

Monetary and Fiscal Policy According to HANK-IO

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Abstract

This paper studies monetary and fiscal policy transmission in a multi-sector heterogeneous-agent New Keynesian model with an input-output network (“HANK-IO”). We document systematic household-sector linkages in micro data and calibrate our model to match them. To identify when these linkages have implications for policy transmission, we analytically characterize an as-if benchmark that features a strict decoupling between household and sectoral heterogeneity. Away from this benchmark, novel earnings and expenditure heterogeneity channels emerge that govern the propagation of demand and supply shocks. Quantitatively, these channels shape the transmission of stabilization policy to aggregates, as well as its distributional and sectoral consequences.

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1 Introduction

Economists have made substantial progress in documenting and modeling the implications of cross-sectional heterogeneity for policy transmission in disaggregated economies. Recent work on household behavior has emphasized the importance of heterogeneity in spending propensities and the incidence of shocks. On the production side, a long tradition of research has documented sectoral heterogeneity and studied its importance for the propagation of shocks in multi-sector models. This paper takes as its starting point a set of empirical regularities that point to systematic linkages *between* households and sectors at a disaggregated micro level.

Motivated by these stylized facts, this paper proposes a new framework to study and quantify the implications of household-sector linkages for the transmission of monetary and fiscal policy. We develop a multi-sector heterogeneous-agent New Keynesian model with an input-output production network, which we call the “HANK-IO” model.¹ Our framework is inspired by both the burgeoning heterogeneous-agent New Keynesian (HANK) literature (Kaplan et al., 2018; Auclert et al., 2024) and the long tradition of multi-sector business cycle models (Long and Plosser, 1983). We analytically characterize an *as-if* benchmark that features a strict decoupling between household and sectoral heterogeneity. Away from this benchmark, novel earnings and expenditure heterogeneity channels may shape the propagation of demand and supply shocks. After disciplining our model empirically to match the household-sector linkages we document in micro data, we show that these channels contribute quantitatively to the transmission of policy to aggregates, as well as the distributional and sectoral consequences of policy.

Our baseline economy departs from a canonical HANK model (Auclert et al., 2024) and enriches its production side by introducing multiple sectors and input-output linkages (Baqae and Farhi, 2020). Households face idiosyncratic uncertainty that leads them to make different consumption, savings, and portfolio decisions (*ex-post* heterogeneity). We also allow for permanent differences in household characteristics or types (*ex-ante* heterogeneity). In particular, household types may differ in their preferences over consumption goods and their labor endowments. Production in our economy takes place across a rich network of sectors, and we allow for sectoral heterogeneity across factor and input shares, input-output linkages, competitiveness, and price rigidity. Our key point of departure from the canonical HANK framework is that we allow for systematic links between household types and production sectors in terms of both earnings and expenditure patterns. We discipline these household-sector linkages empirically using micro data.

As-if benchmark. We begin our analysis with an instructive *as-if* benchmark to illustrate conceptually under what conditions household-sector linkages may play a role for policy transmission. Our *as-if* benchmark assumes that all households share the same homothetic preferences over consumption goods and there is a single labor factor. Under these two assumptions, there is no

¹ We choose this name in memory of the late Emmanuel Farhi whose work with David Baqae on heterogeneous-agent economies with input-output production networks (“HA-IO”) has inspired this paper (Baqae and Farhi, 2018).

role for earnings or expenditure heterogeneity as households are symmetrically exposed to price inflation and changes in labor demand. Our first result is that the HANK-IO model features a strict decoupling between household and sectoral heterogeneity under these two assumptions. We show that the macroeconomic dynamics of our as-if benchmark admit an intertemporal IS-LM representation as a system of two forward-looking equations: a dynamic IS curve that determines output as a function of the time path of real interest rates, and a dynamic LM curve that pins down real interest rates as a function of future aggregate demand. The dynamic IS curve is shaped by household but not by sectoral heterogeneity: Given a path of real interest rates, it takes the same form as the IS equation of a canonical one-sector HANK model. Conversely, the dynamic LM curve is shaped by sectoral but not by household heterogeneity: It maps a given path of aggregate demand to the same path of real interest rates as a canonical representative-household multi-sector model. The as-if benchmark therefore highlights that household and sectoral heterogeneity are decoupled in this sense in the absence of earnings and expenditure heterogeneity.

Monetary and fiscal policy transmission in the as-if benchmark is characterized by an Intertemporal Keynesian Cross. When the monetary authority adopts a rule that neutralizes real interest rate effects, fiscal policy transmission is governed by the same Keynesian multiplier as in the one-sector HANK model of [Auclert et al. \(2024\)](#). Intertemporal marginal propensities to consume (iMPCs) remain a sufficient statistic for the output effects of government spending. This result obtains in spite of substantial sectoral heterogeneity that is masked by the dynamic LM equation. For a given path of real interest rates, the aggregate fiscal multiplier is therefore unaffected by sectoral heterogeneity and, in fact, identical to that in any HANK economy that admits the same IS curve representation.

Likewise, for a given change in the path of real interest rates, monetary policy transmission is solely governed by household heterogeneity. The sufficient statistics for policy are again identical to those in a one-sector HANK economy. Sectoral heterogeneity does, however, affect the transmission from nominal to real rates, which is governed by the LM curve. Our result shows that once the monetary authority has implemented a desired path of real rates, the production network structure of the economy no longer matters for policy transmission—the path of real rates is a summary statistic for the effect of monetary policy on aggregate activity. These results are manifestations of our decoupling result under the IS-LM representation: Demand shocks interact with the IS curve and are shaped by household but not by sectoral heterogeneity.

Finally, we unpack the LM curve and derive an aggregation result that traces the macroeconomic effects of sectoral technology shocks in our as-if benchmark. We decompose the impact on real GDP into a pure technology effect—accounting for increased productivity of resources at a given allocation—and changes in allocative efficiency—summarizing the effects on output from a reallocation of resources across firms and workers. Remarkably, our aggregation result for the as-if benchmark is identical to that of [Baqae and Farhi \(2020\)](#). This is despite our economy featuring rich and dynamic household heterogeneity, whereas theirs is a static representative-household setting.

For given changes in markups and factor shares, the aggregate consequences of microeconomic technology shocks are not directly shaped by household heterogeneity. In other words, changes in sectoral markups and the labor income share are sufficient statistics for the implications of household heterogeneity.

The role of household-sector linkages. Away from the as-if benchmark, household-sector linkages shape the transmission of policy and shocks through novel earnings and expenditure heterogeneity channels. Our next analytical result characterizes an Intertemporal Keynesian Cross for monetary and fiscal policy away from the as-if benchmark. When households supply different labor factors, an *earnings heterogeneity channel* emerges. It is captured by a cross-sectional covariance across household types between iMPCs and changes in households' earnings shares. Earnings heterogeneity amplifies the aggregate effects of policy when it leads to a redistribution of income shares to factors (and in states of the world) with large spending propensities.

When households consume different consumption baskets, an *expenditure heterogeneity channel* emerges that comprises two distinct effects. A change in the price path of a household's basket elicits income and intertemporal substitution effects. If relative prices increase for households (and in states of the world) with large spending propensities, then the resulting drop in their effective purchasing power leads to a fall in aggregate consumer spending. This effect is captured by a covariance across household types between iMPCs and changes in relative bundle prices. Moreover, when households experience different rates of inflation in their respective bundle prices, then their effective real rates of return on savings differ as well. The expenditure heterogeneity channel of policy then also comprises a covariance across households between their relative rates of inflation and intertemporal interest rate response elasticities. Intuitively, if relative real savings rates increase for those household types that have large intertemporal elasticities of substitution, then the aggregate effect of policy is dampened.

Finally, we derive an aggregation result for sectoral technology shocks. Unlike in the as-if benchmark, gains from allocative efficiency are now shaped by household-sector linkages. Expenditure heterogeneity has implications for allocative efficiency because changes in purchasing power affect household labor supply and consequently firms' cost structure. This new force is governed by a covariance across household types between factor shares and changes in the relative price of households' consumption baskets: If consumer prices increase especially for those households whose labor factors have large cost-based Domar weights, then output falls and allocative efficiency deteriorates. Intuitively, when the price of a household's consumption bundle increases, a given nominal wage has less purchasing power and the real wage falls. If prices increase for households whose factors play a dominant role in firms' cost structures (large cost-based Domar weights), then output falls. Crucially, the relevant covariance is with respect to *cost-based* factor shares because markups drive a wedge between prices and marginal costs. Earnings heterogeneity also shapes the transmission of technology shocks to allocative efficiency.

When the overall income share falls, households receive less labor income, which elicits a positive labor supply response and increases output. Heterogeneity in factor share responses can amplify or dampen the resulting change in allocative efficiency. When factors with the largest cost-based Domar weights experience a relative decrease in their income share, then resources are reallocated towards the more monopolized and distorted sectors. The strength of this effect is governed by households' labor supply elasticities.

Our results point to an important conceptual distinction between demand and supply shock propagation in HANK-IO. What matters for the aggregation of sectoral technology shocks—in particular their transmission through changes in allocative efficiency—are covariances with respect to cost-based factor shares. Gains from allocative efficiency result from a reallocation of resources to relatively more distorted and monopolized sectors. The cost-based factor share precisely captures those markups that drive a wedge between prices and marginal costs, and it is the appropriate measure to capture whether resources find more productive uses. The transmission of monetary and fiscal policy (demand) shocks, on the other hand, is determined by covariances with respect to iMPCs. Intuitively, demand propagation is governed by spending propensities in response to changes in income and prices.

Empirical and quantitative analysis. Drawing in parts on existing and novel results, we document empirical regularities that speak to systematic linkages between households and sectors at the micro level. On the expenditure side, higher-income households spend relatively more on sectors that (i) have higher labor but (ii) lower capital shares, and (iii) are more central in the investment network. Middle-income households spend relatively more on sectors that (iv) have high intermediates shares, (v) are more flexible, (vi) are more network-central, (vii) have lower markups, and (viii) have a higher government spending share. On the earnings side, we show that higher-income households earn relatively more from sectors that (i) have higher intermediates and (ii) capital shares but (iii) lower labor shares, (iv) are less price rigid, (v) are more network-central, (vi) have lower markups, (vii) a higher government spending share, and (viii) a higher investment share. While these empirical regularities point to a potential role for the interaction of household and sectoral heterogeneity in the propagation of shocks, we currently lack a framework to assess their quantitative importance.

To match these empirical regularities on household-sector linkages, we develop a quantitative model that enriches our analytical framework along two dimensions, introducing capital as an additional production factor and nonhomothetic CES consumption preferences. Allowing households to trade a second, illiquid asset is important to match the intertemporal marginal propensities to consume that govern the new earnings and expenditure heterogeneity channels of policy. Explicitly modeling capital formation also allows us to capture the concentrated investment network documented by [Vom Lehn and Winberry \(2022\)](#). Finally, estimating a nonhomothetic CES demand system allows us to match empirical expenditure patterns across the income distribution ([Comin](#)

et al., 2021). We calibrate our model to match the income and wealth distribution of households, key moments of sectoral heterogeneity across a 22-sector production network, and the household-sector linkages we document empirically.

Our quantitative analysis focuses on a prominent policy question of the post-Covid inflation episode. By mid-2023, core goods inflation had already fallen sharply while services inflation remained stubbornly high. At the same time, consumer demand was unexpectedly strong as households still held large cash balances accumulated from three rounds of fiscal transfers. Against this backdrop, monetary policymakers faced three related questions. First, to what extent did the remaining cash overhang account for unexpectedly strong consumer activity, and through which sectors was it feeding into inflation? Second, how should policymakers judge whether policy was “sufficiently restrictive” after raising the federal funds rate by roughly 5% in little more than a year, given the long and uncertain transmission lags of monetary tightening? Third, in light of the stark divergence in inflation across sectors, which price index—or which sectoral aggregates—should guide policy?

HANK-IO is a useful framework for these questions because it features realistic iMPCs and non-Ricardian household behavior—both necessary to analyze fiscal rebates—alongside sectoral inflation dynamics. We show in Section 5 that fiscal transfers weaken monetary policy transmission overall, and—more importantly—that this weakening is concentrated in precisely the sectors most stimulated by the transfers. In other words, monetary policy becomes relatively least effective at reducing inflation exactly where fiscal transfers generated the most inflationary pressure.

Related literature. Our paper builds on much previous work that has documented and studied the implications of household and sectoral heterogeneity.

HANK. The burgeoning HANK literature studies the implications of household heterogeneity for business cycles and policy transmission.² Our contribution is to unbundle the production side and introduce sectoral heterogeneity to this literature. We extend the Intertemporal Keynesian Cross (Auclert et al., 2024) to an environment with rich sectoral heterogeneity and characterize the implications of household-sector linkages. Monetary and fiscal policy transmission is governed by earnings and expenditure heterogeneity channels, as well as a supply-side misallocation channel.

Sectoral heterogeneity. We contribute to a long tradition of research on the transmission and propagation of shocks in multi-sector business cycle models. Starting with Long and Plosser (1983), much work has employed structural real business cycle models to assess quantitatively whether sectoral technology shocks can account for observed business cycles patterns.³ An important strand of this literature characterizes the aggregation properties of these models.⁴ It has long been

² Important contributions include, among many others, McKay and Reis (2016), McKay et al. (2016), Kaplan et al. (2018), Auclert (2019), and Auclert et al. (2024).

³ Among many others, see Bak et al. (1993), Horvath (2000), Foerster et al. (2011), Atalay (2017). While these papers focus on sectoral input-output linkages, recent work by Vom Lehn and Winberry (2022) studies the role of the investment network in propagating sectoral shocks.

⁴ See Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) for recent surveys, as well as Hulten (1978), Horvath

appreciated that Hulten’s theorem applies to first order in frictionless competitive economies, and the aggregation of sectoral shocks is governed by sales shares (Domar weights). This literature has recently been reinvigorated with renewed focus on misallocation and potential gains in allocative efficiency in inefficient economies with distortions.⁵ Bigio and La’O (2020) and Baqaee and Farhi (2020) derive Hulten-like aggregation results for the macroeconomic effects of sectoral shocks in environments with distortions. See Baqaee and Rubbo (2023) for a recent review.

Another strand of the multi-sector business cycle literature introduces nominal rigidities in the New Keynesian tradition.⁶ Many of these papers investigate the implications of sectoral heterogeneity and input-output linkages for monetary policy and inflation dynamics. Baqaee et al. (2024) show that monetary policy can have supply-side effects and operate through a misallocation channel when policy redirects resources towards more monopolized sectors. Rubbo (2023) studies a general multi-sector representative-agent New Keynesian model and shows that sectoral and aggregate Phillips curve slopes decrease in intermediate input shares. She shows that a divine coincidence price index provides a better fit in Phillips curve regressions than consumer prices.

Relative to existing work, we develop a dynamic structural model featuring rich heterogeneity across both households and sectors. We emphasize the role of systematic household-sector linkages observed in micro data that give rise to novel transmission channels. Analytically, we derive an Intertemporal Keynesian Cross to characterize monetary and fiscal policy in the presence of household-sector linkages, and we extend the aggregation result of Baqaee and Farhi (2020) to a multi-sector heterogeneous-agent New Keynesian model. Quantitatively, we assess the importance of earnings and expenditure heterogeneity for policy transmission.

Household-sector linkages. Our paper contributes to research studying the interaction of household and sectoral heterogeneity. Indeed, we take as our starting point and motivation the large body of empirical work that studies earnings and expenditure heterogeneity across households.⁷ A strand of this literature explicitly documents systematic household-sector linkages through earnings and expenditure patterns.⁸ Relative to this empirical literature, we document several new empirical regularities on household-sector linkages. Our main contribution is to develop a structural model that matches these facts and to establish the quantitative importance of household-sector linkages for policy transmission.

Our paper is naturally related to a small but growing body of research that studies disaggregated economies featuring both household and sectoral heterogeneity, starting with Baqaee and

(1998), Dupor (1999), Gabaix (2011), Acemoglu et al. (2012), Carvalho and Gabaix (2013), Acemoglu et al. (2017).

⁵ See Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Midrigan and Xu (2014), Baqaee and Farhi (2019), Liu (2019), La’O and Tahbaz-Salehi (2022), Dávila and Schaab (2023), and many more.

⁶ See among others Aoki (2001), Bouakez et al. (2009), Pasten et al. (2017, 2020), and Baqaee et al. (2024).

⁷ See among many others Kaplan and Schulhofer-Wohl (2017), Jaravel (2019), Cravino et al. (2020), Jaravel (2021), Comin et al. (2021), and Andersen et al. (2022).

⁸ Cravino et al. (2020) and Clayton et al. (2018) show that higher-income households spend in and earn from relatively price-rigid sectors. Hubmer (2023) documents that high-income households spend in sectors with higher labor shares. Jaravel (2019) shows that high-income households spend in sectors with higher markups.

Farhi (2018).⁹ Olivi et al. (2025) characterize a new marginal consumer price index in a multi-sector New Keynesian model with permanent household heterogeneity and non-homothetic preferences. They show this new price index governs the welfare-relevant output gap and optimal monetary policy. Rubbo (2025) studies the supply-side effects of government spending in a multi-sector environment with rich heterogeneity in factor supply elasticities. The contribution of our paper is to show theoretically and quantitatively that intertemporal marginal propensities to consume govern the interaction between household and sectoral heterogeneity. We develop a new HANK-IO model with both ex ante and ex post household heterogeneity that can match iMPCs in the data. Our results suggest that models featuring ex post household heterogeneity may be more appropriate to study policy questions at the intersection of household and sectoral heterogeneity.

2 A Baseline HANK-IO Model

In this section, we develop a new multi-sector heterogeneous-agent New Keynesian framework with input-output linkages, which we call the “HANK-IO” model. Time is discrete and indexed by $t \in \{0, 1, \dots\}$. We abstract from aggregate uncertainty and focus on one-time, unanticipated shocks. Our model features heterogeneous households and multiple sectors. There are H permanent household types, indexed by h and each with mass μ_h where $\sum_h \mu_h = 1$. There are J production sectors, indexed by j and k , each producing a distinct good.

