = (1-b)R_1 Y_0$ when $R_\ell^s \leq R_1 \leq R_u^s$. Finally, the same continuity argument implies $x_u^s = (1-b)R_u^s Y_0$.

Therefore, we have shown that the socially optimal contract can be implemented by a combination of standard and bail-in debt, with corresponding thresholds R_ℓ^s and R_u^s .⁵⁸

Socially Optimal Thresholds. Finally, we characterize the socially optimal thresholds R_ℓ^s and R_u^s . As in the proof of Proposition 1, we define effort $e_0^*(R_\ell, R_u)$ implicitly as the solution to

$$-B'_0(e_0^*(R_\ell, R_u)) = \int_{R_\ell^s}^{R_u^s} bR(1-\Lambda_1(R_1)^{-1})f_{1H}(R_1)dR_1 + \int_{R_u^s}^{\bar{R}} [R_1 - (1-b)R_u^s](1-\Lambda_1(R_1)^{-1})f_{1H}(R_1)dR_1$$

It is important to note that effort does not depend directly on the liquidation price. The social planner's Lagrangian is

$$\begin{aligned} \mathcal{L} = & \int_{R_\ell^s}^{R_u^s} bR_1 Y_0 f_1(R_1|e_0^*)dR_1 + \int_{R_u^s}^{\bar{R}} [R_1 - (1-b)R_u^s] Y_0 f_1(R_1|e_0^*)dR_1 + B_0(e_0^*)Y_0 \\ & + \lambda^s \left[A_0 - Y_0 + \int_{\underline{R}}^{R_\ell^s} \gamma(\Omega)R_1 Y_0 f_1(R_1|e_0^*)dR_1 + \int_{R_\ell^s}^{R_u^s} (1-b)R_1 Y_0 f_1(R_1|e_0^*)dR_1 \right. \\ & \left. + \int_{R_u^s}^{\bar{R}} (1-b)R_u^s Y_0 f_1(R_1|e_0^*)dR_1 \right] \end{aligned}$$

where $\Omega = \int_{\underline{R}}^{R_\ell^s} R_1 f_1(R_1|e_0^*(R_\ell^s, R_u^s))dR_1$.

The optimality condition for R_ℓ^s is

$$\begin{aligned} 0 = & -bR_\ell^s Y_0 f_1(R_\ell^s|e_0^*) - \lambda^s (1-b-\gamma)R_\ell^s Y_0 f_1(R_\ell^s|e_0^*) + \lambda^s \int x(R_1)(f_{1H}(R_1) - f_{1L}(R_1)) \frac{\partial e_0^*}{\partial R_\ell} dR_1 \\ & + \lambda^s \int_{\underline{R}}^{R_\ell^s} \frac{\partial \gamma}{\partial \Omega} \frac{d\Omega}{dR_\ell^s} R_1 Y_0 f_1(R_1|e_0^*)dR_1 \end{aligned}$$

The first line parallels the terms from the bank's optimum, while the second line accounts for the fire sale externality. Observe that we have

$$\lambda^s \int_{\underline{R}}^{R_\ell^s} \frac{\partial \gamma}{\partial \Omega} \frac{d\Omega}{dR_\ell^s} R_1 Y_0 f_1(R_1|e_0^*)dR_1 = \lambda^s \frac{\partial \gamma}{\partial \Omega} \Omega \frac{d\Omega}{dR_\ell^s} Y_0 = \lambda^s \sigma \gamma \frac{d\Omega}{dR_\ell^s} Y_0$$

which follows from the definition of σ . Finally, we have

$$\frac{d\Omega}{dR_\ell^s} = \frac{d}{dR_\ell^s} \int_{\underline{R}}^{R_\ell^s} R_1 f_1(R_1|e_0^*)dR_1 = R_\ell^s f_1(R_\ell^s|e_0^*) + \int_{\underline{R}}^{R_\ell^s} R_1 (f_{1H}(R_1) - f_{1L}(R_1)) \frac{\partial e_0^*}{\partial R_\ell^s} dR_1$$

⁵⁸As in the proof of Proposition 1, Assumption 2(d) rules out liquidating dividends. Assumptions 2(f) and 3 rule out low effort at date 1 by the same dominance argument as in the proof of Proposition 1.

We can therefore, following the steps of the proof of Proposition 1, rearrange to obtain

$$\frac{1 - \Lambda_1(R_\ell^s)}{(1 - e_0^*) + e_0^* \Lambda_1(R_\ell^s)} \frac{1}{|B_0''(e_0^*)|} b \lambda^s G^s = b + \lambda^s (1 - b - \gamma^s) + \tau_\ell^s$$

where the “wedge” τ_ℓ^s is given by

$$\begin{aligned} \tau_\ell^s &= - \frac{\frac{d\Omega}{dR_\ell^s}}{R_\ell^s Y_0 f_1(R_\ell^s | e_0^*)} \lambda^s \sigma \gamma^s Y_0 \\ &= \left(1 - \frac{1 - \Lambda_1(R_\ell^s)}{(1 - e_0^*) + e_0^* \Lambda_1(R_\ell^s)} \frac{1}{|B_0''(e_0^*)|} b \int_{\underline{R}}^{R_\ell^s} R_1 (f_{1L}(R_1) - f_{1H}(R_1)) dR_1 \right) \lambda^s \sigma \gamma^s \\ &= \left(1 - \frac{1 - \Lambda_1(R_\ell^s)}{(1 - e_0^*) + e_0^* \Lambda_1(R_\ell^s)} \frac{1}{|B_0''(e_0^*)|} b L^s \right) \lambda^s \sigma \gamma^s \end{aligned}$$

where $L^s = \int_{\underline{R}}^{R_\ell^s} R_1 (f_{1L}(R_1) - f_{1H}(R_1)) dR_1$. Given $|B_0''| \geq \underline{B}_0''$, we have $\tau_\ell^s \geq 0$.