Households differ in their permanent characteristics (ex-ante heterogeneity across types), and they face idiosyncratic uncertainty that leads them to make different consumption, savings, and portfolio decisions (ex-post heterogeneity within type). Our description of household behavior for a given type h is deliberately close to Auclert et al. (2024). We depart from the canonical HANK model by unbundling the aggregate production function and allowing for systematic links between household types h and different production sectors j in terms of both earnings and expenditure patterns. We later discipline these *household-sector linkages* empirically using micro data in Section 4.

2.1 Households

Preferences. Consider household i of permanent type h . The household’s preferences are

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_h^t \left[u_h(c_{i,t}) - v_h(N_{h,t}) \right],$$

⁹ Most of these papers study the propagation of shocks in static environments (Flynn et al., 2022; Guerrieri et al., 2022; Andersen et al., 2022). Clayton et al. (2018) assess the importance of heterogeneity in price rigidity across sectors, also emphasizing the importance of earnings and expenditure heterogeneity. Yang (2022) computes optimal monetary policy in a HANK model where inflation has redistributive effects through households’ consumption baskets, nominal wealth positions, and earnings elasticities to business cycles. Bellifemine et al. (2022) characterize an Intertemporal Keynesian Cross in a HANK economy with regional heterogeneity.

where i 's consumption is defined by an aggregator \mathcal{D}_h that is homogeneous of degree 1,

$$c_{i,t} = \mathcal{D}_h(\mathbf{c}_{i,t}).$$

We use bold-faced notation $\mathbf{c}_{i,t} = (c_{i1,t}, \dots, c_{ij,t})$ for the vector of consumption $c_{ij,t}$ of each good j . All households of type h work $N_{h,t}$ hours; labor supply is determined by labor unions as we describe below. We allow for permanent differences in preferences across types in terms of β_h , $u_h(\cdot)$, $v_h(\cdot)$ and $\mathcal{D}_h(\cdot)$. And we assume that households of each type h are endowed with a distinct labor factor, which we simply refer to as factor h .

Budget constraint. Households can trade a risk-free real bond, which is a unit claim to a bundle of goods in the following period with price P_t .¹⁰ We denote the spot price of the bond at date t by Q_t and household i 's end-of-period stock of bonds by $b_{i,t}$. The household's per-period budget constraint is

$$\sum_j p_{j,t} c_{ij,t} + Q_t b_{i,t} = P_t b_{i,t-1} + z_{i,t} E_{h,t} + P_t T_{i,t},$$

where $p_{j,t}$ is the price of good j . We denote by

$$E_{h,t} = (1 - \tau_t)(W_{h,t} N_{h,t} + d_{h,t})$$

type h 's total post-tax non-financial income. It comprises labor income, where $W_{h,t}$ is the nominal wage of labor factor h , and dividends. Households are the owners of firms but equity shares are not traded. We denote by $d_{h,t}$ the dividend allocation to households of type h . The income tax rate is τ_t , and $T_{i,t}$ is a transfer payment from the government to household i in period t (in units of the reference bundle in which the bond pays out). Household i 's individual labor productivity is $z_{i,t}$. It follows a Markov process, normalized so that $\mathbb{E}_i[z_{i,t} | h] = 1$, and represents the source of idiosyncratic risk in our model. We allow for type-specific Markov processes and write $z_{i,t+1} \sim \mathcal{T}_h^z(\cdot | z_{i,t})$. We follow [Auclert et al. \(2024\)](#) and assume for analytical tractability that i 's share of type h 's dividend income is also proportional to $z_{i,t}$.

Cost of living, real wealth, and borrowing limit. Since the household consumption aggregator $\mathcal{D}_h(\cdot)$ is homothetic, there exists an ideal price index $P_{h,t}$ for households of type h that satisfies $P_{h,t} c_{i,t} = \sum_j p_{j,t} c_{ij,t}$. This price index measures the cost of living for households of type h . Adjusting for the cost of living, we can define the end-of-period real wealth of a household i with bond value $Q_t b_{i,t}$ as $a_{i,t} = \frac{Q_t}{P_{h,t}} b_{i,t}$. Finally, we assume that each household faces a borrowing limit in terms of her real wealth given by $a_{i,t} \geq \underline{a}_h$, where \underline{a}_h may differ across types.

¹⁰ Denote by $\kappa_t \in \mathbb{R}^J$ the weights of the reference bundle in which the bond pays out at date t . A unit claim is therefore priced at $P_t \cdot 1 = \sum_j p_{j,t} \kappa_{j,t}$. We will later associate P_t with the GDP deflator.

Recursive representation. The household's problem is to choose consumption $c_{i,t}$ and wealth $a_{i,t}$ to maximize lifetime utility subject to the budget and borrowing constraints. We now characterize this problem recursively in terms of households' state variables, and we drop the expositional reference to the particular household i .

A household of type h with real wealth a and labor productivity z at time t solves the dynamic programming problem

$$\begin{aligned} V_{h,t}(a, z) = \max_{c, a'} & u_h(\mathcal{D}_h(c)) - v_h(N_{h,t}) + \beta_h \mathbb{E}_t \left[V_{h,t+1}(a', z') \right] \\ \text{s.t.} & a' = R_{h,t}a + e_{h,t}z + T_{h,t}(z) - \sum_j \frac{p_{j,t}}{P_{h,t}} c_j \\ & a' \geq \underline{a}_h \\ & z' \sim \mathcal{T}_h^z(\cdot | z) \end{aligned} \quad (1)$$

where $V_{h,t}(a, z)$ is the household's lifetime value at time t . We denote by $R_{h,t} = \frac{P_{h,t-1}}{P_{h,t}} \frac{P_t}{P_{t-1}} R_t$ the household's real rate of return, where we define $R_t = \frac{P_{t-1}}{Q_{t-1}}$ as the aggregate gross real interest rate. Real rates of return differ across household types due to differences in the evolution of the cost of living given by $\frac{P_{h,t-1}}{P_{h,t}}$. We denote by $e_{h,t} = \frac{E_{h,t}}{P_{h,t}}$ the real income of type h . Finally, we assume the household's share of total transfers T_t is a time-invariant and exogenous function $\phi_h(z)$ of her type h and labor productivity z . And we denote by $T_{h,t}(z) = \phi_h(z) \frac{P_t}{P_{h,t}} T_t$ the real fiscal transfer adjusted for purchasing power.

When choosing consumption c and end-of-period wealth a' , households take as given demand-determined hours $N_{h,t}$, the real rate of return $R_{h,t}$, real income $e_{h,t}$, fiscal transfers $T_{h,t}(\cdot)$, goods prices $p_{j,t}$, and the cost of living index $P_{h,t}$. The dynamic programming problem (1) gives rise to the consumption policy functions $c_{hj,t}(a, z)$ and the wealth accumulation policy function $a'_{h,t}(a, z)$.

Aggregation. Households are uniquely identified by their type h , wealth a , and labor productivity z . We denote the joint density over these state variables in period t by $g_{h,t}(a, z)$, which we also refer to as the cross-sectional household distribution. Aggregate household demand for good j in period t is then given by $C_{j,t} = \sum_h \mu_h \iint c_{hj,t}(a, z) g_{h,t}(a, z) da dz$.

2.2 Labor Market Structure

Household labor supply decisions are intermediated by labor unions as in [Erceg et al. \(2000\)](#) and [Auclert et al. \(2024\)](#). There is a distinct labor market for each factor h .

Labor packer. In each market h , a labor packer aggregates labor varieties supplied by a unit mass of unions $\ell \in [0, 1]$ using a CES aggregation technology,

$$N_{h,t} = \left(\int_0^1 N_{h\ell,t}^{\frac{\epsilon_h^w - 1}{\epsilon_h^w}} d\ell \right)^{\frac{\epsilon_h^w}{\epsilon_h^w - 1}},$$

where ϵ_h^w denotes the elasticity of substitution across labor varieties of type h . The labor packer sells the aggregated factor h to firms at the nominal wage rate $W_{h,t}$. Cost minimization implies the demand functions for labor varieties

$$N_{h\ell,t} = \left(\frac{W_{h\ell,t}}{W_{h,t}} \right)^{-\epsilon_h^w} N_{h,t},$$

where $W_{h\ell,t}$ is the wage at which union ℓ sells its differentiated labor service in market h .

Labor unions. A household i of type h works $n_{i\ell,t}$ hours for each union ℓ , which produces a differentiated labor service according to $N_{h\ell,t} = \mathbb{E}_i[z_{i,t} n_{i\ell,t} | h]$. Unions allocate hours across its workers uniformly, so $n_{i\ell,t} = N_{h\ell,t}$.

Wages are flexible. Each union ℓ sets its wage $W_{h\ell,t}$ to maximize the stakeholder value of its workers. Unions are small. They internalize that wage changes directly affect the indirect utility of its workers, but they take as given prices and aggregates. Similar to [Auclert et al. \(2024\)](#) and [Wolf \(2023\)](#), we assume that each union uses $\frac{v'(N_{h,t})}{u'(C_{h,t})}$ as the relevant marginal rate of substitution when trading off changes in consumption and hours of work for its members, where $C_{h,t} = \int \int c_{h,t}(a, z) g_{h,t}(a, z) da dz$ is aggregate consumption of all households of type h . We show in [Appendix A.1](#) that the union problem gives rise to a labor supply schedule for factor h given by

$$w_{h,t} = \frac{\epsilon_h^w}{\epsilon_h^w - 1} \frac{v'(N_{h,t})}{u'(C_{h,t})}.$$

2.3 Production with Input-Output Linkages

The economy has J production sectors indexed by j and k . Each comprises a retailer and a unit mass of intermediate firms indexed by $\omega \in [0, 1]$.

Retailer. Each sectoral good j is bundled by a retailer using intermediate inputs ω according to the CES production technology

$$y_{j,t} = \left(\int_0^1 y_{j\omega,t}^{\frac{\epsilon_j - 1}{\epsilon_j}} d\omega \right)^{\frac{\epsilon_j}{\epsilon_j - 1}},$$

with elasticity of substitution ϵ_j . The retailer's demand for input ω is given by

$$y_{j\omega,t} = \left(\frac{p_{j\omega,t}}{p_{j,t}} \right)^{-\epsilon_j} y_{j,t}, \quad (2)$$

where $p_{j\omega,t}$ is the price of firm ω , and $p_{j,t}$ is the price of sector j 's bundle, which we simply refer to as "good j " going forward.

Production and cost structure. Each firm ω in sector j produces a differentiated good using intermediate inputs and labor according to the production technology

$$y_{j\omega,t} = A_{j,t} F_j(\mathbf{x}_{j\omega,t}, \mathbf{N}_{j\omega,t}), \quad (3)$$

where $y_{j\omega,t}$ is the firm's output, $\mathbf{x}_{j\omega,t} = (x_{j\omega 1,t}, \dots, x_{j\omega J,t})$ denotes the vector of intermediate inputs, and $\mathbf{N}_{j\omega,t} = (N_{j\omega 1,t}, \dots, N_{j\omega H,t})$ the vector of labor inputs. The production functions $F_j(\cdot)$ are homogeneous of degree 1 (constant returns to scale) but may vary across sectors. And $A_{j,t}$ denotes a sectoral productivity shifter.

Each firm's unit cost function is defined by

$$\mathcal{U}_{j\omega,t}(\mathbf{p}_t, \mathbf{W}_t) = \min_{\mathbf{x}, \mathbf{N}} \left\{ \sum_k p_{k,t} x_k + \sum_h W_{h,t} N_h \quad \text{s.t.} \quad A_{j,t} F_j(\mathbf{x}, \mathbf{N}) \geq 1 \right\}.$$

Notice that cost functions are symmetric across all firms within a sector, $\mathcal{U}_{j\omega,t} = \mathcal{U}_{j,t}$. Due to constant returns, sectoral real marginal costs are $mc_{j,t} = \frac{\mathcal{U}_{j,t}}{p_{j,t}}$.

Dynamic firm problem. The firm's objective is to maximize the present discounted value of future profits, choosing production inputs $\mathbf{x}_{j\omega,t}$ and $\mathbf{N}_{j\omega,t}$ as well as its price $p_{j\omega,t}$. When changing prices, firms face a quadratic adjustment cost $\frac{\chi_j}{2} \left(\frac{p_{j\omega,t}}{p_{j\omega,t-1}} - 1 \right)^2 p_{j,t} y_{j,t}$, where χ_j governs the degree of nominal rigidity and may vary across sectors. Profits are therefore equal to revenue net of operating expenses and adjustment costs,

$$\Pi_{j\omega,t} = p_{j\omega,t} y_{j\omega,t} - \sum_k p_{k,t} x_{j\omega k,t} - \sum_h W_{h,t} N_{j\omega h,t} - \frac{\chi_j}{2} \left(\frac{p_{j\omega,t}}{p_{j\omega,t-1}} - 1 \right)^2 p_{j,t} y_{j,t}. \quad (4)$$

We follow [Kaplan et al. \(2018\)](#) and assume that firms discount future profits at the bond price, so firm ω 's objective can be written as

$$\sum_{t=0}^{\infty} Q_{0,t} \Pi_{j\omega,t}, \quad (5)$$

where $Q_{0,t} = \prod_{s=0}^{t-1} Q_s$. The firm problem is to choose $\{\mathbf{x}_{j\omega,t}, \mathbf{N}_{j\omega,t}, p_{j\omega,t}\}_{t=0}^{\infty}$ to maximize (5) subject to (2), (3) and (4), taking as given its initial price $p_{j\omega,-1}$. Each firm is small and takes as given sectoral and macroeconomic aggregates.

Sectoral Phillips curves. We solve the firm problem in Appendix A.2 and show that all firms ω within a sector j will be symmetric ex post if they have the same initial prices $p_{j\omega,-1} = p_{j\omega',-1}$. We maintain this assumption throughout and focus on a symmetric firm equilibrium, where $y_{j\omega,t} = y_{j,t}$ and $p_{j\omega,t} = p_{j,t}$ for all ω . We will refer to $y_{j,t}$ as the output of good j from now on and to $p_{j,t}$ as the price of good j .

The firm's dynamic price-setting problem gives rise to a canonical non-linear Phillips curve at the sector level,

$$\pi_{j,t}(1 + \pi_{j,t}) = Q_t \pi_{j,t+1}(1 + \pi_{j,t+1}) \frac{p_{j,t+1} y_{j,t+1}}{p_{j,t} y_{j,t}} + \frac{\epsilon_j}{\chi_j} \left[mc_{j,t} - \frac{\epsilon_j - 1}{\epsilon_j} \right] \quad (6)$$

The sectoral Phillips curves (6) express current inflation in terms of future inflation and current real marginal cost $mc_{j,t}$. In a zero-inflation steady state, markups are $\frac{\epsilon_j}{\epsilon_j - 1}$ and may vary across sectors.

2.4 Monetary and Fiscal Policy

Monetary policy. There is a second asset in the economy — a nominal bond that we refer to as “reserve balances” — which is in zero net supply and can be traded only by the central bank and a representative financial market arbitrageur. The central bank sets the nominal interest rate on reserve balances (IORB) i_t . Arbitrageurs trade both reserve balances and the real bond, and their objective is to maximize the present value of future profits discounted at the bond price. That is, they choose B_t^{arb} and b_t^{arb} to maximize

$$\sum_{t=0}^{\infty} Q_{0,t} \left[(1 + i_t) B_{t-1}^{\text{arb}} + P_t b_{t-1}^{\text{arb}} - B_t^{\text{arb}} - Q_t b_t^{\text{arb}} \right]$$

subject to the budget constraint $0 = Q_t b_t^{\text{arb}} + B_t^{\text{arb}}$ and taking as given an initial portfolio $(B_{-1}^{\text{arb}}, b_{-1}^{\text{arb}})$. The first-order conditions imply the no-arbitrage condition

$$1 + i_t = \frac{P_t}{P_{t-1}} \frac{P_{t-1}}{Q_{t-1}} = \frac{P_t}{P_{t-1}} R_t. \quad (7)$$

Since arbitrageur portfolio positions are indeterminate as long as this no-arbitrage Fisher equation holds, we initialize $b_{-1}^{\text{arb}} = B_{-1}^{\text{arb}} = 0$ and set $b_t^{\text{arb}} = B_t^{\text{arb}} = 0$ for all t .¹¹ Arbitrageurs therefore pin down the Fisher equation but otherwise play no role in the model.

The central bank sets the path of nominal interest rates according to the policy rule

$$i_{t+1} = i^* + \mathcal{T}_\pi(\boldsymbol{\pi}_t) + \mathcal{T}_y(\mathbf{y}_t, \mathbf{A}_t) + \varepsilon_t, \quad (8)$$

where ε_t is a monetary policy shock, and \mathcal{T}_π and \mathcal{T}_y represent the aggregators of sectoral inflation

¹¹ This ensures there are no capital gains effects in response to an unanticipated MIT shock at date 0.

and output gaps that the central bank uses in its policy rule.¹² We keep these general for now and later compare the implications of different assumptions on \mathcal{T}_π and \mathcal{T}_y . Finally, i^* is the interest rate consistent with a zero-inflation steady state.

Fiscal policy. The fiscal authority finances expenditures $\mathbf{G}_t = (G_{1,t}, \dots, G_{J,t})$ and transfers T_t with income taxes and real bond issuance. We assume there exists a homothetic aggregator that defines $G_t = \mathcal{G}(\mathbf{G}_t)$ with associated price index $P_{g,t}$. Denoting the government's real bond position by B_t , the per-period government budget constraint is

$$Q_t B_t + \sum_j p_{j,t} G_{j,t} + P_t T_t = P_t B_{t-1} + \tau_t \left(\sum_h \mu_h W_{h,t} N_{h,t} + \Pi_t \right), \quad (9)$$

taking as given an initial bond position B_{-1} . The government faces a no-Ponzi condition that ensures long-run fiscal debt sustainability, given by $\lim_{t \rightarrow \infty} Q_{0,t} B_t = 0$. We consider joint fiscal perturbations that comprise bounded and convergent sequences $(d\mathbf{G}, d\tau, dT)$ that satisfy long-run debt sustainability, with taxes adjusting according to

$$\tau_t = \tau - \phi^\tau \frac{B_t - B}{Y}, \quad (10)$$

as in [Bardóczy et al. \(2024\)](#). Variables without t subscripts denote steady state values.

2.5 Markets and Equilibrium

Equilibrium requires that the markets for each sectoral good j , for each labor factor h , and for bonds clear. Goods market clearing in sector j requires that output equals total use,

$$y_{j,t} = C_{j,t} + \sum_k x_{kj,t} + G_{j,t} + \frac{\chi_j}{2} \pi_{j,t}^2 y_{j,t}. \quad (11)$$

Labor markets clear when hours worked by all households of type h are equal to firms' labor demand for factor h ,

$$\mu_h N_{h,t} = \sum_j N_{jh,t}. \quad (12)$$

Finally, the real bond market clears when the net bond position of households is equal to the government's outstanding debt,

$$0 = \sum_h \mu_h \iint \frac{P_{h,t}}{Q_t} a g_{h,t}(a, z) da dz + B_t. \quad (13)$$

The market for reserve balances clears under our assumption of 0 arbitrageur positions.

¹² We allow \mathcal{T}_y to depend on sectoral output y_t and productivities A_t since output gaps depend on both.

Definition (Competitive Equilibrium). *Given a symmetric initial price distribution $\{p_{j\omega,-1}\}$, initial government debt B_{-1} , and an initial cross-sectional distribution $\{g_{h,-1}(a, z)\}$, and taking as exogenously given paths for sectoral productivities $\{A_{h,t}\}$ as well as monetary and fiscal policy $\{\varepsilon_t, G_{j,t}, \tau_t\}$, competitive equilibrium consists of sequences of prices $\{i_t, p_{j,t}, P_{h,t}, W_{h,t}\}$, sectoral allocations $\{y_{j,t}, x_{jk,t}, N_{jh,t}, \Pi_{j,t}\}$, union labor supply $\{N_{h,t}\}$, policy functions $\{c_{hj,t}(a, z), a'_{h,t}(a, z)\}$, and cross-sectional distributions $\{g_{h,t}(a, z)\}$ such that: households optimize, unions optimize, firms optimize, markets clear, the government budget constraint and no-Ponzi condition are satisfied, monetary policy follows the Taylor rule, and the evolution of the cross-sectional distribution is consistent with household behavior.*

2.6 GDP, Network Objects, and iMPCs

To conclude our description of the model, we introduce several accounting objects, as well as sufficient statistics that govern our analytical results in Section 3.

GDP and expenditure shares. We define nominal GDP as total value added at producer prices net of adjustment costs, which is also equal to the sum of all expenditures on goods for final use,

$$Y_t^n = \sum_j p_{j,t} y_{j,t} - \sum_j \sum_k p_{j,t} x_{kj,t} - \sum_j \frac{\chi_j}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2 p_{j,t} y_{j,t} = \sum_j p_{j,t} (C_{j,t} + G_{j,t}).$$

Final expenditure shares are then defined as the share of a good j in nominal GDP, $b_{j,t} = \frac{p_{j,t} C_{j,t} + p_{j,t} G_{j,t}}{Y_t^n}$. And we define the expenditure share of households of type h as well as the government by $b_{h,t} = \frac{P_{h,t} C_{h,t}}{Y_t^n}$ and $b_{g,t} = \frac{P_{g,t} G_t}{Y_t^n}$ respectively.

Defining aggregate real GDP in levels is conceptually ambiguous since our environment features heterogeneity in consumption preferences. We instead characterize our results in terms of changes in real GDP, using alternatively Laspeyres and Divisia indices. We take the zero-inflation steady state at date $t = -1$ as our base year and define Laspeyres real GDP changes in levels as

$$dY_t = \sum_j \frac{p_j}{\bar{P}} (dC_{j,t} + dG_{j,t}),$$

where variables without time subscripts like p_j denote steady state values and \bar{P} is the Laspeyres base-year GDP deflator (whose normalization pins down units). We also define real GDP changes in logs using the standard Divisia index as

$$d \log Y_t = \sum_j b_j \frac{C_j}{C_j + G_j} d \log C_{j,t} + \sum_j b_j \frac{G_j}{C_j + G_j} d \log G_{j,t}.$$

Changes in the GDP deflator are similarly defined as $d \log P_t = \sum_j \frac{p_j (C_j + G_j)}{Y_t^n} d \log p_{j,t}$, so that $d \log Y_t^n = d \log Y_t + d \log P_t$. We define inflation in the GDP deflator around a zero-inflation

steady state as $d\pi_t = d \log P_t - d \log P_{t-1}$. To first order, real GDP changes expressed in either the Laspeyres or the Divisia indices coincide.

Domar weights. We define the revenue-based Domar weight or sales share of sector j as

$$\lambda_{j,t} = \frac{p_{j,t}y_{j,t}}{Y_t^n}.$$

As in [Baqaee and Farhi \(2020\)](#), the vector of Domar weights also satisfies $\lambda_t = \Psi_t' \mathbf{b}_t$, where \mathbf{b}_t is the $J \times 1$ vector of final expenditure shares and $\Psi_t = (I - \Omega_t)^{-1}$ is the $J \times J$ standard Leontief inverse matrix. Here, Ω_t is the standard (revenue-based) input-output matrix, with $\Omega_{jk,t} = \frac{p_{k,t}x_{jk,t}}{p_{j,t}y_{j,t}}$. We also define cost-based Domar weights as $\tilde{\lambda}_t = \tilde{\Psi}_t' \mathbf{b}_t$, where $\tilde{\Psi}_t = (I - \tilde{\Omega}_t)^{-1}$ is the cost-based Leontief inverse. Here, $\tilde{\Omega}_t$ is the cost-based input-output matrix, with $\tilde{\Omega}_{jk,t} = \frac{\partial \log C_j}{\partial \log p_k}$.

Earnings and expenditure heterogeneity. An important measure of earnings heterogeneity is the non-financial income share of households of type h , which we define as

$$\zeta_{h,t} = \frac{W_{h,t}N_{h,t} + d_{h,t}}{Y_t^n}.$$

Notice that $\sum_h \mu_h \zeta_{h,t} = 1$, so $\zeta_{h,t}$ is a cross-sectional dispersion measure of households' non-financial income before taxes and transfers. We also introduce relative price inflation as a measure of expenditure heterogeneity. We define

$$\rho_{h,t} = \frac{P_{h,t}}{P_t}$$

as the change in household type h 's relative price (over the bond price index) in response to a shock. Similarly, we define

$$\pi_{h,t} = \frac{\rho_{h,t}}{\rho_{h,t-1}} - 1$$

as relative price inflation faced by households of type h .

Intertemporal MPCs. We adopt sequence space notation and use bold-faced notation $\mathbf{Y} = \{Y_t\}_{t=0}^\infty$ to denote sequences that are elements of ℓ^∞ , the space of bounded sequences with the sup norm. Only three aggregate sequences matter for the household's dynamic optimization problem (1): the type-specific rate of return \mathbf{R}_h on savings, type-specific real earnings \mathbf{e}_h , and type-specific real transfer income \mathbf{T}_h . Aggregating households' consumption policy functions then yields an aggregate consumption function of the form

$$C_{h,t} = C_{h,t}(\mathbf{R}_h, \mathbf{e}_h, \mathbf{T}_h), \tag{14}$$

which maps the sequences $(\mathbf{R}_h, \mathbf{e}_h, \mathbf{T}_h)$ to aggregate consumption of households of type h .¹³

Following [Auclert et al. \(2024\)](#), we consider bounded perturbations in the shocks $(dA, \varepsilon, \mathbf{G}, \boldsymbol{\tau}, \mathbf{T})$ around the zero-inflation steady state of our economy. Assuming that the aggregate consumption function $\mathcal{C}_h : \ell^\infty \times \ell^\infty \rightarrow \ell^\infty$ is Fréchet-differentiable, we collect its partial derivatives in three matrices, $\mathbf{M}_h, \mathbf{M}_h^T$ and \mathbf{M}_h^r , with elements

$$M_{h,ts} = \frac{\partial \mathcal{C}_{h,t}}{\partial e_{h,s}} \quad \text{and} \quad M_{h,ts}^T = \frac{\partial \mathcal{C}_{h,t}}{\partial T_{h,s}} \quad \text{and} \quad M_{h,ts}^r = \frac{\partial \mathcal{C}_{h,t}}{\partial R_{h,s}}. \quad (15)$$

$\mathbf{M}_h, \mathbf{M}_h^T$ and \mathbf{M}_h^r are sequence-space Jacobians. We refer to $M_{h,ts}$ and $M_{h,ts}^r$ as type h 's intertemporal marginal propensity to consume (iMPC) and interest rate response elasticity.

3 Analytical Results

Policy transmission in HANK-IO works through a rich set of mechanisms that load on the interaction of both household and sectoral heterogeneity. Before turning to our quantitative experiments, it is useful to inspect these mechanisms analytically. The results in this section will help us organize and interpret the policy counterfactuals later in the paper.

We proceed in three steps. Section 3.1 characterizes an “as-if benchmark” in which household and sectoral heterogeneity decouple and the HANK-IO economy admits a dynamic IS–LM representation. Section 3.2 extends the Intertemporal Keynesian Cross to HANK-IO and shows how earnings and expenditure heterogeneity give rise to new transmission channels for monetary and fiscal policy. Section 3.3 derives an aggregation result for sectoral technology shocks, highlighting when household-sector linkages matter for allocative efficiency and the propagation of supply shocks. Throughout, we focus on shocks that are most relevant for the 2023 policy debate: fiscal transfers, monetary policy, and sectoral supply shocks.

Our results make frequent reference to two benchmark economies that are nested by the HANK-IO model.

HANK. Our model nests a canonical heterogeneous agent New Keynesian (HANK) model with a single production sector and a single good, $J = 1$. All households face the same price index, $P_{h,t} = P_t = p_{1,t}$, but households of different types h may differ in preferences, labor supply, and wages. As in [Auclert et al. \(2024\)](#), this benchmark admits an aggregate consumption function $C_t = \mathcal{C}_t^{\text{HANK}}(\mathbf{R}, \mathbf{Y}; \boldsymbol{\tau}, \mathbf{T})$, which maps sequences of real interest rates \mathbf{R} , aggregate income \mathbf{Y} , and fiscal policy $(\boldsymbol{\tau}, \mathbf{T})$ into aggregate consumption. Combining this with goods market clearing yields

$$Y_t = \mathcal{C}_t^{\text{HANK}}(\mathbf{R}, \mathbf{Y}; \boldsymbol{\tau}, \mathbf{T}) + G_t. \quad (16)$$

¹³ Intertemporal consumption functions of the form (14) are also used in [Auclert et al. \(2024\)](#), [Kaplan et al. \(2018\)](#), [Auclert et al. \(2021\)](#), and many other papers.

Equation (16) represents a fixed point in output: given sequences $(\mathbf{R}, \boldsymbol{\tau}, \mathbf{T})$ it pins down a path for \mathbf{Y} . Like in the standard New Keynesian model, it plays the role of a dynamic IS equation, except that $\mathcal{C}_t^{\text{HANK}}(\cdot)$ captures rich household heterogeneity.

RANK-IO. Our model also nests a canonical multi-sector representative agent New Keynesian (RANK-IO) model (Bouakez et al., 2009; Pasten et al., 2020; Rubbo, 2023; Baqaee et al., 2024). There is a representative household who supplies a homogeneous labor factor and faces the consumption price index P_t . Aggregate consumption satisfies the standard Euler equation $u'(C_t) = \beta R_{t+1} u'(C_{t+1})$. The model's supply side, monetary policy rule, and Fisher equation can be summarized by a "dynamic LM" equation,

$$R_t = \mathcal{R}_t^{\text{RANK-IO}}(\mathbf{Y}; \mathbf{A}, \boldsymbol{\varepsilon}, \mathbf{G}). \quad (17)$$

Equation (17) determines the level of the real interest rate as a function of the sequences of aggregate output \mathbf{Y} and exogenous shocks $(\mathbf{A}, \boldsymbol{\varepsilon}, \mathbf{G})$. It is the dynamic analogue of the LM curve in the textbook IS-LM model: just like the static LM curve maps output to the real interest rate, (17) maps a path for output into a path for real interest rates.

3.1 The "As-If" Benchmark

There is a particularly tractable version of our HANK-IO model that obtains under three assumptions on functional forms and parameters. This benchmark is a useful starting point for understanding how shocks propagate in HANK-IO. We begin with this benchmark in the present subsection and then show in Sections 3.2 and 3.3 how departures from it give rise to new transmission mechanisms.

Assumption A1. There exists a common homothetic aggregator \mathcal{D} used by all households and the government, with $\mathcal{D} = \mathcal{D}_h = \mathcal{G}$.

Under Assumption A1, there exists a price index P_t such that $P_t C_{h,t} = \sum_j p_{j,t} C_{hj,t}$ for all household types h and $P_t G_t = \sum_j p_{j,t} G_{j,t}$ for the government. This allows us to define real GDP in levels by $P_t Y_t = Y_t^n$. Using goods market clearing, we obtain $Y_t = C_t + G_t$, where aggregate consumption is $C_t = \sum_h \mu_h C_{h,t}$.

Assumption A2. Every sector's production function is weakly separable in intermediates and a common labor nest \mathcal{N} that is Cobb-Douglas,

$$F_j(\mathbf{x}_{j,t}, \mathbf{N}_{j,t}) = \tilde{F}_j(\mathbf{x}_{j,t}, \mathcal{N}(\mathbf{N}_{j,t})),$$

with $\mathcal{N}(\mathbf{N}_{j,t}) = \prod_h v_h^{-v_h} N_{jh,t}^{v_h}$. Furthermore, aggregate profits $\Pi_t = \sum_j \Pi_{j,t}$ are allocated as dividends

in proportion to labor income shares, $d_{h,t} = \omega_{h,t}\Pi_t$.

Assumption A2 implies that labor can be summarized by an effective factor $N_{j,t} = \mathcal{N}(N_{j,t})$ with an associated wage index $W_{j,t}$ such that $W_{j,t}N_{j,t} = \sum_h W_{h,t}N_{jh,t}$. Cobb–Douglas homogeneity then delivers labor demand $N_{jh,t} = \nu_h(W_{j,t}/W_{h,t})N_{j,t}$. Because the labor nest is symmetric across sectors, the effective wage index is common, $W_{j,t} = W_t$, and each household type’s labor income share,

$$\omega_{h,t} = \frac{W_{h,t}N_{h,t}}{\sum_{\tilde{h}} \mu_{\tilde{h}} W_{\tilde{h},t} N_{\tilde{h},t}}$$

is constant over time, $\omega_{h,t} = \omega_h$. By setting each type’s profit share equal to its labor income share, total income of type h is simply $e_{h,t} = \omega_h(1 - \tau_t)Y_t$. Thus, type- h income is a time-invariant share ω_h of aggregate disposable income $(1 - \tau_t)Y_t$, and movements in the aggregate profit share do not change relative incomes across types.

Assumption A3. Labor disutility is isoelastic with $v'_h(n) = \kappa_h n^{\phi_h}$ and labor unions in all factor markets use $\frac{v'_h(N_{h,t})}{u'(C_t)}$ as the effective marginal rate of substitution between consumption and hours.

Under Assumption A3, unions evaluate labor supply decisions using the marginal utility of aggregate consumption $u'(C_t)$, so that *relative* changes in households’ consumption do not generate income effects on labor supply. The assumption does not rule out heterogeneous labor supply schedules, which may arise from $v'_h(N_{h,t})$ and factor-specific wage markups ε_h^w . What it rules out is a direct link from heterogeneity in household income to heterogeneity in income effects on labor supply.

Proposition 1 (Dynamic IS-LM Representation). *Under Assumptions A1–A3 as well as standard regularity and determinacy conditions, sequences of real GDP Y and interest rates R form part of a competitive equilibrium in HANK-IO if and only if they satisfy*

$$Y = \mathcal{C}^{\text{HANK-IO}}(Y, R; \tau, T) + G \quad (\text{DIS})$$

$$R = \mathcal{R}^{\text{HANK-IO}}(Y; A, \varepsilon, G). \quad (\text{DLM})$$

Moreover, (DIS) coincides with the IS curve (16) of the HANK economy and (DLM) coincides with the LM curve (17) of the RANK-IO economy,

$$\mathcal{C}^{\text{HANK-IO}} = \mathcal{C}^{\text{HANK}} \quad \text{and} \quad \mathcal{R}^{\text{HANK-IO}} = \mathcal{R}^{\text{RANK-IO}}.$$

Under Assumptions A1–A3, equilibrium in HANK-IO can be summarized by a *dynamic IS-LM representation*. The *dynamic IS curve* (DIS) is a demand block: it maps a path of real interest rates R (together with exogenous fiscal policy) into a path of aggregate demand Y . The *dynamic LM curve*

summarizes the model’s supply block, asset market, and Taylor rule: it maps a path of aggregate activity Y (together with sectoral shocks and monetary policy) into the corresponding path of real interest rates R . Taken together, (DIS) and (DLM) pin down the joint evolution of (Y, R) .

An “as-if” benchmark. The main content of Proposition 1 is that, under Assumptions A1–A3, household heterogeneity and sectoral heterogeneity affect equilibrium through *separate* blocks of the system. The dynamic IS curve inherits its shape from the household side: for any given path of real rates R , (DIS) takes the same form as the one-sector HANK IS equation (16). Conversely, the dynamic LM curve inherits its shape from the multi-sector supply side: (DLM) maps any given path of aggregate demand Y to the same path of real rates R as the representative-household RANK-IO model would.

In that sense, the economy behaves *as if* the demand block was that of a one-sector HANK model given a path of the real rate R , while the supply block was that of a multi-sector RANK model given a path of aggregate demand Y . The determination of aggregate demand is unaffected by sectoral heterogeneity taking as given a path of real interest rates; while the evolution of real interest rates is independent from household heterogeneity taking as given a path of aggregate activity. There is a *decoupling* between household and sectoral heterogeneity.