Analogously, the optimality condition for R_u^s is

$$\begin{aligned} 0 &= (\lambda^s - 1)(1 - b) \int_{R_u^s}^{\bar{R}} Y_0 f_1(R_1 | e_0^*) dR_1 + \lambda^s \int x(R_1) (f_{1H}(R_1) - f_{1L}(R_1)) \frac{\partial e_0^*}{\partial R_u} dR_1 \\ &\quad + \lambda^s \int_{\underline{R}}^{R_u^s} \frac{\partial \gamma}{\partial \Omega} \frac{d\Omega}{dR_u^s} R_1 Y_0 f_1(R_1 | e_0^*) dR_1 \end{aligned}$$

Following the same steps as the proof of Proposition 1, we thus have

$$\frac{F_{1L}(R_u^s) - F_{1H}(R_u^s)}{|B_0''(e_0^*)|} \lambda^s G^s = (\lambda^s - 1)(1 - F_1(R_u^s | e_0^*)) - \tau_u^s$$

where we have defined

$$\tau_u^s = - \frac{1}{(1 - b) Y_0} \lambda^s \int_{\underline{R}}^{R_u^s} \frac{\partial \gamma}{\partial \Omega} \frac{d\Omega}{dR_u^s} R_1 Y_0 f_1(R_1 | e_0^*) dR_1.$$

Substituting from here, we have

$$\tau_u^s = \frac{\frac{d\Omega}{dR_u^s}}{1 - b} \lambda^s \sigma \gamma^s.$$

Finally, we have

$$\frac{d\Omega}{dR_u^s} = \frac{d}{dR_u^s} \int_{\underline{R}}^{R_u^s} R_1 f_1(R_1 | e_0^*) dR_1 = \int_{\underline{R}}^{R_u^s} R_1 (f_{1H}(R_1) - f_{1L}(R_1)) \frac{\partial e_0^*}{\partial R_u^s} dR_1 = -L^s \frac{\partial e_0^*}{\partial R_u^s}.$$

Thus we obtain

$$\tau_u^s = -\frac{\partial e_0^*}{\partial R_u^s} \frac{1}{1-b} \lambda^s \sigma \gamma^s L^s = \frac{F_{1L}(R_u^s) - F_{1H}(R_u^s)}{|B_0''(e_0^*)|} \lambda^s \sigma \gamma^s L^s.$$

Since MLRP implies first order stochastic dominance, $\tau_u^s \geq 0$, concluding the proof.

A.6.1 Decentralizing Social Optimum with Caps

We now show that the social optimum can be implemented with caps on standard and total debt, that is requiring banks to set $R_\ell \leq R_\ell^s$ and $R_u \leq R_u^s$ (rather than directly mandating these allocations).

Consider a bank that chooses its debt, (R_ℓ, R_u) , subject to the caps $R_\ell \leq R_\ell^s$ and $R_u \leq R_u^s$. We internalize the investor participation constraint into bank utility to write bank utility over the debt levels (R_ℓ, R_u) and the liquidation price γ ,

$$U(R_\ell, R_u, \gamma) = \frac{\int_{R_\ell}^{R_u} b R_1 f_1(R_1 | e_0^*) dR_1 + \int_{R_u}^{\bar{R}} [R_1 - (1-b)R_u] f_1(R_1 | e_0^*) dR_1 + B_0(e_0^*)}{1 - \int_{\underline{R}}^{R_\ell} \gamma R_1 f_1(R_1 | e_0^*) dR_1 - \int_{R_\ell}^{R_u} (1-b) R_1 f_1(R_1 | e_0^*) dR_1 - \int_{R_u}^{\bar{R}} (1-b) R_u f_1(R_1 | e_0^*) dR_1} A_0$$

where $e_0^* = e_0^*(R_\ell, R_u)$ is the effort level determined by the incentive constraint. The proof strategy is to show that

$$(R_\ell^s, R_u^s) \in \arg \max_{R_\ell, R_u} U(R_\ell, R_u, \gamma^s) \quad s.t. \quad R_\ell \leq R_\ell^s, R_u \leq R_u^s$$

where $\gamma^s = \gamma(\Omega(R_\ell^s, R_u^s))$ is the liquidation price under the socially optimal contract, and where we throughout leave implicit that $R_\ell \leq R_u$. The above is equivalent to verifying

$$U(R_\ell^s, R_u^s, \gamma(\Omega(R_\ell^s, R_u^s))) \geq U(R_\ell, R_u, \gamma(\Omega(R_\ell^s, R_u^s))) \quad \forall R_\ell \leq R_\ell^s, R_u \leq R_u^s \quad (34)$$

Verifying Equation 34: Because (R_ℓ^s, R_u^s) is socially optimal, then

$$U(R_\ell^s, R_u^s, \gamma(\Omega(R_\ell^s, R_u^s))) \geq U(R_\ell, R_u, \gamma(\Omega(R_\ell, R_u))) \quad \forall R_\ell \leq R_\ell^s, R_u \leq R_u^s$$

As a first step, for any $R_\ell < R_\ell^s$, from Assumption 2 we have $\Omega(R_\ell^s, R_u^s) \geq \Omega(R_\ell, R_u^s)$. Thus since U increases in γ and γ decreases in Ω , we have

$$U(R_\ell, R_u^s, \gamma(\Omega(R_\ell^s, R_u^s))) \leq U(R_\ell, R_u^s, \gamma(\Omega(R_\ell, R_u^s))) \leq U(R_\ell^s, R_u^s, \gamma(\Omega(R_\ell^s, R_u^s))),$$

where the last inequality follows from the social planner's optimality. Therefore, the bank prefers (R_ℓ^s, R_u^s) to (R_ℓ, R_u^s) for any $R_\ell < R_\ell^s$.