Importantly, this decoupling does *not* eliminate either source of heterogeneity. Households may still differ in preferences, wealth and labor productivity, and sectors may still differ in technology, markups, price rigidities, and input–output linkages.

We now leverage Proposition 1 to characterize the transmission of stabilization policy.

Corollary 1 (As-If Benchmark: Fiscal Policy). *Consider a bounded fiscal perturbation $(dG, dT, d\tau)$ under a monetary policy rule that stabilizes the real interest rate, $dR = 0$. The impulse response of output must satisfy*

$$(I - M) dY = M^\tau d\tau + M^T dT + dG, \quad (18)$$

where M has entries $M_{ts} = \frac{\partial C_t^{\text{HANK}}}{\partial Y_s}$, and $M^\tau = Y(1 - \tau)^{-1}M$, and M^T has entries $M_{ts}^T = \frac{\partial C_t^{\text{HANK}}}{\partial T_s}$.

The as-if benchmark of our economy admits the same Intertemporal Keynesian Cross characterization of fiscal policy shocks as the canonical one-sector HANK model (Auclert et al., 2024). In particular, the iMPC matrix M remains a sufficient statistic for the effects of fiscal policy. It is observationally equivalent to that in the one-sector HANK model.

This result obtains in spite of substantial sectoral heterogeneity that is masked by the dynamic LM equation. When monetary policy neutralizes indirect effects through the real interest rate, untargeted fiscal policy is entirely unaffected by sectoral heterogeneity. In particular, fiscal multipliers in our as-if benchmark are identical to any HANK economy without sectoral heterogeneity that admits a representation of the aggregate consumption function as in (DIS).

Sectoral heterogeneity does, however, affect the monetary policy response $d\mathbf{i}$ that is necessary to neutralize real rates since this policy rule is governed by the dynamic LM equation. In fact, the nominal interest rate rule $d\mathbf{i}$ is determined solely by the dynamic LM curve, taking as given aggregate demand \mathbf{Y} , which implies that it is governed by sectoral but not by household heterogeneity.

Corollary 2 (As-If Benchmark: Monetary Policy). *Consider a monetary shock $d\boldsymbol{\varepsilon}$ that induces a bounded perturbation in real interest rates $d\mathbf{R}$. The impulse response of output must satisfy*

$$(\mathbf{I} - \mathbf{M}) d\mathbf{Y} = \mathbf{M}^r d\mathbf{R}. \quad (19)$$

where \mathbf{M}^r has entries $M_{ts}^r = \frac{\partial \mathcal{C}_t^{\text{HANK}}}{\partial R_s}$.

For a given change in the real interest rate, $d\mathbf{R}$, the effect of monetary policy on aggregate activity is again independent of sectoral heterogeneity and solely shaped by household heterogeneity. In particular, the iMPC and interest rate response matrices that appear in Corollary 2, \mathbf{M} and \mathbf{M}^r , are equivalent to their counterparts in the one-sector HANK economy.

Sectoral heterogeneity does, however, affect the transmission of nominal interest rate shocks, $d\mathbf{i}$, to the real interest rate, $d\mathbf{R}$. In particular, heterogeneity in price rigidities and other sectoral variables does shape the strength of interest rate policy, i.e., monetary non-neutrality, but only through its transmission to real rates. Corollary 2 demonstrates that the path of real interest rates is a summary statistic for the effect of monetary policy on aggregate activity. Once the monetary authority has implemented a desired path of real rates, the production network structure of the economy no longer matters for transmission.

3.2 An Intertemporal Keynesian Cross with Earnings and Expenditure Heterogeneity

Our next result characterizes an Intertemporal Keynesian Cross for monetary and fiscal policy in the baseline HANK-IO model of Section 2.

Proposition 2 (Intertemporal Keynesian Cross in HANK-IO). *Consider a bounded perturbation in*

demand and supply shocks ($dA_j, d\tau, dT, dG, d\epsilon$). The impulse response of output must satisfy

$$\begin{aligned}
 \underbrace{(\mathbf{I} - \mathbf{M})}_{\text{Multiplier}} d\mathbf{Y} &= \underbrace{\rho_g d\mathbf{G} - \mathbf{M}^\tau d\boldsymbol{\tau} + \mathbf{M}^T dT + \mathbf{M}^r d\mathbf{R}}_{\text{As-If Benchmark}} \\
 &+ \underbrace{Y \text{Cov}_h(\mathbf{M}_h, d\boldsymbol{\xi}_h)}_{\text{Earnings Heterogeneity}} - \underbrace{\text{Cov}_h\left(\kappa_h \mathbf{M}_h, \frac{b_h}{\rho_h} d\rho_h\right)}_{\text{Expenditure Heterogeneity (Static)}} - \underbrace{R \text{Cov}_h\left(\mathbf{M}_h^r, b_h d\pi_h^o\right)}_{\text{Expenditure Heterogeneity (Intertemporal)}}
 \end{aligned}$$

where $\mathbf{M} = (1 - \tau) \sum_h \mu_h \xi_h \mathbf{M}_h$, $\mathbf{M}^\tau = Y(1 - \tau)^{-1} \mathbf{M}$ and $\mathbf{M}^T = \sum_h \mu_h \phi_h \mathbf{M}_h$, as well as $\mathbf{M}_h^r = \frac{P_h}{b_h P} \frac{\partial \bar{C}_{h,t}}{\partial R_{h,s}}$ and $\mathbf{M}^r = \sum_h \mu_h b_h \mathbf{M}_h^r$.

Proposition 2 characterizes the implications of household-sector linkages for policy transmission. Three new effects emerge relative to the transmission channels already operative in our as-if benchmark.¹⁴

The first new effect is an *earnings heterogeneity channel* that emerges when household income shares are no longer constant. It is captured by a cross-sectional covariance across household types between iMPCs \mathbf{M}_h and changes in income shares $d\boldsymbol{\xi}_h$. Earnings heterogeneity therefore dampens the aggregate effects of contractionary monetary and fiscal policy when policy redistributes income to households with large iMPCs.

The next two terms of our decomposition capture the implications of *expenditure heterogeneity*. A change in the price of household type h 's consumption basket elicits a static and an intertemporal effect. Holding fixed nominal income, an increase in the relative cost of living $d\rho_h$ of households of type h will reduce their effective purchasing power. Households respond to this income effect in proportion to their intertemporal marginal propensities to consume. This first new transmission channel due to expenditure heterogeneity is therefore a covariance across household types between iMPCs \mathbf{M}_h and changes in relative bundle prices $d\rho_h$ weighted by expenditure shares. The iMPC matrix \mathbf{M}_h is scaled by $\kappa_h = [(1 - \tau) \frac{\xi_h}{b_h} Y + \frac{\phi_h}{b_h} T]$, which translates a change in relative price into income. The relative price change $d\rho_h$ is scaled by b_h / ρ_h so that this term only appears as a cross-sectional covariance, since $\sum_h \mu_h b_h d \log \rho_h = 0$. When policy leads to a relative price increase in the consumption basket of households (and in states of the world) with large iMPCs, then aggregate consumer spending falls.

A change in the path of relative prices also elicits an intertemporal effect. The relative bundle price governs a household's intertemporal decision between consuming in period t and saving for consumption in future periods. When households experience *relative* price inflation,

¹⁴ We refer to Proposition 2 as an Intertemporal Keynesian Cross because the impulses on the RHS are amplified by a Keynesian-cross-like multiplier (Auclert et al., 2024). As in our as-if benchmark, this multiplier is governed by households' iMPCs. This Keynesian multiplier is not directly shaped by sectoral heterogeneity.

their effective real rate of return on savings $d \log R_h$ falls disproportionately. And when relative price inflation is especially large for those households with strong intertemporal interest rate responses M_h^r , then aggregate demand decreases, in proportion to the cross-sectional covariance $-\text{Cov}_h(M_h^r, b_h d \log \pi_h)$.

3.3 Beyond Hulten: An Aggregation Result for Sectoral Shocks

We now derive an aggregation result for the macroeconomic effects of sectoral technology shocks in HANK-IO. It is again useful to start with the as-if benchmark in Proposition 3 and then inspect how allocative efficiency is shaped by earnings and expenditure heterogeneity in Proposition 4.

Proposition 3 (Aggregating Sectoral Technology Shocks: As-If Benchmark). *Consider a bounded perturbation in sectoral technology $d \log A = (d \log A_1, \dots, d \log A_N)$. The impulse response of output must satisfy*

$$d \log Y_t = \underbrace{\frac{1 + \phi}{\gamma + \phi} \tilde{\lambda}' d \log A_t}_{\text{Pure Technology Effect}} - \underbrace{\frac{1 + \phi}{\gamma + \phi} \tilde{\lambda}' d \log \mu_t - \frac{\phi}{\gamma + \phi} d \log \Lambda_t^L}_{\text{Change in Allocative Efficiency}} \quad (20)$$

where

$$\phi = \frac{\sum_h \nu_h \frac{\phi_h}{1 + \phi_h}}{\sum_h \nu_h \frac{1}{1 + \phi_h}} \quad \text{and} \quad \gamma = -\frac{1}{b_C} \frac{C u''(C)}{u'(C)}.$$

Proposition 3 is an aggregation result that traces the macroeconomic effects of microeconomic sectoral technology shocks. As in Baqaee and Farhi (2020), equation (20) decomposes the impact on real GDP into two effects. The pure technology effect holds fixed the resources employed by firms and workers, and measures the change in output that results from the increased productivity of given resources. Changes in allocative efficiency summarize the effect on output from a reallocation of resources across firms and workers. We denote by $\mu_{j,t} = \frac{p_{j,t}}{mc_{j,t}}$ the time-varying markup in sector j , so that $d \log \mu = (d \log \mu_1, \dots, d \log \mu_N)$ captures the endogenous response of sectoral markups across time and sectors. Finally, we denote by $\Lambda_t^L = \frac{W_t N_t}{Y_t}$ the aggregate labor income share.

It is remarkable that equation (20) is virtually identical to the aggregation result of Baqaee and Farhi (2020).¹⁵ This is despite our economy featuring rich and dynamically evolving household heterogeneity, whereas theirs is a static representative-household setting. Proposition 3 underscores that our as-if benchmark features a decoupling of household and sectoral heterogeneity: For given

¹⁵ In their baseline model, Baqaee and Farhi (2020) derive the aggregation result $d \log Y = \tilde{\lambda} d \log A - \tilde{\lambda} d \log \mu - \tilde{\Lambda} d \log \Lambda$. There are minor differences between our result and theirs. First, our setting is dynamic and equation (20) solves for the sequence of output changes. Second, we allow for elastic factor supply, which accounts for the presence of the elasticities η and γ . Baqaee and Farhi (2020) extend their main result to elastic factor supply in Appendix H.2 and equation (20) mirrors their extended formula. Finally, markups in our setting are endogenous and result from nominal rigidities. Nonetheless, Proposition 3 underscores that the importance of changes in markups is captured in reduced form by $\frac{1 + \eta}{\gamma + \eta} \tilde{\lambda}$, as in Baqaee and Farhi (2020). Also notice that cost-based factor shares always sum to 1, and so $\tilde{\Lambda}^L = 1$ with a single factor.

changes in markups and factor shares, the aggregate consequences of microeconomic technology shocks do not directly interact with household heterogeneity. Changes in sectoral markups $d \log \mu$ and the labor income share $d \log \Lambda^L$ are sufficient statistics for the implications of household heterogeneity. In particular, the effect of a given perturbation $(d \log A_t, d \log \mu_t, d \log \Lambda_t^L)$ on output $d \log Y_t$ is determined *as if* in a representative household model with a modified elasticity of labor supply ϕ that aggregates type-specific labor supply elasticities ϕ_h and factor shares v_h .

The importance of allocative efficiency is tightly linked to nominal rigidities. In the flexprice limit of our as-if benchmark, where markups are positive but constant, equation (20) becomes

$$d \log Y_t^{\text{flex}} = \frac{1 + \eta}{\gamma + \eta} \tilde{\lambda}' d \log A_t. \quad (21)$$

This flexprice aggregation result still features two key departures from the canonical Hulten's theorem, according to which the macroeconomic effect of a sectoral technology shock is proportional to that sector's revenue-based Domar weight. First, the importance of sectoral technology shocks is governed by the cost-based Domar weight $\tilde{\lambda}_j = \mu_j \lambda_j = \frac{\epsilon_j}{\epsilon_j - 1} \lambda_j$. Second, the standard Hulten's theorem applies in settings with inelastic factor supply. The multiplier $\frac{1 + \eta}{\gamma + \eta}$ accounts for elastic labor supply, with the elasticities η and γ governing the household labor supply curve. When we allow for appropriate sectoral employment subsidies to offset steady state markups, $\mu_j = 1$ for all j , a variant of Hulten's theorem extended to elastic labor supply applies in the as-if benchmark with flexible prices (Bigio and La'O, 2020). In this case, production is efficient and aggregation on the production side of the economy is again governed by revenue-based Domar weights despite rich household heterogeneity.

This simple benchmark is no longer valid in the presence of systematic household-sector linkages. Away from our as-if benchmark and Assumptions A1–A3 the Baqaee and Farhi (2020) aggregation result must be augmented to account for earnings and expenditure heterogeneity as we show next.

Proposition 4 (Aggregating Sectoral Technology Shocks). *Assuming symmetric $\gamma_h = \gamma$ and $\phi_h = \phi$, the aggregate effect of sectoral technology shocks is given by*

$$d \log Y = \overbrace{\frac{1 + \eta}{\gamma + \eta} \tilde{\lambda}' d \log A - \frac{1 + \eta}{\gamma + \eta} \tilde{\lambda}' d \log \mu - \frac{\eta}{\gamma + \eta} d \log \Lambda^L}^{\text{As-If Benchmark}} - \underbrace{\frac{1}{\gamma + \eta} \text{Cov}_h \left(\tilde{\Lambda}_h, d \log \rho_h \right)}_{\text{Expenditure Heterogeneity}} - \underbrace{\frac{\eta}{\gamma + \eta} \text{Cov}_h \left(\tilde{\Lambda}_h, d \log \Lambda_h^L \right)}_{\text{Earnings Heterogeneity}} - \underbrace{\frac{\gamma}{\gamma + \eta} \text{Cov}_h \left(\tilde{\Lambda}_h, d \log \delta_h \right)}_{\text{Income Effect on Labor Supply}}$$

where $d \log \delta_h = d \log C_h - d \log C$ is a measure of consumption dispersion.

Proposition 4 generalizes our aggregation result beyond the as-if benchmark. The aggregate output response is again governed by a pure technology effect and changes in allocative efficiency. The pure technology effect, given by $\frac{1+\eta}{\gamma+\eta} \tilde{\lambda}' d \log A$, summarizes the increased productivity of resources holding fixed the current allocation. It is remarkable that pure technology gains are again unaffected by household heterogeneity. They are proportional to sectors' cost-based Domar weights, $\tilde{\lambda}$, and to the elasticities governing household labor supply, γ and η .

The remaining terms of our decomposition summarize changes in allocative efficiency. As in the as-if benchmark, changes in endogenous markups and factor shares have implications for allocative efficiency. These terms are exactly as in Proposition 3. More surprisingly, household heterogeneity and, in particular, cross-sectional household-sector linkages now also shape allocative efficiency. Three additional channels emerge that all work through the factor supply equations given by $d \log W_h - d \log P_h = \eta d \log N_h + \gamma d \log C_h$. Households of type h adjust their labor supply either in response to changes in their real wage, $d \log W_h - d \log P_h$, or when their income and consequently their consumption changes, $d \log C_h$.

The first novel determinant of allocative efficiency is the effect of expenditure heterogeneity on household labor supply. Firms' cost structure is governed by nominal factor prices $W_{h,t}$. In other words, what matters for the production side is the marginal cost at which firms can hire additional labor. Labor supply, however, is governed by real wages. When the price of a household's consumption bundle increases, $d \log \rho_{h,t} > 0$, a given nominal wage for household h has less purchasing power and the real wage falls. At a given nominal wage, this household now supplies fewer hours and labor becomes more expensive for firms. The new effect on allocative efficiency is proportional to the cross-sectional covariance across household types or labor factors $\text{Cov}_h(\tilde{\Lambda}_h, d \log \rho_{h,t})$: If consumer prices *increase* for those households whose labor factors have *large* cost-based Domar weights, then output falls and allocative efficiency deteriorates. Intuitively, labor factors that play a dominant role in firms' cost structures become relatively more expensive because those household types experience a relative drop in their purchasing power. Crucially, the relevant covariance is with respect to *cost-based* factor shares $\tilde{\Lambda}_h$ because markups drive a wedge between prices and marginal costs.

Earnings heterogeneity also shapes the transmission of technology shocks to allocative efficiency. The overall effect of changes in factor shares on allocative efficiency is captured by

$$-\frac{\eta}{\gamma+\eta} \sum_h \tilde{\Lambda}_h d \log \Lambda_{h,t} = \underbrace{\frac{\eta}{\gamma+\eta} d \log \Lambda_t^L}_{\text{Labor Income Share}} - \underbrace{\frac{\eta}{\gamma+\eta} \text{Cov}_h(\tilde{\Lambda}_h, d \log \Lambda_{h,t})}_{\text{Earnings Heterogeneity}}$$

where we denoted the average change in factor share by $d \log \Lambda_t = \mathbb{E}_h(d \log \Lambda_{h,t})$. Notice that the aggregate labor income share is not weighted by cost-based Domar weights because $\mathbb{E}_h \tilde{\Lambda}_h = 1$: All production costs must ultimately be accounted for by factor costs. The aggregate effect is also operative in the as-if benchmark. Intuitively, when the total labor income share falls,

$d \log \Lambda_t < 0$, then households receive less labor income. This elicits a positive labor supply response governed by the elasticities $\frac{\eta}{\gamma+\eta}$. Heterogeneity in factor share responses can amplify or dampen the resulting change in allocative efficiency. When factors with the *largest* cost-based Domar weights $\tilde{\Lambda}_h$ experience a relative *decrease* in their income share, then resources are reallocated towards the more monopolized and distorted sectors. The strength of this effect is again proportional to $\frac{\eta}{\gamma+\eta}$, which governs the endogenous labor supply response to the fall in income.