We now extend this argument. Consider any point (R_ℓ, R_u) with $R_u < R_u^s$. We begin by showing

that $\Omega(R_\ell, R_u) \leq \Omega(R_\ell, R_u^s)$. As derived in the proof of Proposition 1,

$$\begin{aligned} \frac{\partial e_0^*}{\partial R_u} &= -\frac{1}{|B_0''(e_0^*(R_\ell, R_u))|} (1-b) \int_{R_u}^{\bar{R}} (f_{1H}(R_1) - f_{1L}(R_1)) dR_1 \\ &= -\frac{1}{|B_0''(e_0^*(R_\ell, R_u))|} (1-b) \left[F_{1L}(R_u) - F_{1H}(R_u) \right] \\ &\leq 0 \end{aligned}$$

where the final inequality follows since MLRP implies first order stochastic dominance. Since $e_0^*(R_\ell, R_u)$ decreases in R_u , then $\Omega(R_\ell, R_u)$ increases in R_u , so that

$$\Omega(R_\ell, R_u) \leq \Omega(R_\ell, R_u^s) \leq \Omega(R_\ell^s, R_u^s),$$

where the last inequality follows from the first step. But then once again since U increases in γ and γ decreases in Ω , we have

$$U(R_\ell, R_u, \gamma(\Omega(R_\ell^s, R_u^s))) \leq U(R_\ell, R_u, \gamma(\Omega(R_\ell, R_u))) \leq U(R_\ell^s, R_u^s, \gamma(\Omega(R_\ell^s, R_u^s))),$$

where the last inequality follows from social planner optimization. Therefore, the bank prefers (R_ℓ^s, R_u^s) to (R_ℓ, R_u) for any $(R_\ell, R_u) \leq (R_\ell^s, R_u^s)$. This concludes the proof.

A.7 Proof of Corollary 2

We need only to verify that the ex post bail-in authority achieves the same outcome as the contractual liabilities of the social optimum. In the region $R_1 < R_\ell^s$, the non-bail-inable senior debt exceeds asset values, and the bail-in authority is unable to resolve the bank. The bank is liquidated.

In the region $R_\ell^s \leq R_1 < R_u^s$, if the bail-in authority does not intervene then the bank gets 0, senior non-bail-inable debt gets $x^S(R_1) = \min\{(1-b)R_\ell^s, \gamma R_1\}Y_0$, and junior bail-inable debt gets $\max\{\gamma R_1 Y_0 - x^S(R_1), 0\}$. If by contrast the bail-in authority intervenes, it recapitalizes the bank with any haircut $R' - R_\ell^s \leq R_1 - R_\ell^s \leq R_u^s - R_\ell^s$ to bail-inable junior debt. Senior non-bail-inable debt gets fully repaid and is weakly better off. The bank gets payment bR' and is better off. Junior bail-inable debt gets $(1-b)(R' - R_\ell^s)$, and is better off local to $R' = R_1$ because $\gamma < 1-b$. Therefore, there is a Pareto efficient haircut. The haircut that maximizes total recovery value to creditors is $R' = R_1$, which is the same outcome as contractual bail-in debt.

In the region $R_u^s \leq R_1$, the bank is able to repay all debt in full while maintaining its minimum agency rent. A haircut on bail-inable debt is not Pareto efficient, and the bail-in authority does not act.

Hence, the bail-in authority implements the social optimum.

A.8 Proof of Proposition 4

We adopt the following proof strategy. We will consider a contract that results in bailouts, and show that it is equivalent to a contract that: (1) features bail-ins (rather than bailouts) ex post; and, (2) implements an ex ante lump sum transfer from taxpayers to the bank. Thus, all contracts with bailouts are equivalent to contracts without bailouts combined with lump sum transfers.

Suppose that there is a liability structure with a bailout at return R_1 , so that $L(R_1) > (1-b)R_1Y_0$ and $T_1(R_1) = L(R_1) - (1-b)R_1Y_0$. This generates consumption profile $c(R_1) = bR_1Y_0$ and a repayment to investors $x(R_1) = L(R_1) = (1-b)R_1Y_0 + T_1(R_1) = \hat{x}(R_1) + T_1(R_1)$, where $\hat{x}(R_1)$ is repayment out of bank resources. We denote repayment out of bank resources by $\hat{x}(R_1)$ also at points in which $T_1(R_1) = 0$, so that $\hat{x}(R_1) = x(R_1)$ if $T_1(R_1) = 0$. Substituting into the participation constraint, we have

$$Y_0 - A_0 - \mathbb{E}[T_1(R_1)|e_0 = e_0^*] \leq \mathbb{E}[\hat{x}(R_1)|e_0 = e_0^*].$$

The problem is otherwise identical. Hence, from the bank perspective the bailout rule $T_1(R_1)$ is equivalent to moving from capital structure x to capital structure \hat{x} , and also conducting a lump-sum transfer $T_0 = \mathbb{E}[T_1(R_1)|e_0 = e_0^*]$ from taxpayers to the bank at date 0 (note that there is no change in the liquidation rule, since banks that were bailed out are now able to repay investors).⁵⁹ There is no change in taxpayer welfare from this change in contract. Thus we can trace out the Pareto frontier in terms of the socially optimally contract with no bailouts (Proposition 3), with movements along the Pareto frontier only involving changes in the ex ante distribution of resources between banks and taxpayers, represented by T_0 . To select the point on the Pareto frontier associated with the existing distribution of resources, we need merely to characterize the Pareto weight that equalizes marginal utility and so rule out desirability of the lump sum transfer. Defining the social welfare weight on taxpayers to be $\omega^T = \lambda^s$, then the social planner is indifferent to transfers between banks and taxpayers at date 0. Thus we have equalized marginal utilities, meaning we have a Pareto efficient contract of Proposition 3 without bailouts. This concludes the proof.

⁵⁹Note that since x was monotone, then \hat{x} is also monotone. In particular, suppose that $\exists R'_1 < R''_1$ with $\hat{x}(R''_1) < \hat{x}(R'_1)$. If there were no bailouts at either R'_1 or R''_1 , this contradicts that x was monotone. If there were bailouts at R'_1 but not R''_1 , then $x(R''_1) = \hat{x}(R''_1) < \hat{x}(R'_1) \leq x(R'_1)$, contradicting that x was monotone. If there were bailouts at R''_1 but not R'_1 , then $\hat{x}(R'_1) \leq (1-b)R'_1Y_0 < (1-b)R''_1Y_0 = \hat{x}(R''_1)$, a contradiction. If there were bailouts at both R'_1 and R''_1 , then $\hat{x}(R'_1) = (1-b)R'_1Y_0 < (1-b)R''_1Y_0 = \hat{x}(R''_1)$, a contradiction.

A.9 Proof of Proposition 5

First, suppose that banks indeed set $R_\ell = R_u^s$. Then, the liquidation threshold is R_ℓ^s and the total debt threshold is R_u^s , as in Proposition 3. Therefore, as in the proof of Proposition 4, we can define investor repayment as $x(R_1) = \hat{x}(R_1) + T_1(R_1)$, where $\hat{x}(R_1)$ is repayment out of bank resources. Substituting into the investor participation constraint, we therefore have

$$Y_0 - (A_0 - T_0) - \mathbb{E}[T(R_1)|e_0 = e_0^*] \leq \mathbb{E}[\hat{x}(R_1)|e_0 = e_0^*].$$

Thus by instituting a transfer $T_0 = \mathbb{E}[T(R_1)|e_0 = e_0^*]$, the investor participation constraint reduces to

$$Y_0 - A_0 \leq \mathbb{E}[\hat{x}(R_1)|e_0 = e_0^*],$$

so that given the choice (R_ℓ^s, R_u^s) of the bank, then project scale is at the socially optimal level, $Y_0 = Y_0^s$. Therefore, banks and investors achieve the same outcome as in the social optimum of Proposition 3, where we note that investors are no better off because they have received bailouts but have also contributed more ex ante. Taxpayers have paid T_0 at date 1 and received T_0 ex ante, and are no better or worse off. Finally, banks are strictly better off because they have moved to the planner's optimum. Thus we have implemented the social planner's optimum in a Pareto efficient manner, provided that banks indeed choose $R_\ell = R_u^s$.

Lastly, consider a bank's optimal choice of R_ℓ , holding fixed R_u^s . Any choice $R_\ell \in [R_\ell^s, R_u^s)$ loses bailout funds and reduces project scale, while leaving the threshold R_ℓ^s for liquidation unchanged. Therefore, the bank prefers $R_\ell = R_u^s$ to any $R_\ell \in [R_\ell^s, R_u^s)$. It remains only to verify the bank would not choose $R_\ell < R_\ell^s$. A sufficient condition is to show that $R_\ell < R_\ell^s$ is dominated by $R_\ell = R_u^s$, even if by selecting $R_\ell < R_\ell^s$ the bank would additionally received a transfer ex ante of the expected bailout funds $\mathbb{E}[T(R_1)|e_0 = e_0^*]$ that it had foregone by selecting $R_\ell < R_\ell^s$ (i.e., it is as-if the bank neither benefits from bailouts nor has to pay for the bailout fund). Formally, we can write the bank's utility in this case as

$$U(R_\ell, R_u^s, \gamma^s) = \frac{\int_{R_\ell}^{R_u^s} bR_1 f_1(R_1|e_0^*) dR_1 + \int_{R_u^s}^{\bar{R}} [R_1 - (1-b)R_u^s] f_1(R_1|e_0^*) dR_1 + B_0(e_0^*)}{1 - \int_{\underline{R}}^{R_\ell} \gamma^s R_1 f_1(R_1|e_0^*) dR_1 - \int_{R_\ell}^{R_u^s} (1-b)R_1 f_1(R_1|e_0^*) dR_1 - \int_{R_u^s}^{\bar{R}} (1-b)R_u^s f_1(R_1|e_0^*) dR_1} A_0$$

which internalizes the participation constraint into the objective function.

Because R_ℓ^s, R_u^s is socially optimal, then

$$U(R_\ell^s, R_u^s, \gamma(\Omega(R_\ell^s, R_u^s))) \geq U(R_\ell, R_u^s, \gamma(\Omega(R_\ell, R_u^s))) \quad \forall R_\ell < R_\ell^s$$

Because U is an increasing function of γ , γ is a decreasing function of Ω , and because Ω is an

increasing function of R_ℓ (holding fixed R_u^s , and given Assumption 2), then we have

$$U(R_\ell^s, R_u^s, \gamma(\Omega(R_\ell^s, R_u^s))) \geq U(R_\ell, R_u^s, \gamma(\Omega(R_\ell, R_u^s))) \geq U(R_\ell, R_u^s, \gamma(\Omega(R_\ell^s, R_u^s))) \quad \forall R_\ell < R_\ell^s$$

so that banks prefer $R_\ell = R_\ell^s$ to $R_\ell < R_\ell^s$ at price γ^s .

Thus, $R_\ell = R_\ell^s$ is optimal for banks, concluding the proof.