Finally, there may be a heterogeneous income effect on labor supply across factors. This effect is summarized by a covariance across household types between cost-based factor shares $\tilde{\Lambda}_h$ and changes in consumption dispersion $d \log \delta_{h,t}$. Intuitively, if households h experience a relative drop in consumption, $d \log \delta_{h,t} < 0$, then they supply more labor for a given real wage. This income effect is governed by the elasticities $\frac{\gamma}{\gamma+\eta}$. As a result, firms can hire a given amount of labor factor h more cheaply and marginal cost falls. If the factors that become relatively cheaper also have high cost-based Domar weights, then output increases due to gains in allocative efficiency. Intuitively, marginal costs then fall in the most monopolized sectors.

Our discussions in Sections 3.2 and 3.3 point to an important conceptual takeaway. What matters for the aggregation of sectoral technology shocks—in particular their transmission through changes in allocative efficiency—are covariances with respect to cost-based factor shares $\tilde{\Lambda}_h$. Gains from allocative efficiency result from a reallocation of resources to relatively more distorted and monopolized sectors. The cost-based factor share $\tilde{\Lambda}_h$ precisely captures those markups that drive a wedge between prices and marginal costs. The covariance terms with respect to $\tilde{\Lambda}_h$ that appear in Proposition 4 therefore indicate whether resources and labor find more productive uses. The transmission of monetary and fiscal policy (demand) shocks, on the other hand, is determined by covariances with respect to iMPCs M_h . Intuitively, demand propagation is governed by spending propensities in response to changes in income and prices. Mechanically, this distinction emerges because our results in Section 3.2 take as their starting point the goods market clearing condition and the aggregate consumption function, whereas our derivations in Section 3.3 focus on the supply and production equations.

4 Taking a Quantitative HANK-IO Model to the Data

This section develops a quantitative HANK-IO model that matches key empirical regularities on household-sector linkages. After discussing our data sources in Section 4.1, we present these empirical regularities in Section 4.2. To match these motivating empirical moments, we enrich the baseline HANK-IO model of Section 2 along two dimensions. The quantitative model we present in Section 4.3 features capital as an additional production factor and nonhomothetic CES consumption preferences. Allowing households to trade a second, illiquid asset is important to match the intertemporal marginal propensities to consume that govern the new earnings and expenditure heterogeneity transmission channels of monetary policy (Auclert et al., 2024), which

we characterized in Section 3 . Adding capital as a factor of production also allows us to capture the importance of the investment network that is an important part of the economy’s network structure (Vom Lehn and Winberry, 2022). Finally, nonhomothetic preferences allow us to match empirical expenditure patterns across the income distribution (Comin et al., 2021).

4.1 Data

In this section, we assemble our dataset, which draws on six sources, and explain the construction of key variables. The details of data construction are reported in Appendix B. We conduct both our empirical and quantitative analysis at the 2-digit NAICS level corresponding to the Bureau of Economic Analysis’ (BEA) Detailed Input-Output (I-O) Tables, which is also the number of sectors for our estimation of the nonhomothetic CES demand system.

Input-output linkages. We use the BEA I-O Use Tables to measure input-output linkages across 71 industries from 1997 to 2015. The Use Tables specifies the nominal amount of inputs used by each industry. After eliminating industry categories related to federal, state, and local government, we are left with 66 private sectors. We aggregate the BEA sectors to the 2-digit NAICS level, arriving at 22 sectors as our baseline.

Household expenditure characteristics: CEX-IO data. We use the Consumer Expenditure Survey (CEX) conducted by the U.S. Bureau of Labor Statistics from 1997 to 2015 to obtain expenditure shares for households across the income distribution. The survey respondents report their consumption expenditures for the full consumption basket of goods and services, across 668 detailed categories called “UCCs”. Sample selection and other data treatment details can be found in Appendix B.9. After the sample construction, we match the CEX spending categories to 22 industries in the IO table, constructing a dataset with household expenditure shares across the income distribution in 22 final IO industries for each year in the sample period. The mapping is based on a manual concordance assembled by Levinson and O’Brien (2019). The dataset contains 948,049 households across the sample period, each with nominal expenditure data in each IO industry and household characteristics.

Household earnings characteristics: ACS-IO data. We obtain cross-sectional household occupation and payroll data from the American Community Survey (ACS). The ACS is a survey administered by the U.S. Census Bureau and answered by a random 1% sample of the U.S. population each year. The dataset is made available by IPUMS (Ruggles et al., 2015) and offers demographical and labor information about all survey respondents. In particular, the ACS provides consistent industry identifiers for 320 industries in the private sector from 2000 to 2015. Matching ACS’s 6-digit NAICS industry categories to industries in the IO table, we obtain a dataset with sector-specific payroll shares for households in various income quantiles.

Sectoral price rigidity. To measure price rigidity across industries, we map the sector-specific monthly price adjustment frequency from [Pasten et al. \(2017\)](#) to the 22 final IO industries. [Pasten et al. \(2017\)](#) use the data underlying the Producer Price Index (PPI) for 754 industries (defined by 6-digit NAICS codes) from the U.S. Bureau of Labor Statistics, from 2005 to 2011. The PPI measures changes in selling prices from the perspective of producers and covers all industrial and service sectors, including the product of intermediate inputs. Compared with earlier estimates by [Nakamura and Steinsson \(2008\)](#) with a focus on CPI data, the PPI measures are more suitable both for our sample period and our emphasis on intermediate inputs. We also show robustness using other price rigidity measures in Appendix C.

Factor shares. We compute the sector-specific factor shares in production from the BEA's GDP by Industry dataset with 66 private industries. The labor share in primary factors is computed as the compensation of employees as a percentage of value added, adjusted for taxes and subsidies, and averaged over 1997-2015. The intermediate inputs share is computed as total intermediate inputs as a percentage of gross output, averaged over the same sample period. We then map factor share data for 66 sectors into the 2-digit NAICS industry specification, where we weight the concordance by sector-specific gross output levels.

Capital investment shares. In our baseline, we use the Investment Flows Data for 41 Sector Partition from [Vom Lehn and Winberry \(2022\)](#) from 1997-2015 to calculate the share of capital investment inputs from each sector. We first calculate the share of total capital purchase from each of the 41 sectors each year in the Investment Flow tables. We then crosswalk the 41 sectors based on their NAICS codes to 22 sectors. Finally we take the average of each year's share.

Government spending shares. The BEA's industry input-output "Use" table allows us to compute the share of government spending on goods and services from different sectors. Government spending shares are calculated as total government expenditure in sector j as a percentage of total overall government spending. "Government" includes the federal government, federal government enterprises, state and local government, and state and local government enterprises. We average the annual share across 1997-2015, and then map the 66 sectors in which the government spends to the 2-digit NAICS level.

Markups. We use sectoral markup data from [Baqae and Farhi \(2020\)](#) in our model calibration. They estimate three alternative measures of markups across 66 sectors from 1997 to 2015. The average markup for each sector in any particular year is computed as the harmonic sales-weighted average of firm markups, which are taken from Compustat and assigned to BEA sectors. In our baseline model, we use the average of their benchmark estimates following the accounting profits approach because the average markup is then around 10% and thus closer to the standard

markup assumed in the HANK literature. Then we map the 66 industries to the 22-sector 2-digit IO categories, weighting the concordance by sector-specific gross output levels.

Network centrality. A reduced-form measure of a sector’s centrality in the input-output production network is the Katz-Bonacich centrality measure discussed by [Carvalho \(2014\)](#). This measure of network centrality is defined as $c = \eta(\mathbf{I} - \lambda\alpha_{jk}^x)^{-1}\mathbf{1}$, where we set $\eta = \frac{1-\theta}{N} = \frac{1-0.5}{22}$ and $\lambda = 0.5$. We denote by α_{jk}^x the share of sector j ’s spending on inputs from sector k in sector j ’s total value of the intermediate input bundle. We also calculated the “outdegree” of sectors, defined as $d_k = \sum_j \alpha_{jk}^x$, that is, the sum over all the weights of the network in which sector k appears as an input-supplying sector.

4.2 Empirical Regularities

We start by documenting empirical evidence of systematic household-sector linkages at the micro level. These empirical regularities suggest that households with different income levels are systematically exposed to different sectors through both expenditure and earnings patterns. Our results draw on much previous empirical work, which we review at the end of this subsection.

Expenditure heterogeneity. For each sectoral feature θ , we compute an expenditure-share-weighted measure for each household percentile hp in each year t as

$$\theta_t^{exp, hp} = \sum_{j \in S} w_{j,t}^{exp, hp} \theta_{j,t}$$

where $\theta_{j,t}$ corresponds to our empirical measure of sectoral feature θ for sector j in year t , and $w_{j,t}^{exp, hp}$ denotes the share of expenditures on goods in sector j accounted for by households of income percentile hp in year t .

The blue dots in [Figure 1](#) represent expenditure-weighted sectoral features in the data, plotted across the household income distribution by linking cross-sectional household spending data from the CEX to the BEA IO table. We find that higher-income households consume relatively more in sectors that are more labor-intensive, less capital-intensive, and contribute more to capital production. Middle-income households consume more in sectors that are more intermediates-intensive, more price-flexible, more central in the production network, have lower markups, and have larger government spending shares. The red lines in [Figure 1](#) demonstrate the fit of our quantitative model, which specifically targets these empirical regularities.

The five largest sectors in households’ consumption baskets are (1) housing, (2) non-durable manufacturing goods such as food, beverages, apparels, and textile products, (3) retail trade, (4) durable manufacturing goods such as appliances, cars, electronic devices, furniture, and (5) utilities. [Table 2](#) shows that higher-income households spend relatively less on non-durable manufacturing goods, utilities, retail and wholesale trades, and information products such as broadcasting and

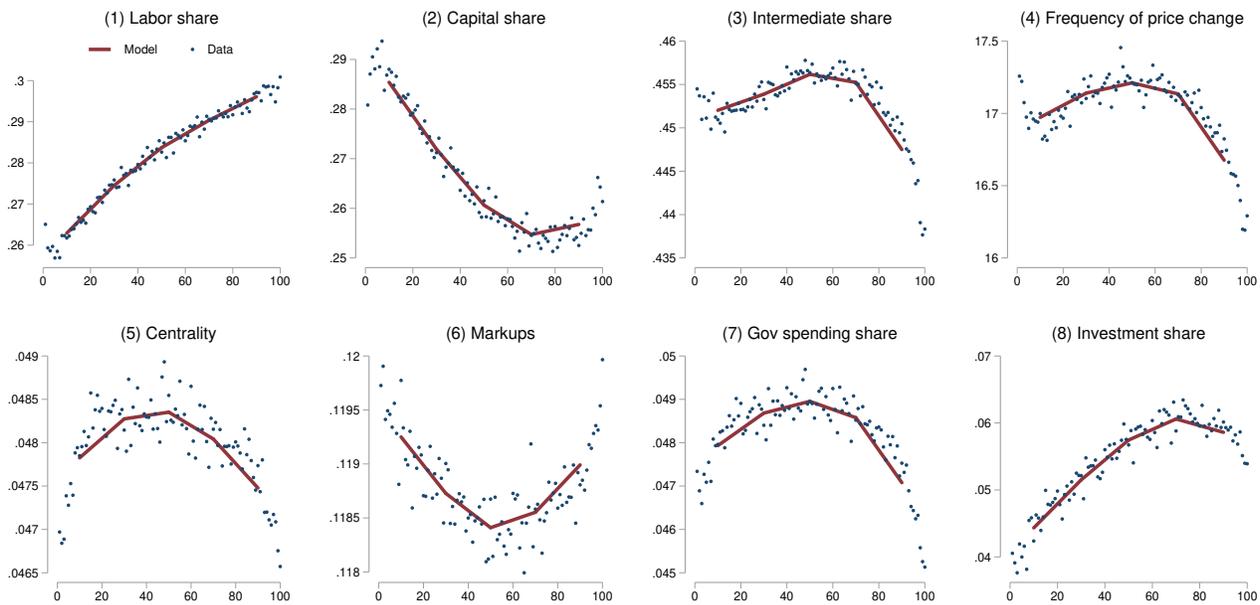


Figure 1. Expenditure-share Weighted Sectoral Heterogeneity for Income Percentiles

Note. The blue dots represent expenditure-weighted sectoral features as a function of households' after-tax income percentiles in the CEX-IO data, averaged over the sample period. The horizontal axis in each panel corresponds to household income percentiles, each bin representing 1% of the population. The vertical axis reports average sectoral features weighted by expenditure shares across household income percentiles. The red line summarizes the fit of our quantitative HANK-IO model, plotting the same measure of average sectoral features across the household income distribution using model-generated data.

telecommunications. They spend relatively more on other services, hotels and restaurants, and home construction. Middle-income households have the smallest expenditure shares for housing and education services, and the largest for durable manufacturing, finance, and insurance.

Earnings heterogeneity. Next, we document systematic household-sector linkages on the earnings side. We link cross-sectional household occupation and payroll data from the American Community Survey (ACS) with data from the BEA IO Tables. Similar to expenditure heterogeneity, we define payroll-share-weighted sectoral features as

$$\theta_t^{pay, hp} = \sum_{j \in S} w_{j,t}^{pay, hp} \theta_{j,t}$$

where $w_{j,t}^{pay, hp}$ is the payroll share in sector j in year t accounted for by households in income percentile hp .

The blue dots in Figure 2 plot earnings-weighted sectoral features across 20 income bins. We find that higher-income households earn relatively more in sectors that are less labor-intensive,

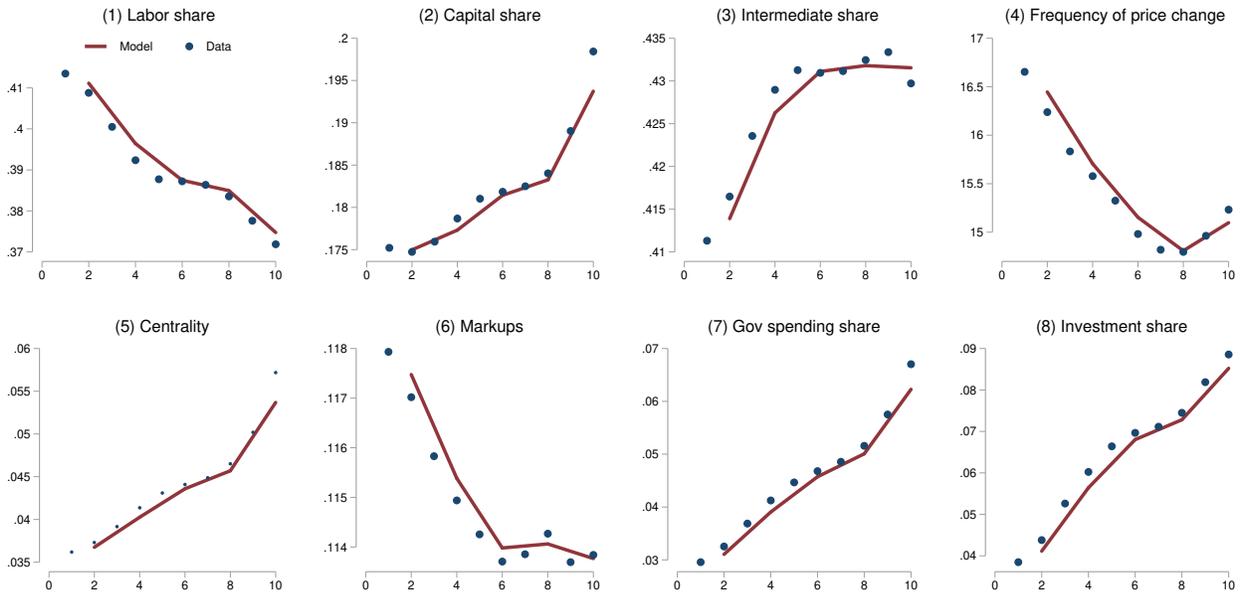


Figure 2. Earnings-share Weighted Sectoral Heterogeneity for Income Percentiles

Note. The blue dots represent payroll-weighted sectoral features as a function of households' income percentiles in the ACS-IO data, averaged over the sample period. The horizontal axis in each panel corresponds to household income percentiles, each dot representing 10% of the population. The vertical axis reports average sectoral features weighted by payroll shares across household income percentiles. The red line summarizes the fit of our quantitative HANK-IO model, plotting the same measure of average sectoral features across the household income distribution using model-generated data.

more capital-intensive, more central in the production network, have lower markups, have larger government spending shares, and contribute more to capital formation. Middle-income households work relatively more in sectors that are more price-rigid.

The sectors that account for the largest share of household earnings are accommodation and food services, retail trade, and healthcare for the bottom 10% of earners; healthcare, retail trade, and education services for the middle-income group; and professional and technical services, healthcare, and durable manufacturing for the top 10% of the income distribution. Table 3 presents summary statistics. Lower-income households earn more from sectors such as accommodation and food services, retail trade and other services, which are less capital-intensive and more labor-intensive. Higher-income households earn more from sectors such as professional and technical services, finance, and manufacturing, which have higher network centrality and government spending shares.

4.2.1 Relation to the Literature

Many of these empirical regularities are already well known. [Hubmer \(2023\)](#) documents that higher-income households spend relatively more on sectors with high labor shares. In related work, [Faber and Fally \(2022\)](#) show that higher-income households spend relatively more on high-quality goods, which is in turn positively correlated with sectoral labor intensity ([Jaimovich et al., 2019](#)). Relative to these papers, we show that higher-income households spend relatively more on sectors that also have high intermediates shares and work relatively more in sectors that have high intermediates and capital shares, but low labor shares.

[Cravino et al. \(2020\)](#) document that higher-income households spend relatively more in sectors with stickier prices. Similarly, [Clayton et al. \(2018\)](#) show that more educated households both spend and work more in sectors with more price rigidity. Our results suggest that the expenditure-weighted frequency of price changes is hump-shaped in household income, while higher-income households earn more in price-rigid sectors.

Government spending is granular and concentrated in sectors with relatively sticky prices ([Cox et al., 2020](#)). We show that government spending also tends to be high in those sectors where high-income households earn and middle-income households spend relatively more. [Jaravel \(2019\)](#) documents that higher-income households spend relatively more in high-markup sectors. Finally, [Vom Lehn and Winberry \(2022\)](#) document that the capital investment network is granular and dominated by a small set of concentrated investment hubs.

4.3 Key Model Elements and Calibration

In this section, we present the key new elements of our quantitative model and discuss its calibration.

4.3.1 Production Network

We calibrate the production network of our quantitative model to match data on 22 sectors. As described in Section 2.3, each sector is modeled as comprising a retailer and intermediate firms that face price adjustment costs. While this structure allows us to derive sectoral Phillips curves (6), we otherwise focus on a symmetric equilibrium in which all firms within a sector are identical. We can therefore proceed as if sectoral production decisions are taken by a representative firm.

The production function of sector j is CES over intermediate inputs and a primary factor that combines capital and labor, given by

$$y_{j,t} = A_{j,t} \left((1 - \theta_j)^{\frac{1}{\eta_{f,j}}} f_{j,t}^{\frac{\eta_{f,j}-1}{\eta_{f,j}}} + \theta_j^{\frac{1}{\eta_{f,j}}} x_{j,t}^{\frac{\eta_{f,j}-1}{\eta_{f,j}}} \right)^{\frac{\eta_{f,j}}{\eta_{f,j}-1}}, \quad (22)$$

where $A_{j,t}$ is a Hicks-neutral technology shifter. We denote by θ_j the CES weight on intermediate

inputs in sector j 's production and by $\eta_{f,j}$ the elasticity of substitution between the primary factor and intermediate inputs.

The primary factor is a Cobb-Douglas aggregate of capital and labor, given by

$$f_{j,t} = K_{j,t}^{\alpha_j} N_{j,t}^{1-\alpha_j}, \quad (23)$$

where α_j is the share of capital in total factors.¹⁶ We denote by $K_{j,t}$ the capital rented by sector j in period t and by $N_{j,t}$ a CES aggregate over all I labor factors used by sector j in production, given by

$$N_{j,t} = \left(\sum_i \left(\Gamma_{ji}^w \right)^{\frac{1}{\eta_{w,j}}} N_{ji,t}^{\frac{\eta_{w,j}-1}{\eta_{w,j}}} \right)^{\frac{\eta_{w,j}}{\eta_{w,j}-1}}, \quad (24)$$

where $N_{ji,t}$ is sector j 's demand for labor factor i , Γ_{ji}^w is the relative CES weight on factor i , and $\eta_{w,j}$ is a sector-specific elasticity of substitution across labor factors in production.

Sector j uses a CES basket of intermediate inputs, $x_{j,t}$, given by

$$x_{j,t} = \left(\sum_k \left(\Gamma_{jk}^x \right)^{\frac{1}{\eta_{x,j}}} x_{jk,t}^{\frac{\eta_{x,j}-1}{\eta_{x,j}}} \right)^{\frac{\eta_{x,j}}{\eta_{x,j}-1}}, \quad (25)$$

where $\eta_{x,j}$ is sector j 's constant elasticity of substitution across intermediate inputs. Γ_{jk}^x denotes how important is good k in sector j 's intermediate input bundle production function. As in Section 2, $x_{jk,t}$ is the demand for good k as an input by sector j .

The standard demand functions for intermediate inputs from sector i is given by

$$x_{jk,t} = \Gamma_{jk}^x \left(\frac{p_{k,t}}{p_{jx,t}} \right)^{-\eta_{x,j}} x_{j,t} \quad (26)$$

where p_t^k is the producer price index (PPI) in sector k and $p_{jx,t}$ is the price of intermediate input bundle in sector j . $x_{jk,t}$ is the unit of goods from sector k used by sector j , and the $x_{j,t}$ is the unit of intermediate input bundle for sector j . The relationship between intermediate input prices and the bundle price is given by

$$p_{jx,t} = \left[\sum_k \Gamma_{jk}^x (p_{k,t})^{1-\eta_{x,j}} \right]^{\frac{1}{1-\eta_{x,j}}}$$

Firms rent capital in an integrated and competitive market at the nominal rental rate i_t^K . Likewise firms hire labor of each factor in non-segmented labor markets at nominal wage rates $W_{i,t}$.

¹⁶ Horvath (2000), Carvalho (2014), Atalay (2017), Carvalho et al. (2021) and Ferrante et al. (2022) all aggregate primary factors using a Cobb-Douglas calibration. Vom Lehn and Winberry (2022) use Cobb-Douglas in their main calibration and explore deviations from Cobb-Douglas in a sensitivity analysis, where they show there are no sizable quantitative implications.

Sectoral profits are now given by $\Pi_{j,t} = p_{j,t}y_{j,t} - \sum_i W_{i,t}N_{ji,t} - i_t^K K_{j,t} - p_{jx,t}x_{j,t}$. We call the share of expenditure on goods from sector k in the nominal value of the intermediate input bundle α_{jk}^x , given by

$$\alpha_{jk}^x = \frac{p_{k,t}x_{jk,t}}{p_{jx,t}x_{j,t}}. \quad (27)$$

Elasticities. We follow much of the literature and set the elasticity between the primary factor and intermediate inputs to $\eta_{f,j} = 1$ across sectors.¹⁷ Consensus on the appropriate calibration of $\eta_{x,j}$, the elasticity of substitution across intermediate inputs, has evolved over time. While most prior work uses a Cobb-Douglas calibration, [Atalay \(2017\)](#) argues that this elasticity should be much smaller. We follow [Atalay \(2017\)](#) and set $\eta_{x,j} = 0.1$. Finally, we calibrate the elasticity of substitution between different labor factors to $\eta_{w,j} = 1$.

Factor shares. Sectors differ in the share of intermediate inputs in production, θ_j , and the share of capital in primary factors, α_j . We compute 22 sector-specific factor shares from the BEA GDP-by-Industry dataset. The intermediate input share θ_j is computed as input expenditures as a percentage of gross output, averaged over 1997-2015. We compute the labor share $1 - \alpha_j$ as total compensation of employees as a percentage of value added, adjusted for taxes and subsidies, averaged over the same period. We show in [Appendix B.1](#) that there is substantial heterogeneity in factor shares across sectors.

Input-output network. We obtain the input-output matrix Γ_{jk}^x from equations (26) and (27),

$$\Gamma_{jk}^x = \alpha_{jk}^x \left(\frac{p_{jx,t}}{p_{k,t}} \right)^{1-\eta_{x,j}}.$$

Every column of Γ_{jk}^x sums to 1.

We use data from the BEA Input Output “Use” Table to calculate the input-output share α_{jk}^x as sector j ’s (columns) nominal expenditure on intermediate inputs from sector k (rows) as a share of j ’s total expenditure on intermediate inputs. Then we average these ratios across 1997-2015. See [Appendix B.6](#) for further discussion on input-output linkages and the IO tables.

Sectoral price rigidities. We compute the sectoral price rigidities χ_j in two steps. First, we use data made publicly available by [Pasten et al. \(2017\)](#) who estimate the frequency of price changes using the U.S. Bureau of Labor Statistics (BLS)’s data underlying the Producer Price Index (PPI) for 754 industries (defined by 6-digits NAICS codes) from 2005 to 2011. Since these estimates are more granular than our production network, we follow [Clayton et al. \(2018\)](#) and use their many-to-one merge to our 22 production sectors. From these estimates, we obtain the monthly

¹⁷ [Atalay \(2017\)](#), [Vom Lehn and Winberry \(2022\)](#), and most prior work use this calibration.

price adjustment frequency. Additional details can be found in Appendix B.5. These estimates of price adjustment frequencies naturally map into sectoral Calvo parameters. In a second step, we analytically characterize the concordance between Calvo and Rotemberg parameters χ_j in our model to first order.

Payroll shares. To measure sectoral payroll shares—and consequently the share of salary expenditures paid to different household types h —we use data from the linked ACS-IO dataset (see Section 4.2). The details of how we link different datasets are provided in Appendix B.8. We define household earnings shares across sectors as the ratio of total earnings paid to household type h in sector j over total earnings of type h , averaged across our sample period.

Markups. Steady state markups across sectors are given by $\mu_j = \frac{\epsilon_j}{\epsilon_j - 1}$. We calibrate ϵ_j directly to match sectoral markups using data from Baqaee and Farhi (2020). Appendix B.4 shows that there is substantial heterogeneity in markups across sectors.

4.3.2 Households

Our quantitative model features multiple household types indexed by h . To match the empirical regularities documented in Figures 1 and 2, we allow types to differ in their permanent income levels, their earnings shares across sectors, as well as their consumption preferences.

Expenditure heterogeneity. Household i of type h consumes a non-homothetic CES basket of goods $c_{i,t}$, implicitly defined via

$$1 = \sum_j \left(\Omega_{hj} c_{i,t}^{\epsilon_j} \right)^{\frac{1}{\eta_c}} c_{ij,t}^{\frac{\eta_c - 1}{\eta_c}}, \quad (28)$$

where $c_{ij,t}$ denotes i 's consumption of good j (Comin et al., 2021). η_c is the elasticity of substitution across consumer goods produced in different sectors and ϵ_j is the relative income elasticity for goods produced in sector j . Ω_{hj} is the taste parameter for good j by households of type h .

Following Comin et al. (2021) and Hubmer (2023), we estimate a non-homothetic CES demand system to obtain the parameters $(\eta_c, \epsilon_1, \dots, \epsilon_J)$ using our linked CEX-IO dataset. We calibrate the taste parameters Ω_{hj} so that expenditure shares $\omega_{hj,t}$ for each household type match those in the data.

Earnings heterogeneity. As in our benchmark model, unions intermediate households' labor supply decisions and ration labor across all households within a type. Earnings heterogeneity emerges because production sectors differ in their demand for different labor factors, as discussed in Section 4.3.1.

Two accounts. We allow households to hold two accounts, a liquid checking account a and an illiquid investment account b held with a bank. Households can move funds between these two accounts subject to a transaction cost paid out of the liquid account. The liquid account bears a relatively low real rate of return $r_{i,t}^a$. Households can accumulate liquid debt up to a borrowing constraint, $a_{i,t} \geq \underline{a}$. The illiquid account bears a higher return $r_{i,t}^b$ and is subject to a short-sale constraint $b_{i,t} \geq 0$. As in our baseline model, real rates of return (in terms of households' purchasing power) depend on the relative rates of inflation households face in their type-specific consumption bundles.

When transferring funds $\iota_{i,t}$ from the liquid to the illiquid account, households incur a transaction cost $\psi(\iota_{i,t}, b_{i,t})$ that may be proportional to the size of the illiquid account. We adopt the functional form for $\psi(\cdot)$ used in [Kaplan et al. \(2018\)](#) and calibrate this transaction cost to match important moments of the household wealth distribution.

4.3.3 Financial Sector

We model a financial sector that consists of a representative financial intermediary, the “bank”, which has two activities: (1) a banking activity, performing maturity transformation by collecting real liquid assets from households and investing them in government bonds, subject to an intermediation spread; and (2) a mutual fund activity, collecting illiquid funds and intermediating them in the form of physical capital to firms.

Banking activity. The bank fully passes through the intermediation cost to households. In addition, the bank applies a borrowing wedge to the prevailing after-intermediation-cost interest rate. We choose the intermediation spread and the borrowing wedge for the model's steady state to match the aggregate and distributional moments in [Kaplan et al. \(2018\)](#) and [Auclert et al. \(2024\)](#).

Mutual fund activity. Illiquid assets are equity claims on the bank. The bank owns the economy's capital stock and makes capital investments. It rents capital to firms in a competitive rental market. We assume the bank operates an investment technology that transforms sectoral goods into gross capital investment. The capital stock then evolves according to $K_{t+1} = I_t + (1 - \delta)K_t$, where investment is given by the CES aggregator

$$I_t = \left(\sum_j (\Gamma_j^{inv})^{\frac{1}{\eta_I}} I_{j,t}^{\frac{\eta_I-1}{\eta_I}} \right)^{\frac{\eta_I}{\eta_I-1}},$$

where η_I is the elasticity of substitution across sectoral goods used for capital investment. Our quantitative model takes seriously the sectoral and network implications of investment spending, as emphasized by [Vom Lehn and Winberry \(2022\)](#). We calibrate the “investment use” parameter Γ_j^{inv} to match data from the BEA 1997 Capital Flow table, where we compute sector j 's total contribution

to capital production as a share of the economy's total inputs to capital production.

4.3.4 Government

Government spending takes the form of a homothetic CES aggregate over sectoral goods given by

$$G_t = \left(\sum_k \left(\Gamma_j^g \right)^{\frac{1}{\eta_g}} G_{j,t}^{\frac{\eta_g-1}{\eta_g}} \right)^{\frac{\eta_g}{\eta_g-1}}.$$

We calibrate Γ_j^g to match the share of government spending across sectors using the BEA industry input-output “use” table, computing government expenditures in sector j as a percentage of total government spending. Government expenditures include spending by the federal government, federal government enterprises, state and local government, and state and local government enterprises.

We assume the government balances its budget at steady state. Any surplus or deficit is rebated to households according to a rescaling rule that is designed to neutralize the quantitative implications of potentially counterfactual lump-sum transfers. The proportion of the aggregate rebate distributed to households of type i is equal to their income share in stationary equilibrium.

The monetary authority follows a standard Taylor rule with weights λ_π and λ_y on inflation and output, respectively. We assume the monetary authority uses the counterpart of the empirical CPI to measure inflation in this context.

4.3.5 Remaining Calibration Parameters

In addition to the parameters already discussed in this section, we calibrate the household discount rate ρ to match average MPCs, and we calibrate the capital depreciation rate δ to match the aggregate capital-output ratio. We set the capital adjustment cost parameter κ so that our baseline model matches VAR evidence on the relative response of investment and output to a monetary policy shock. We take the remaining parameters—including Taylor rule coefficients and the labor income tax rate—from the literature. We calibrate the borrowing wedge to match the share of borrowers. We summarize the calibration of our baseline HANK-IO model in Table 1.

Table 1. List of Calibrated Parameters of HANK-IO Full

Parameters	Value	Target / Source
<i>Preferences</i>		
$\bar{\rho}$	Average discount rate (p.q.)	6.876 % Internally calibrated
γ	Relative risk aversion	2 Standard
ϕ	Inverse Frisch elasticity	1 Standard
η_c	Elasticity of substitution between sectors in consumption	0.349 Estimation using CEX-IO
η_x	Elasticity of substitution between sectors in intermediate inputs	0.1 Atalay (2017)
η_l	Elasticity of substitution between types in labor supply	1 Standard
η_f	Elasticity of substitution between factors in production	1 Standard
η_i	Elasticity of substitution between sectors in capital investment	1 Standard
<i>Household portfolio choice</i>		
\underline{a}	Borrowing constraint	-0.25 0.631 quarter of average income
δ	Capital depreciation (p.q.)	1 % Internally calibrated
ψ_0	Linear adjustment cost	0.044 Internally calibrated
ψ_1	Convex adjustment cost 1	0.956 Internally calibrated
ψ_2	Convex adjustment cost 2	1.402 Internally calibrated
<i>Financial Intermediary</i>		
ω	Intermediation cost	0.5 % Internally calibrated
θ	Borrowing wedge	3.106 % Internally calibrated
<i>Firms</i>		
κ	Aggregate capital adjustment cost	5 Δy after shock
<i>Nominal rigidities</i>		
ϵ	Average elasticity of substitution for goods in retailer bundling	12.347 Baqaee & Farhi (2020)
χ_j	Average price adjustment cost	96.08 Pasten (2017)
ϵ^w	Elasticity of substitution for labor	10 CEE (2005)
χ^w	Avg. duration of wage contracts	0 Flexible-wage limit
<i>Government</i>		
λ_π	Taylor rule weight on inflation	1.5 Standard
λ_Y	Taylor rule weight on output	0 Standard
τ^{lab}	Income tax rate	27.512 % Standard

5 Policy Analysis

Our quantitative application focuses on the interaction between fiscal and monetary policy in the determination of sectoral inflation dynamics. In 2022, U.S. inflation reached its highest level since 1980. By mid-2023, core goods inflation had already fallen sharply from its peak, but services inflation—especially in non-housing services—remained stubbornly high. Consumer demand remained unexpectedly strong as households still held large cash balances accumulated from three rounds of fiscal transfers and forced saving. Against this backdrop, monetary policymakers faced an important question: To what extent did the remaining cash overhang account for unexpectedly strong consumer activity, and through which sectors was it feeding into inflation? This question is inherently counterfactual—it asks what inflation and activity would have looked like had household balance sheets been weaker and had monetary policy tightened more or less—and addressing it therefore requires a structural model.