A.10 Proof of Proposition 6

The proof of the first part follows by parallel arguments to the proof of Proposition 4. Consider debt levels (R_ℓ, R_u) that violates equation (29), and let $L(R_1)$ be the promised repayments associated with these debt levels. Since equation (29) is violated, all banks are bailed out ex post, and a bank with return $R_1 \leq R_\ell$ receives a bailout of size $T(R_1) = L(R_1) - (1 - b)R_1Y_0$, has bank consumption $c(R_1) = bR_1Y_0$, and has investor repayment $x(R_1) = L(R_1) = (1 - b)R_1Y_0 + T_1(R_1) = \hat{x}(R_1) + T_1(R_1)$, where $\hat{x}(R_1)$ is repayment out of bank resources. Substituting into the participation constraint, we have

$$Y_0 - A_0 - \mathbb{E}[T_1(R_1)|e_0 = e_0^*] + T_0 \leq \mathbb{E}[\hat{x}(R_1)|e_0 = e_0^*].$$

writing $\hat{T}_0 = T_0 - \mathbb{E}[T_1(R_1)|e_0 = e_0^*]$, this then reduces to

$$Y_0 - A_0 + \hat{T}_0 \leq \mathbb{E}[\hat{x}(R_1)|e_0 = e_0^*].$$

The problem is otherwise identical. Hence, from the perspective of bank welfare (ignoring fixed costs), the debt levels (R_ℓ, R_u) , bailouts $T_1(R_1)$, and transfer T_0 are welfare-equivalent to instead setting debt levels $(0, R_u)$, having no bailouts, and implementing a lump-sum transfer of \hat{T}_0 from taxpayers to the bank at date 0. Because bailouts have a fixed cost $F \geq 0$, choosing debt levels (R_ℓ, R_u) with bailouts is therefore welfare-reducing relative to the no-bailouts debt levels $(0, R_u)$. Thus, each point on the Pareto frontier is characterized by a debt level (R_ℓ^s, R_u^s) that satisfies equation 29, but is associated with different distributions of ex ante resources between banks and taxpayers (represented by the transfer T_0). To select the point on the Pareto frontier associated with the existing distribution of resources, we need merely to characterize the Pareto weight that equalizes marginal utility and so rule out desirability of the lump sum transfer. As in Proposition 4, this is $\omega^T = \lambda^s$. This proves part (a).

Part (b) is immediate from implementability. Since an increase in F relaxes the no-bailouts constraint (equation 29), the optimal no-bailout contract $(R_\ell^s(F), R_u^s(F))$ at fixed cost F can always be implemented at any higher bailout fixed cost $F' \geq F$. This holds for any point on the Pareto

frontier, concluding the proof.

B Extensions

Appendix B.1 allows for positive arbitrageur welfare weights. Appendix B.2 studies the trade-off between bailouts and bail-ins in protecting insured deposits when banks are allowed to issue insured deposits as part of their standard debt. Appendix B.3 provides additional results described in text under linear private benefits of effort.

B.1 Pareto Efficiency

We now study Pareto efficient social contracts, accounting for positive welfare weight ω^A on arbitrageurs. Recall that we have assumed that $u'(\bar{A}) > 1$. We obtain the following result.

Proposition 7. *The socially optimal contract is as in Proposition 3, but with wedges*

$$\tau_\ell^s = \left(1 - \frac{1 - \Lambda_1(R_\ell^s)}{(1 - e_0^*) + e_0^* \Lambda_1(R_\ell^s)} \frac{1}{|B_0''(e_0^*)|} bL^s \right) \lambda^s \omega^s \sigma \gamma^s \geq 0$$

$$\tau_u^s = \frac{F_{1L}(R_u^s) - F_{1H}(R_u^s)}{|B_0''(e_0^*)|} L^s \lambda^s \omega^s \sigma \gamma^s$$

where $\omega^s = 1 - \frac{1}{u'(\bar{A} - A_0)} > 0$.

Pareto efficient improvements arise because arbitrageurs are borrowing constrained, so that their marginal utility at date 0 exceeds that at date 1 (i.e., there is a distributive externality). Efficiency is achieved by transferring resources to arbitrageurs at date 0 in order to compensate them for resource losses from lower surplus from bank liquidations. When we take $\omega^A \rightarrow 0$, the associated point on the Pareto frontier is $A_0 \rightarrow \bar{A}$ and $u'(\bar{A} - A_0) \rightarrow +\infty$, and we obtain the first order conditions of Proposition 3.

B.1.1 Proof of Proposition 7

We can characterize a Pareto efficient contract by adopting the welfare function

$$\mathbb{E}[c|e_0 = e_0^*] + B_0(e_0^*)Y_0 + \omega^A [u(\bar{A} - A_0) + (\mathcal{F}(\Omega) - \gamma(\Omega)\Omega)Y_0],$$

where ω^A is the welfare weight on arbitrageurs. The optimality of standard and bail-in debt follows the same steps as in the proofs of Propositions 1 and 3. However, observe there is an additional

spillover from an increase in aggregate liquidations onto arbitrageur utility,

$$\frac{d\omega^A [u(\bar{A} - A_0) + (\mathcal{F}(\Omega) - \gamma(\Omega)\Omega) Y_0]}{d\Omega} = -\omega^A \frac{d\gamma}{d\Omega} \Omega Y_0 = \omega^A \gamma \sigma Y_0$$

Thus we obtain the wedges

$$\tau_\ell^s = \left(1 - \frac{1 - \Lambda_1(R_\ell^s)}{(1 - e_0^*) + e_0^* \Lambda_1(R_\ell^s)} \frac{1}{|B_0''(e_0^*)|} bL^s \right) (\lambda^s - \omega^A) \sigma \gamma^s$$

$$\tau_u^s = \frac{F_{1L}(R_u^s) - F_{1H}(R_u^s)}{|B_0''(e_0^*)|} L^s (\lambda^s - \omega^A) \sigma \gamma^s$$

which accounts for the modified social cost of liquidations. Finally, the optimality condition for A_0 is given by $\lambda^s = \omega^A u'(\bar{A} - A_0)$. Substituting in completes the proof.

B.2 Safety Premia and Insured Deposits

In this appendix, we study the interaction between our unobservable effort model and a safety premium story. This synergizes well with the observation that protecting insured deposits is another goal of bail-in regimes. It also allows us to shed further light on bail-ins versus bailouts as means of protecting insured deposits.

To streamline the model, we assume there are no fire sales (fixed γ) and that there is a linear private benefit to effort, $B_0(e_0) = B_0 \cdot (1 - e_0)$.

There are special depositors who place an excess value $\beta > 0$ on a completely safe bank deposit at date 0, that is they are willing to pay $1 + \beta$ for a safe deposit. At date 1, special depositors will withdraw their funds and be replaced by regular investors if rollover occurs. The number of special depositors that show up to a given bank is $(1 - b)R_d Y_0$ for a fixed $R_d > \underline{R}$. We thus abstract away from the optimal level R_d and focus on the residual capital structure and how the planner protects special depositors. We assume that the planner extends deposit insurance to special depositors, so that banks can treat special deposits as completely safe. The bank is always unable to repay its liabilities and meet its minimum agency rent if $R_1 < R_d$, absent intervention, regardless of its other liabilities. Because deposits are insured, the planner is liable for any shortfall relative to the face value $(1 - b)R_d Y_0$. Insured deposits are always at the top of the creditor hierarchy in liquidation.⁶⁰

The planner chooses bailouts with commitment. Bailouts have a constant variable cost $\tau > 0$. Thus a bailout that recapitalizes a distressed bank costs

$$\text{Cost}_{\text{No Liquidation}} = \tau (L(R_1) - (1 - b)R_1 Y_0)$$

⁶⁰In practice, banks may issue wholesale funding which is not insured but runs prior to resolution.

where $L(R_1)$ is total liabilities including insured deposits. When the planner instead allows the bank to fail, insured deposits receive the entire liquidation value and are covered by deposit insurance, so that the cost in taxpayer funds is

$$\text{Cost}_{\text{Liquidation}} = \min\{\tau((1-b)R_d Y_0 - \gamma R_1 Y_0), 0\}$$

Note that even when $L(R_1) = (1-b)R_d Y_0$ and there are only insured deposits remaining, the cost of rescuing the bank with a bailout is lower than the cost of rescuing the bank under liquidation, due to the loss of pledgeable income in liquidation.

The planner solves for the optimal contract, which includes the rescue decision (either via bailout or via liquidation and repayment by insurance).⁶¹ We constrain bank consumption to be *monotone*, that is $c(R_1)$ must be nondecreasing in R_1 ,⁶² which was satisfied by optimal contracts in the baseline model. This implies that bailouts must be monotone: if a bank with return R_1 is bailed out, then all banks $R'_1 \geq R_1$ must also be bailed out. This rules out the possibility that the planner bails out a bank with $R_1 < R_d$ to protect depositors but liquidates a bank with $R_1 > \frac{1-b}{\gamma}R_d$ for incentive reasons.

Proposition 8. *The socially optimal contract consists of insured deposits R_d , standard (uninsured) debt $R_\ell^s \geq R_d$, and bail-in debt $R_u^s \geq R_\ell^s$. The following are true:*

1. *If $R_\ell^s > R_d$, there is deposit insurance but no bailouts. The bank is liquidated when $R_1 \leq R_\ell^s$.*
2. *If $R_\ell^s = R_d$, there is a threshold $R_L^s \leq R_d$ such that the bank is liquidated when $R_1 \leq R_L^s$ and bailed out when $R_L^s \leq R_1 \leq R_d$.*

Proposition 8 illustrates the capital structure decision and method of protecting insured deposits. If $R_\ell^s > R_d$, the optimal contract combines insured deposits and uninsured deposits. Intuitively, this will tend to occur when liquidation costs γ and bailout costs τ are not too high. If these costs are high, then $R_\ell^s = R_d$ and there are only insured deposits. This arises due to the trade-off between deposit insurance and bailouts for protecting special depositors. Bailing out the bank reduces the taxpayer cost of deposit insurance, but provides worse incentives for the bank. Whenever the planner allows use of standard debt in excess of insured deposits, that is $R_\ell^s > R_d$, then necessarily the planner will commit to rescue depositors but not the bank. In this case, there is deposit insurance but no bailouts. If $R_\ell^s = R_d$ and $R_L^s < R_d$, the planner uses bailouts ex post in order to reduce the cost of protecting depositors. Interestingly, this is a case where special depositors play both roles.

⁶¹A technical aside is that it is possible that the planner does not find it optimal to allow the bank to scale up as much as possible due to the cost of insuring deposits. We assume this is not the case, for example if R_d is close to \underline{R} .

⁶²If $c(R_1) > c(R'_1)$ but $R_1 < R'_1$, the bank could increase its payoff ex post by destroying assets to bring its return down to R_1 . We look for contracts where value destruction is not ex post optimal.

B.2.1 Proof of Proposition 8

Due to consumption monotonicity, there is a threshold $R_L^s \geq \underline{R}$ for bank liquidation, with $R_L = \underline{R}$ corresponding to no liquidations. As in the proof of Proposition 4, there are no bailouts above R_d , due to the taxpayer burden. We can thus split the problem into two parts.

First, suppose that the liquidation threshold satisfies $R_L^s > R_d$, and suppose that the planner finds it optimal to engage in bailouts in a states $R_1 < R_d$. By consumption monotonicity, there are also bailouts for $R_d \leq R_1 \leq R_L^s$. But then because transfers to regular investors are inefficient, it is optimal to set $R_L^s = R_d$, as in the proof of Proposition 4. The optimal contract does not feature both $R_L^s > R_d$ and bailouts. Consider then the form of the optimal contract when $R_L^s > R_d$. Because there are no bailouts and because we have linear private benefits, we have a corner solution $e_0^* = 1$ and the social objective function is

$$\int c(R_1) f_{1H}(R_1) dR_1 - \int_{R_1 \leq R_L^s} \tau \max\{(1-b)R_d - \gamma R_1, 0\} Y_0 f_{1H}(R_1) dR_1$$

while the corresponding investor participation constraint is

$$Y_0 - A_0 = \beta(1-b)R_d Y_0 + \int_{\underline{R}}^{R_L^s} \max\{(1-b)R_d, \gamma R_1\} Y_0 f_{1H}(R_1) dR_1 + \int_{R_1 \geq R_L^s} ((1-b)R_d + x(R_1)) f_{1H}(R_1) dR_1$$

where x is repayment to regular investors, and where incentive compatibility is the same as in the baseline model. From here, note that the trade-off in liability structure above R_L^s is the same as in the baseline model. The model again combines standard and bail-in debt, as in the baseline model.