Such a model must satisfy two requirements. First, it has to deliver realistic intertemporal

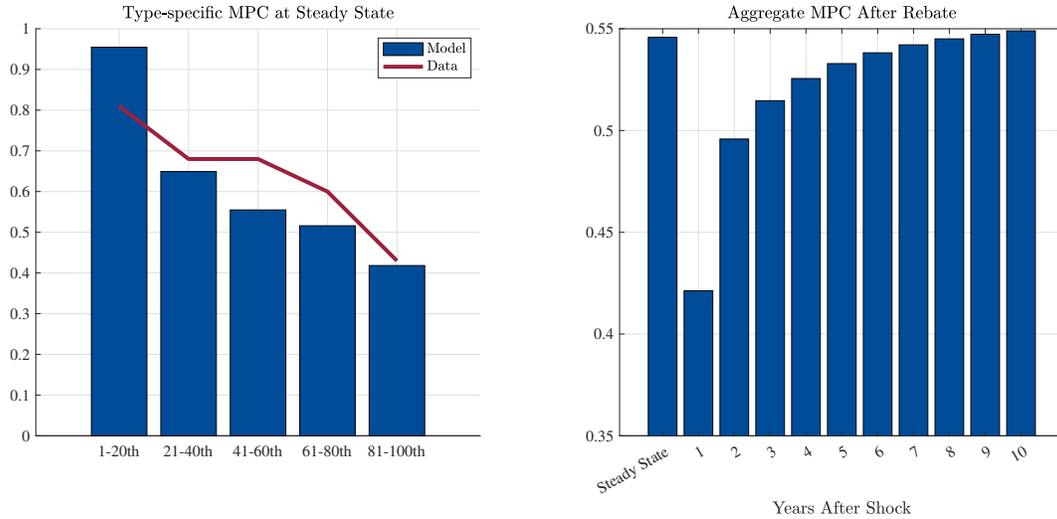


Figure 3. MPCs at Steady State and After the Rebate Shock

Note. Panel (a) plots the steady state distribution of MPCs in the model (blue bars) and compares it to data (red line) from Auclert et al. (2025). Panel (b) plots the impulse response of the economy’s aggregate MPC to the rebate shock.

marginal propensities to consume (iMPCs) across households. In representative-agent models fiscal rebates are neutral, and in simple two-agent models they have at most a one-period effect once transfers are absorbed by Ricardian households (Bardóczy et al., 2024); neither can speak to why payments in 2021 were still influencing demand in 2023. A heterogeneous-agent environment with incomplete markets is needed to capture those persistent, liquidity-driven spending responses. Second, the model must feature sectoral heterogeneity. The 2021-23 episode was defined by large and persistent differences in sectoral inflation patterns, and the FOMC itself described the economy in terms of a simple four-sector decomposition. A one-sector model—even with realistic iMPCs—would wash out exactly the sectoral heterogeneity that motivated the policy debate. The quantitative HANK-IO model we introduced in Section 4 satisfies both of these requirements and is therefore a useful framework to study this policy question.

The experiment. We study a simple sequence of counterfactuals designed to isolate *state dependence* in policy transmission. We initialize the economy at date $t = -1$ in its zero-inflation steady state. At date $t = 0$, households receive a one-time, unanticipated fiscal transfer that is calibrated to match the cumulative magnitude of the three major rounds of U.S. pandemic-era transfers. Because this shock is large, we compute the transition dynamics non-linearly.

We then measure how an otherwise standard monetary tightening transmits *from this post-transfer state*. Concretely, we consider an unanticipated contractionary 25 basis point monetary policy shock. We feed this same shock into (i) the economy along the non-linear transition path generated by the fiscal transfer, and (ii) the economy starting from steady state without the transfer. Comparing the two impulse responses isolates how the cash overhang—and the induced change

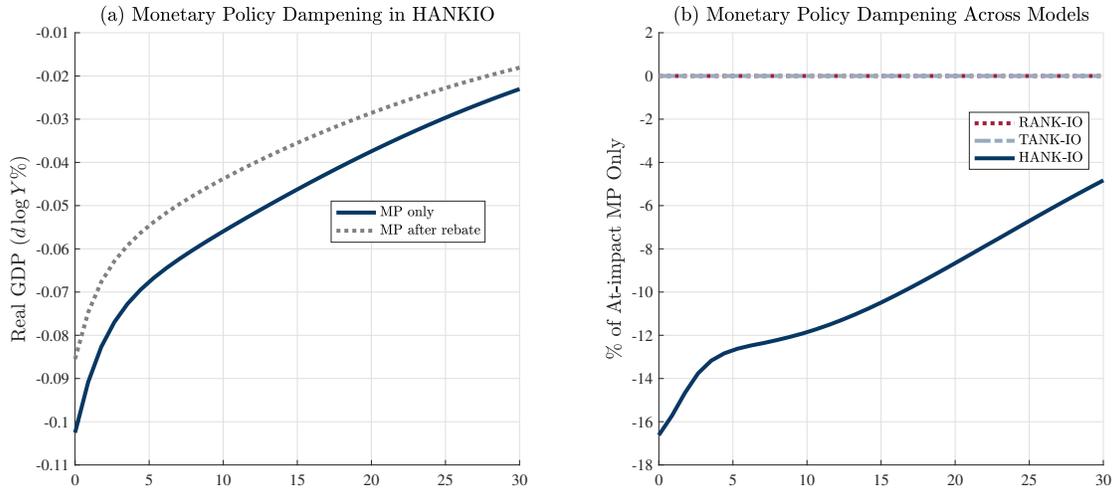


Figure 4. Monetary Policy Dampening

Note. Panel (a) plots the change in real GDP ($d \log Y$) after a 25 basis point contractionary monetary policy shock. The solid blue line initializes the economy at steady state, and the dashed grey line initializes the economy after the fiscal rebate shock. Panel (b) plots the relative dampening of monetary policy in % of the at-impact real GDP response across RANK-IO (red), TANK-IO (grey) and HANK-IO (blue).

in household balance sheets—alters both the *overall* potency of monetary policy and the *sectors* through which it works.

Marginal propensities to consume. The fiscal transfer immediately and materially lowers household MPCs. Intuitively, the transfer pushes a large mass of households away from binding or near-binding liquidity constraints. When more households have buffers in liquid wealth, their MPCs fall. In our two-asset model, the effect is large and persistent: As we show in Figure 3, the economy’s aggregate MPC falls from 55% in steady state to 42% after the fiscal rebate. Even after three years the aggregate MPC remains depressed at 51%. This decline in MPCs is consistent with related work by [Beraja and Zorzi \(2025\)](#) who show that fiscal transfers on the order of \$3000 can reduce MPCs by as much as half in an environment with durables.

Monetary policy dampening. Lower MPCs dampen monetary policy transmission through indirect income effects. In HANK models, the contractionary effect of higher interest rates on aggregate demand is mediated by who bears the incidence of the tightening and how strongly those households adjust spending. When *i*MPCs are compressed, shifts in real rates and incomes translate into smaller changes in consumption, which in turn weakens the response of output and inflation.

Quantitatively, this effect is large and persistent in the full HANK-IO model. Figure 4 shows that the at-impact effect of a 25 basis point tightening is about 16% weaker after the fiscal transfer than in steady state. This dampening fades only gradually. Put differently, a given path for the

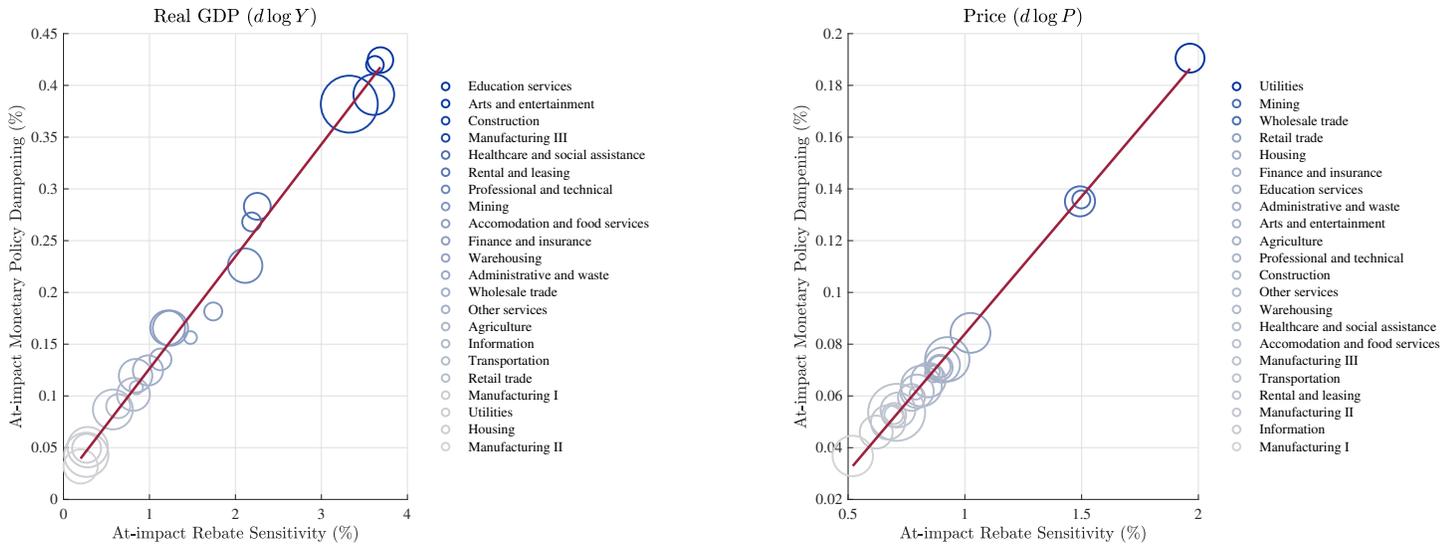


Figure 5. Monetary Policy Dampening and Rebate Sensitivity

Note. Panel (a) shows a scatter plot that illustrates the relationship between sectors’ at-impact sensitivity to the fiscal rebate shock (x-axis) and the change in their sensitivity to monetary policy (y-axis). Circle size corresponds to sector size. Panel (b) shows the same relationship for sectoral inflation.

policy rate delivers less restraint when households enter the tightening cycle with unusually strong balance sheets. Multi-sector models with no household heterogeneity (RANK-IO) or only permanent household heterogeneity (TANK-IO) cannot speak to this mechanism because fiscal rebates have no effect in those environments (Bardóczy et al., 2024).

Sectoral implications. The interaction between fiscal and monetary policy has sharp sectoral consequences. Fiscal transfers do not provide a uniform demand impulse across sectors: households differ in their spending shares, and non-homothetic preferences imply a sizable reallocation of consumption toward income-elastic sectors. As a result, transfers generate disproportionately strong inflationary pressure in some sectors. These sectors experience a pronounced, demand-driven inflation surge.

Our main finding is that monetary policy transmission is most dampened in *those very sectors* where fiscal policy generated the most inflation. The cash overhang created by fiscal transfers reduces the sensitivity of marginal demand to interest rates in transfer-exposed sectors. Consequently, the sectoral contraction induced by a policy hike is muted precisely where transfers previously pushed demand up the most.

We document this result in Figure 5. The post-transfer economy is therefore not just one in which monetary policy is weaker on average; it is one in which policy is *relatively least effective* at reducing inflation in exactly the sectors where fiscal transfers raised inflation the most. This helps explain why disinflation can proceed quickly in some sectors while remaining stubbornly slow in others, even under aggressive monetary policy tightening.

The result is a sectoral “double whammy”: fiscal transfers generate the largest inflationary pressure in certain sectors, and monetary policy then becomes relatively least effective at fighting inflation in precisely those sectors.

6 Conclusion

This paper is motivated by empirical evidence of systematic household-sector linkages in disaggregated micro data. We build a quantitative framework to assess their implications for policy transmission and the aggregation of sectoral shocks. Our “HANK-IO” model brings together a heterogeneous-agent New Keynesian model with a multi-sector business cycle model with input-output linkages in the tradition of [Long and Plosser \(1983\)](#). We analytically characterize an as-if benchmark that features a strict decoupling between household and sectoral heterogeneity. Away from this benchmark, however, novel earnings and expenditure heterogeneity channels emerge through which household-sector linkages have important implications for policy transmission.

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Online Appendix

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A Additional Model Details and Proofs

A.1 Union Problem

Under flexible wages, the problem of union ℓ can be written as

$$\max_{W_{h\ell,t}} \int \left[c_{i,t} - \frac{v'(N_{h,t})}{u'(C_{h,t})} n_{i,t} \right] di,$$

taking as given $N_{h,t}$ and $C_{h,t}$. The optimality condition is $0 = \int_h \left[\frac{\partial c_{i,t}}{\partial W_{h\ell,t}} + \frac{v'(N_{h,t})}{u'(C_{h,t})} \frac{\partial n_{i,t}}{\partial W_{h\ell,t}} \right] di$. Following the argument in [Auclert et al. \(2024\)](#), the envelope theorem implies that the effect of wage $W_{h\ell,t}$ on household i 's indirect consumption utility can be evaluated as if all income from wage changes is consumed directly,

$$\begin{aligned} \frac{\partial c_{i,t}}{\partial W_{h\ell,t}} &= z_{i,t} \frac{\partial e_{h,t}}{\partial W_{h\ell,t}} \\ &= z_{i,t} \frac{\partial}{\partial W_{h\ell,t}} \frac{1}{P_{h,t}} \int_0^1 W_{h\ell,t} n_{i\ell,t} d\ell \\ &= z_{i,t} \frac{\partial}{\partial W_{h\ell,t}} \frac{1}{P_{h,t}} \int_0^1 W_{h\ell,t} \left(\frac{W_{h\ell,t}}{W_{h,t}} \right)^{-\epsilon_h^w} N_{h,t} d\ell \\ &= z_{i,t} \frac{1}{P_{h,t}} W_{h,t}^{\epsilon_h^w} N_{h,t} \int_0^1 (1 - \epsilon_h^w) W_{h\ell,t}^{-\epsilon_h^w} d\ell \\ &= z_{i,t} \frac{1}{P_{h,t}} N_{h,t} (1 - \epsilon_h^w) \end{aligned}$$

where the last line follows after focusing on the symmetric union equilibrium with $W_{h\ell,t} = W_{h,t}$ for all ℓ . Similarly, a change in wages affects labor demand and therefore household i 's labor supply,

$$\frac{\partial n_{i,t}}{\partial W_{h\ell,t}} = \frac{\partial}{\partial W_{h\ell,t}} \int_0^1 n_{i\ell,t} d\ell = \frac{\partial}{\partial W_{h\ell,t}} \int_0^1 N_{h\ell,t} d\ell = \frac{\partial}{\partial W_{h\ell,t}} \int_0^1 \left(\frac{W_{h\ell,t}}{W_{h,t}} \right)^{-\epsilon_h^w} N_{h,t} d\ell = -\epsilon_h^w \frac{1}{W_{h,t}} N_{h,t}$$

where the last line again follows from symmetry. Putting these two expressions into the first-order condition for $W_{h\ell,t}$ yields the labor supply schedule for factor h .

A.2 Firm Problem

The Lagrangian is

$$L = \sum_{t=0}^{\infty} Q_{0,t} \left[(p_{j\omega,t} - MC_{j,t}) y_{j\omega,t} - \frac{\chi_j}{2} \left(\frac{p_{j\omega,t}}{p_{j\omega,t-1}} - 1 \right)^2 p_{j,t} y_{j,t} \right]$$

and therefore

$$L = \sum_{t=0}^{\infty} Q_{0,t} \left[(p_{j\omega,t} - MC_{j,t}) \left(\frac{p_{j\omega,t}}{p_{j,t}} \right)^{-\epsilon_j} y_{j,t} - \frac{\chi_j}{2} \left(\frac{p_{j\omega,t}}{p_{j\omega,t-1}} - 1 \right)^2 p_{j,t} y_{j,t} \right]$$

The FOC with respect to $p_{j\omega,t}$ is

$$0 = Q_{0,t} \left[\left(\frac{p_{j\omega,t}}{p_{j,t}} \right)^{-\epsilon_j} y_{j,t} - \epsilon_j (p_{j\omega,t} - MC_{j,t}) \left(\frac{p_{j\omega,t}}{p_{j,t}} \right)^{-\epsilon_j - 1} \frac{1}{p_{j,t}} y_{j,t} - \chi_j \pi_{j\omega,t} \frac{1}{p_{j\omega,t-1}} p_{j,t} y_{j,t} \right] \\ + Q_{0,t+1} \chi_j \pi_{j\omega,t+1} \frac{p_{j\omega,t+1}}{p_{j\omega,t}^2} p_{j,t+1} y_{j,t+1}$$

Applying symmetry with $p_{j\omega,t} = p_{j,t}$ yields

$$0 = y_{j,t} - \epsilon_j (p_{j,t} - MC_{j,t}) \frac{1}{p_{j,t}} y_{j,t} - \chi_j \pi_{j,t} \frac{1}{p_{j,t-1}} p_{j,t} y_{j,t} \\ + Q_t \chi_j \pi_{j,t+1} \frac{p_{j,t+1}}{p_{j,t}^2} p_{j,t+1} y_{j,t+1}$$

And rearranging,

$$0 = (1 - \epsilon_j) + \epsilon_j mc_{j,t} - \chi_j \pi_{j,t} (1 + \pi_{j,t}) + Q_t \chi_j \pi_{j,t+1} (1 + \pi_{j,t+1}) \frac{p_{j,t+1} y_{j,t+1}}{p_{j,t} y_{j,t}}$$

where $mc_{j,t} = \frac{MC_{j,t}}{p_{j,t}}$. Rearranging, we arrive at the canonical non-linear form

$$\pi_{j,t} (1 + \pi_{j,t}) = Q_t \pi_{j,t+1} (1 + \pi_{j,t+1}) \frac{p_{j,t+1} y_{j,t+1}}{p_{j,t} y_{j,t}} + \frac{\epsilon_j}{\chi_j} \left[mc_{j,t} - \frac{\epsilon_j - 1}{\epsilon_j} \right]$$

The sectoral Phillips curves express current inflation in terms of future inflation and current real marginal cost $mc_{j,t}$.