Consider next the optimal contract when $R_L^s < R_d$. R_L^s then also corresponds to the bailout threshold, such that there are bailouts when $R_L^s \leq R_1 \leq R_d$, and where $R_L^s = R_d$ corresponds to no bailouts. The resulting social objective function is

$$\int c(R_1) f_{1H}(R_1) dR_1 - \int_{\underline{R}}^{R_L^s} \tau [(1-b)R_d - \gamma R_1] Y_0 f_{1H}(R_1) dR_1 - \int_{R_L^s}^{R_d} \tau (1-b) (R_d - R_1) Y_0 f_{1H}(R_1) dR_1$$

while investor repayment is given by

$$Y_0 - A = (1 + \beta)(1-b)R_d Y_0 + \int_{R_d}^{\bar{R}} x(R_1) f_{1H}(R_1) dR_1$$

reflecting that depositors are always repaid. Finally, incentive compatibility is as in the baseline model. Thus while optimal contracts combine standard and bail-in debt, we necessarily have $R_L^s = R_d$.

B.3 Additional Results with Linear Private Benefit

In this Appendix, we provide additional results referenced in main text for the case with linear private benefit of effort at date 0, $B_0(e_0) = b_0(1 - e_0)$.

Standard and Total Leverage Decrease in Liquidation Discount. The following result shows that in the private optimum, both standard and total debt increase in the liquidation price, potentially helping to understand the relatively low leverage and high resolvability of nonfinancials.⁶³

Corollary 3. *With linear private benefit of effort, then R_ℓ^P and R_u^P are increasing in γ .*

Social Optimum Features Less Standard and Total Debt. We obtain the following corollary to Proposition 3.

Corollary 4. *With linear private benefit of effort, then $R_\ell^S \leq R_\ell^P$ and $R_u^S \leq R_u^P$, that is the social optimum features less standard debt and less total debt than the private optimum.*

B.3.1 Proof of Corollary 3

In the case where $B_0(e_0) = (1 - e_0)b_0$, equation (12) implies a corner solution in effort at $e_0^* = 1$. Since equation (12) binds⁶⁴, we can use equation (12) to determine the function $R_u(R_\ell)$ implicitly as solving

$$b_0 = \int_{R_\ell}^{R_u(R_\ell)} bR_1(1 - \Lambda_1^{-1}(R_1))f_{1H}(R_1)dR_1 + \int_{R_u(R_\ell)}^{\bar{R}} [R_1 - (1 - b)R_u(R_\ell)](1 - \Lambda_1^{-1}(R_1))f_{1H}(R_1)dR_1.$$

In the relevant region, we then have $\frac{\partial R_u}{\partial R_\ell} = \frac{bR_\ell(\Lambda_1(R_\ell)^{-1} - 1)f_{1H}(R_\ell)}{(1 - b)\int_{R_u}^{\bar{R}} (1 - \Lambda_1^{-1})f_{1H}dR_1} \geq 0$. Now, we can substitute in $R_u(R_\ell)$ and the participation constraint into the bank's objective function to obtain the bank's objective over R_ℓ and the parameter γ ,

$$U(R_\ell, \gamma) = \frac{\int_{R_\ell}^{R_u(R_\ell)} bR_1f_{1H}(R_1)dR_1 + \int_{R_u(R_\ell)}^{\bar{R}} [R_1 - (1 - b)R_u(R_\ell)]f_{1H}(R_1)dR_1}{1 - \int_{\underline{R}}^{R_\ell} \gamma R_1f_{1H}(R_1)dR_1 - \int_{R_\ell}^{R_u(R_\ell)} (1 - b)R_1f_{1H}(R_1)dR_1 - \int_{R_u(R_\ell)}^{\bar{R}} (1 - b)R_u(R_\ell)f_{1H}(R_1)dR_1} A_0$$

⁶³Formally to undertake variations in γ , we assume that arbitrageurs have a linear technology, $\mathcal{F} = \gamma\Omega$, and vary the slope of the linear technology.

⁶⁴If the corner solution were strict and the RHS of equation (12) exceeded the LHS, then the bank could borrow an extra unit from investors while pledging a sufficient increase in bail-in debt to ensure it is repaid. Such a perturbation would be incentive compatible and hence optimal given the project is positive NPV.

Taking logs, the log objective function is

$$\begin{aligned} \log U(R_\ell, \gamma) = & \log \left(\int_{R_\ell}^{R_u(R_\ell)} bR_1 f_{1H}(R_1) dR_1 + \int_{R_u(R_\ell)}^{\bar{R}} [R_1 - (1-b)R_u(R_\ell)] f_{1H}(R_1) dR_1 \right) + \log A_0 \\ & - \log \left(1 - \int_{R_\ell}^{R_\ell} \gamma R_1 f_{1H}(R_1) dR_1 - \int_{R_\ell}^{R_u(R_\ell)} (1-b)R_1 f_{1H}(R_1) dR_1 \right. \\ & \left. - \int_{R_u(R_\ell)}^{\bar{R}} (1-b)R_u(R_\ell) f_{1H}(R_1) dR_1 \right) \end{aligned}$$

From here, the proof strategy is to show that $\log U$ has increasing differences in (R_ℓ, γ) over the relevant range $R_\ell \in [\underline{R}, \bar{R}_\ell]$, where \bar{R}_ℓ is defined implicitly by $\Lambda_1(\bar{R}_\ell) = 1$. Having verified increasing differences, we can invoke standard monotone comparative statics to conclude that $R_\ell(\gamma)$ is increasing in γ . Since $R_u(R_\ell)$ is an increasing function, then R_u is also increasing in γ .

Let us define

$$X(R_\ell, \gamma) = \int_{R_\ell}^{R_\ell} \gamma R_1 f_{1H}(R_1) dR_1 + \int_{R_\ell}^{R_u(R_\ell)} (1-b)R_1 f_{1H}(R_1) dR_1 + \int_{R_u(R_\ell)}^{\bar{R}} (1-b)R_u(R_\ell) f_{1H}(R_1) dR_1$$

Differentiating $\log U$ in γ , we have

$$\frac{\partial \log U}{\partial \gamma} = \frac{\frac{\partial X(R_\ell, \gamma)}{\partial \gamma}}{1 - X(R_\ell, \gamma)}$$

Differentiating again in R_ℓ , we have

$$\frac{\partial^2 \log U}{\partial \gamma \partial R_\ell} = \frac{\frac{\partial^2 X(R_\ell, \gamma)}{\partial \gamma \partial R_\ell} (1 - X(R_\ell, \gamma)) + \frac{\partial X}{\partial R_\ell} \frac{\partial X(R_\ell, \gamma)}{\partial \gamma}}{(1 - X(R_\ell, \gamma))^2}$$

As we aim to show increasing differences, the strategy from here is to show $\frac{\partial^2 X(R_\ell, \gamma)}{\partial \gamma \partial R_\ell} (1 - X(R_\ell, \gamma)) + \frac{\partial X}{\partial R_\ell} \frac{\partial X(R_\ell, \gamma)}{\partial \gamma} \geq 0$. Evaluating the corresponding derivatives, we have

$$\frac{\partial X}{\partial \gamma} = \int_{R_\ell}^{R_\ell} R_1 f_{1H}(R_1) dR_1$$

$$\frac{\partial^2 X}{\partial \gamma \partial R_\ell} = R_\ell f_{1H}(R_\ell)$$

$$\frac{\partial X}{\partial R_\ell} = (\gamma + b - 1)R_\ell f_{1H}(R_\ell) + \int_{R_u(R_\ell)}^{\bar{R}} (1-b) \frac{\partial R_u(R_\ell)}{\partial R_\ell} f_{1H}(R_1) dR_1 \geq (\gamma + b - 1)R_\ell f_{1H}(R_\ell)$$

where the inequality follows from $\frac{\partial R_u}{\partial R_\ell} \geq 0$. So then, we have

$$\begin{aligned}
\frac{\partial^2 X}{\partial \gamma \partial R_\ell} (1 - X) + \frac{\partial X}{\partial R_\ell} \frac{\partial X}{\partial \gamma} &\geq R_\ell f_{1H}(R_\ell) (1 - X(R_\ell, \gamma)) + (\gamma + b - 1) R_\ell f_{1,H}(R_\ell) \int_{\underline{R}}^{R_\ell} R_1 f_{1H}(R_1) dR_1 \\
&= R_\ell f_{1H}(R_\ell) \left[1 - X(R_\ell, \gamma) - (1 - b - \gamma) \int_{\underline{R}}^{R_\ell} R_1 f_{1H}(R_1) dR_1 \right] \\
&= R_\ell f_{1H}(R_\ell) \left[1 - X(R_\ell, 1 - b) \right] \\
&\geq R_\ell f_{1H}(R_\ell) \left[1 - (1 - b) \mathbb{E}[R_1 | e_0^* = 1] \right] \\
&\geq R_\ell f_{1H}(R_\ell) \left[1 - (1 - B_1) \mathbb{E}[R_1 | e_0^* = 1] \right] \\
&\geq 0
\end{aligned}$$

where the last line follows from Assumption 2(b). Thus we have verified increasing differences, concluding the proof.

B.3.2 Proof of Corollary 4

Following the same steps as the proof of Corollary 3, we have

$$U(R_\ell, \gamma) = \frac{\int_{R_\ell}^{R_u(R_\ell)} b R_1 f_{1H}(R_1) dR_1 + \int_{R_u(R_\ell)}^{\bar{R}} [R_1 - (1 - b) R_u(R_\ell)] f_{1H}(R_1) dR_1}{1 - \int_{\underline{R}}^{R_\ell} \gamma R_1 f_{1H}(R_1) dR_1 - \int_{R_\ell}^{R_u(R_\ell)} (1 - b) R_1 f_{1H}(R_1) dR_1 - \int_{R_u(R_\ell)}^{\bar{R}} (1 - b) R_u(R_\ell) f_{1H}(R_1) dR_1} A_0.$$

As usual, we can define γ as a decreasing function of R_ℓ (this follows immediately given effort is constant at 1).

To clarify notation, let R_ℓ^p be the privately optimal choice of R_ℓ and $\gamma^p = \gamma(R_\ell^p)$, then by bank optimization

$$U(R_\ell^p, \gamma^p) \geq U(R_\ell, \gamma^p) \quad \forall R_\ell$$

Now, consider the social planner's problem,

$$R_\ell^s \in \arg \max_{R_\ell} U(R_\ell, \gamma(R_\ell)).$$

Since γ is a decreasing function of R_ℓ and U is an increasing function of γ , then

$$U(R_\ell^p, \gamma^p) \geq U(R_\ell, \gamma^p) \geq U(R_\ell, \gamma(R_\ell)) \quad \forall R_\ell \geq R_\ell^p$$

Therefore, $R_\ell^s \leq R_\ell^p$. Finally since, as in the proof of Corollary 3, $R_u(R_\ell)$ is an increasing function, then $R_u^s \leq R_u^p$. This concludes the proof.