A.3 Proof of Proposition 1

We prove Proposition 1 in several steps, first building up LM curve (DLM) and then the IS curve (DIS). Under Assumption A1, the relative demand for good j by households and the government is summarized by an expenditure share $b_j(\mathbf{p}_t)$, satisfying $C_{hj,t} = b_j(\mathbf{p}_t) C_{h,t}$. We can then summarize

the supply side of the HANK-IO economy in terms of the equations

$$\begin{aligned}
y_{j,t} &= \sum_h \mu_h s_j(\mathbf{p}_t) C_{h,t} + \sum_k x_{kj,t} + s_j(\mathbf{p}_t) G_t + \frac{\chi_j}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2 y_{j,t} \\
\frac{p_{j,t}}{p_{j,t-1}} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right) &= \frac{P_{t+1}}{1+i_{t+1}} \frac{p_{j,t+1}}{p_{j,t}} \left(\frac{p_{j,t+1}}{p_{j,t}} - 1 \right) \frac{p_{j,t+1} y_{j,t+1}}{p_{j,t} y_{j,t}} + \frac{\epsilon_j}{\chi_j} \left[mc_{j,t}(\mathbf{p}_t, \mathbf{W}_t) - \frac{\epsilon_j - 1}{\epsilon_j} \right] \\
x_{jk,t} &= y_{j,t} \frac{\partial \mathcal{U}_j}{\partial p_{k,t}} \\
N_{jh,t} &= y_{j,t} \frac{\partial \mathcal{U}_j}{\partial W_{h,t}} \\
W_{h,t} &= P_t \frac{\epsilon_h^w}{\epsilon_h^w - 1} \frac{1}{u'_h(C_{h,t})} v'_h(N_{h,t}) \\
i_{t+1} &= i^* + \mathcal{T}_\pi(\boldsymbol{\pi}_t) + \mathcal{T}_y(\mathbf{y}_t, \mathbf{A}_t) + \varepsilon_t \\
N_{h,t} &= \frac{1}{\mu_h} \sum_j N_{jh,t}
\end{aligned}$$

Step 1: production scale. Under [A1](#), we get a level definition of real GDP with $Y_t = C_t + G_t$. Also using $\sum_h \mu_h C_{h,t} = C_t$, we can write

$$y_{j,t} = s_j(\mathbf{p}_t) Y_t + \sum_k \mathcal{X}_{kj,t}(\mathbf{p}_t, \mathbf{W}_t) y_{k,t} + \frac{\chi_j}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2 y_{j,t}$$

where we denote by $\mathcal{X}_{jk,t}(\mathbf{p}_t, \mathbf{W}_t) = \frac{\partial \mathcal{U}_j}{\partial p_{k,t}}$. Stacking, we can write

$$\mathbf{y}_t = \mathbf{s}_t Y_t + \boldsymbol{\mathcal{X}}(\mathbf{p}_t, \mathbf{W}_t)' \mathbf{y}_t + \boldsymbol{\Delta}(\mathbf{p}_t, \mathbf{p}_{t-1}) \mathbf{y}_t$$

where $\boldsymbol{\Delta}$ is the matrix with $\frac{\chi_j}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2$ on the diagonal. And so we have

$$(\mathbf{I} - \boldsymbol{\mathcal{X}}(\mathbf{p}_t, \mathbf{W}_t)' - \boldsymbol{\Delta}(\mathbf{p}_t, \mathbf{p}_{t-1})) \mathbf{y}_t = \mathbf{s}_t Y_t,$$

where \mathbf{I} is the identity matrix. Let's define

$$\boldsymbol{\Omega}_t = \boldsymbol{\mathcal{X}}(\mathbf{p}_t, \mathbf{W}_t)' + \boldsymbol{\Delta}(\mathbf{p}_t, \mathbf{p}_{t-1})$$

Then we have $\mathbf{y}_t = (\mathbf{I} - \boldsymbol{\Omega}_t)^{-1} \mathbf{s}_t Y_t$ whenever $(\mathbf{I} - \boldsymbol{\Omega}_t)$ is invertible. Notice that $(\mathbf{I} - \boldsymbol{\Omega}_t)^{-1}$ is simply the standard Leontief inverse augmented by the Rotemberg adjustment cost. We proceed under the assumption that $(\mathbf{I} - \boldsymbol{\Omega}_t)$ is invertible. That implies the solution for sectoral production scale admits the representation

$$y_{j,t} = \mathcal{Y}_j(Y_t, \mathbf{p}_t, \mathbf{p}_{t-1}, \mathbf{W}_t).$$

Conditional on Y_t , the function \mathcal{Y}_j is identical to RANK-IO.

Step 2: labor supply curve. Under Assumptions A1 and A3, the labor supply schedule in factor market h can be written as

$$W_{h,t} = P_t \frac{\epsilon_h^w}{\epsilon_h^w - 1} \frac{1}{u'(Y_t - G_t)} \kappa_h N_{h,t}^{\phi_h}.$$

Exploiting the Cobb-Douglas structure of Assumption A2,

$$W_{h,t} = P_t \frac{\epsilon_h^w}{\epsilon_h^w - 1} \frac{1}{u'(Y_t - G_t)} \left(\frac{1}{\mu_h} v_h \frac{W_t}{W_{h,t}} N_t \right)^{\phi_h}$$

where aggregate effective labor is defined as $\sum_j N_{j,t} = N_t$. This follows from

$$\begin{aligned} W_{h,t} N_{jh,t} &= v_h W_{j,t} N_{j,t} \\ W_{h,t} \sum_j N_{jh,t} &= v_h W_t \sum_j N_{j,t} \\ \mu_h W_{h,t} N_{h,t} &= v_h W_t N_t \end{aligned}$$

Rearranging, we have

$$W_{h,t} = \left[P_t \frac{\epsilon_h^w}{\epsilon_h^w - 1} \frac{\kappa_h}{u'(Y_t - G_t)} \left(\frac{v_h}{\mu_h} W_t N_t \right)^{\phi_h} \right]^{\frac{1}{1+\phi_h}}$$

Next, we have the common effective wage index across sectors

$$W_t = \prod_{h=1}^H \left(\frac{W_{h,t}}{v_h} \right)^{v_h}.$$

Therefore, we have

$$\begin{aligned} \log W_t &= \sum_h \log \left(\frac{1}{v_h^{v_h}} W_{h,t}^{v_h} \right) \\ &= - \sum_h v_h \log v_h + \sum_h v_h \log W_{h,t} \end{aligned}$$

Now we have

$$\begin{aligned}
\log W_{h,t} &= \frac{1}{1+\phi_h} \log \left[P_t \frac{\epsilon_h^w}{\epsilon_h^w - 1} \frac{\kappa_h}{u'(C_t)} \left(\frac{v_h}{\mu_h} W_t N_t \right)^{\phi_h} \right] \\
&= \frac{1}{1+\phi_h} \log P_t + \frac{1}{1+\phi_h} \log \frac{\epsilon_h^w}{\epsilon_h^w - 1} + \frac{1}{1+\phi_h} \log \kappa_h - \frac{1}{1+\phi_h} \log u'(C_t) + \frac{1}{1+\phi_h} \log \left(\frac{v_h}{\mu_h} W_t N_t \right)^{\phi_h} \\
&= A_h + \frac{1}{1+\phi_h} \log P_t - \frac{1}{1+\phi_h} \log u'(C_t) + \frac{\phi_h}{1+\phi_h} \log W_t + \frac{\phi_h}{1+\phi_h} \log N_t
\end{aligned}$$

where

$$A_h = \frac{1}{1+\phi_h} \log \frac{\epsilon_h^w}{\epsilon_h^w - 1} + \frac{1}{1+\phi_h} \log \kappa_h + \frac{\phi_h}{1+\phi_h} \log \frac{v_h}{\mu_h}$$

Thus, we have

$$\log W_t = - \sum_h v_h \log v_h + \sum_h v_h \left(A_h + \frac{1}{1+\phi_h} \log P_t - \frac{1}{1+\phi_h} \log u'(C_t) + \frac{\phi_h}{1+\phi_h} \log W_t + \frac{\phi_h}{1+\phi_h} \log N_t \right)$$

Or rewriting and collecting terms

$$\left(1 - \sum_h v_h \frac{\phi_h}{1+\phi_h} \right) \log W_t = \mathcal{A} + \left(\sum_h v_h \frac{1}{1+\phi_h} \right) \log P_t - \left(\sum_h v_h \frac{1}{1+\phi_h} \right) \log u'(C_t) + \left(\sum_h v_h \frac{\phi_h}{1+\phi_h} \right) \log N_t$$

where $\mathcal{A} = - \sum_h v_h \log v_h + \sum_h v_h A_h$.

Now define

$$\zeta = \sum_h v_h \frac{\phi_h}{1+\phi_h} \quad \text{and} \quad \gamma = \sum_h v_h \frac{1}{1+\phi_h}$$

so that $\gamma + \zeta = 1$. Then we can write

$$(1 - \zeta) \log W_t = \gamma \log W_t = \mathcal{A} + \gamma \log P_t - \gamma \log u'(C_t) + (1 - \gamma) \log N_t$$

Dividing through we get

$$\log W_t = \frac{1}{\gamma} \mathcal{A} + \log P_t - \log u'(C_t) + \frac{1-\gamma}{\gamma} \log N_t$$

Exponentiating, we get the following labor supply schedule for HANK-IO under A1 – A3:

$$W_t = \mathcal{B} P_t \frac{1}{u'(C_t)} N_t^{\frac{1-\gamma}{\gamma}}$$

where $\mathcal{B} = \exp(\mathcal{A}/\gamma)$.

Now compare this to the RANK-IO labor supply curve,

$$W_t = P_t \frac{\epsilon_w}{\epsilon_w - 1} \frac{\kappa N_t^\phi}{u'(C_t)}.$$

So we get observational equivalence to RANK-IO by setting

$$\begin{aligned} \phi &= \frac{\sum_h v_h \frac{\phi_h}{1+\phi_h}}{\sum_h v_h \frac{1}{1+\phi_h}} \\ \kappa &= \exp \left\{ \frac{1}{\gamma} \sum_h \left[\frac{v_h}{1+\phi_h} \log \kappa_h + \frac{v_h}{1+\phi_h} \phi_h \log \frac{v_h}{\mu_h} - v_h \log v_h \right] \right\} \\ \frac{\epsilon_w}{\epsilon_w - 1} &= \exp \left\{ \frac{1}{\gamma} \sum_h \frac{v_h}{1+\phi_h} \log \frac{\epsilon_h^w}{\epsilon_h^w - 1} \right\} \end{aligned}$$

Finally, we can solve for N_t using firm labor demand and arrive at the representation

$$W_t = \mathcal{W}(Y_t, \mathbf{p}_t, \mathbf{p}_{t-1}; G_t)$$

where G_t only enters due to its income effect on labor supply.

Step 3: sectoral Phillips curves. We have characterized $y_{j,t}$ and W_t . Now we look at prices. Using

$$\pi_{j,t} = \frac{p_{j,t}}{p_{j,t-1}} - 1,$$

we can rewrite the non-linear sectoral Phillips curves in price levels as

$$\frac{p_{j,t}}{p_{j,t-1}} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right) = \frac{P_{t+1}}{1+i_{t+1}} \frac{p_{j,t+1}}{p_{j,t}} \left(\frac{p_{j,t+1}}{p_{j,t}} - 1 \right) \frac{p_{j,t+1} y_{j,t+1}}{p_{j,t} y_{j,t}} + \frac{\epsilon_j}{\chi_j} \left[mc_j(\mathbf{p}_t, W_t; A_{j,t}) - \frac{\epsilon_j - 1}{\epsilon_j} \right]$$

We can rewrite this equation in levels as

$$\begin{aligned} 0 &= \mathcal{F}_j(p_{j,t-1}, p_{j,t+1}, \mathbf{p}_t, W_t, i_{t+1}, y_{j,t}, y_{j,t+1}; A_{j,t}) \\ &= \mathcal{F}_j(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{p}_{t+1}, W_t, i_{t+1}, Y_t, Y_{t+1}; A_{j,t}) \\ &= \mathcal{F}_j(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{p}_{t+1}, i_{t+1}, Y_t, Y_{t+1}; A_{j,t}, G_t). \end{aligned}$$

Stacking, $0 = \mathcal{F}$ becomes a system of J second-order forward-looking difference equations in sectoral price levels \mathbf{p}_t , taking as given initial prices \mathbf{p}_{-1} , sequences of real GDP Y and interest rates i , and sequences of exogenous sectoral productivities A and government spending G .

We proceed under the assumption that the system of second-order difference equations $0 = \mathcal{F}$ has a unique solution. Under this assumption, we can solve for the level of sectoral prices in terms

of the sequence-space representation

$$p_{j,t} = \mathcal{P}_{j,t}(\mathbf{i}, \mathbf{Y}; \mathbf{A}, \mathbf{G}, \mathbf{p}_{-1}).$$

Step 4: Taylor rule and Fisher equation. Abusing notation slightly, we can write the Taylor rule and Fisher equations as

$$\begin{aligned} i_{t+1} &= i^* + \mathcal{T}_\pi(\mathbf{p}_t, \mathbf{p}_{t-1}) + \mathcal{T}_y(\mathbf{y}_t, \mathbf{A}_t) + \varepsilon_t \\ R_t &= \frac{1 + i_t}{1 + \pi_t}. \end{aligned}$$

Plugging our previous results into the Taylor rule, we have system of non-linear equations

$$0 = \mathcal{T}(\mathbf{i}, \mathbf{Y}; \mathbf{A}, \mathbf{G}, \boldsymbol{\varepsilon}, \mathbf{p}_{-1}).$$

We proceed under the assumption that the non-linear equations $0 = \mathcal{T}$ have a unique solution. Under this assumption, we can solve for the level of nominal interest rates as

$$i_{t+1} = \mathcal{T}_t(\mathbf{Y}; \mathbf{A}, \mathbf{G}, \boldsymbol{\varepsilon}, \mathbf{p}_{-1}).$$

Plugging into the Fisher equation completes the proof of (DLM), yielding the LM curve

$$R_{t+1} = \mathcal{R}_t(\mathbf{Y}; \mathbf{A}, \mathbf{G}, \boldsymbol{\varepsilon}, \mathbf{p}_{-1}).$$

And since \mathbf{p}_{-1} is invariant to the shocks that we consider, we suppress explicit dependence of \mathcal{R} on the initial sectoral price levels.

Step 5: demand side. Under Assumption A1, the household problem can be written as

$$\begin{aligned} V_{h,t}(a, z) &= \max_{c, a'} u_h(c) - v_h(N_{h,t}) + \beta_h \mathbb{E}_t \left[V_{h,t+1}(a', z') \right] \\ \text{s.t.} \quad a' &= R_t a + e_{h,t} z + T_{h,t} z - c \\ a' &\geq \underline{a}_h \end{aligned}$$

where

$$\begin{aligned} e_{h,t} &= \frac{1}{P_t} (1 - \tau_t) (W_{h,t} N_{h,t} + \zeta_h \sum_j \Pi_{j,t}) \\ \text{and} \quad T_{h,t} &= \phi_h T_t \end{aligned}$$

and where we used $R_{h,t} = R_t$ under Assumption A1. Notice that we can write profits as

$$\begin{aligned}
\Pi_t &= \sum_j \Pi_{j,t} = \sum_j \left[p_{j,t} y_{j,t} - \sum_h W_{h,t} N_{jh,t} - \sum_k p_{k,t} x_{jk,t} - \frac{\chi_j}{2} \pi_{j,t}^2 p_{j,t} y_{j,t} \right] \\
&= \sum_j \left[p_{j,t} y_{j,t} - \sum_k p_{k,t} x_{jk,t} - \frac{\chi_j}{2} \pi_{j,t}^2 p_{j,t} y_{j,t} \right] - \sum_h W_{h,t} \sum_j N_{jh,t} \\
&= P_t Y_t - \sum_h \mu_h W_{h,t} N_{h,t}
\end{aligned}$$

So we have

$$\begin{aligned}
e_{h,t} &= \zeta_h (1 - \tau_t) Y_t + (1 - \tau_t) \frac{1}{P_t} (W_{h,t} N_{h,t} - \zeta_h \sum_h \mu_h W_{h,t} N_{h,t}) \\
&= \zeta_h (1 - \tau_t) Y_t + (1 - \tau_t) (\omega_{h,t} \Lambda_t^L - \zeta_h \Lambda_t^L) Y_t
\end{aligned}$$

where the second line follows after introducing the aggregate labor share and the type- h labor income share

$$\Lambda_t^L = \frac{\sum_h \mu_h W_{h,t} N_{h,t}}{P_t Y_t} \quad \text{and} \quad \omega_{h,t} = \frac{W_{h,t} N_{h,t}}{\sum_h \mu_h W_{h,t} N_{h,t}}$$

Under Assumption A2, we have

$$\begin{aligned}
\frac{\mu_h}{\nu_h} W_{h,t} N_{h,t} &= \sum_h W_{h,t} \sum_j N_{jh,t} \\
\frac{\mu_h}{\nu_h} W_{h,t} N_{h,t} &= \sum_h \mu_h W_{h,t} N_{h,t}
\end{aligned}$$

And so we have a constant type- h labor income share

$$\omega_{h,t} = \omega_h = \frac{\nu_h}{\mu_h}.$$

This is constant across time. By now setting the dividend share to the constant labor income share $\zeta_h = \omega_h$, we get

$$\begin{aligned}
e_{h,t} &= \zeta_h (1 - \tau_t) Y_t + (1 - \tau_t) (\omega_h \Lambda_t^L - \omega_h \Lambda_t^L) Y_t \\
&= \zeta_h (1 - \tau_t) Y_t.
\end{aligned}$$

The household problem therefore becomes

$$\begin{aligned}
V_{h,t}(a, z) &= \max_{c, a'} u_h(c) - v_h(N_{h,t}) + \beta_h \mathbb{E}_t [V_{h,t+1}(a', z')] \\
\text{s.t. } a' &= R_t a + \omega_h(1 - \tau_t) Y_t z + \phi_h T_t z - c \\
a' &\geq \underline{a}
\end{aligned}$$

And it implies the aggregate consumption function for type h

$$C_{h,t} = C_{h,t}(\mathbf{R}, \mathbf{Y}; \boldsymbol{\tau}, \mathbf{T}).$$

Finally, we define the aggregate consumption function as

$$C_t(\mathbf{R}, \mathbf{Y}; \boldsymbol{\tau}, \mathbf{T}) = \sum_h \mu_h C_{h,t}(\mathbf{R}, \mathbf{Y}; \boldsymbol{\tau}, \mathbf{T}),$$

completing the proof.

A.4 Proof of Proposition 2

Laspeyres. We define level changes in real GDP using the Laspeyres index. Since the HA literature commonly works with level changes in GDP, it is useful to do so as well in this part of the paper to compare our results to earlier benchmarks (Auclert et al., 2024). We initialize the economy at the zero-inflation steady state and take $t = -1$ as our base year. Real GDP in period t is the quantity of goods produced in period t valued at base-period prices, up to a choice of units. We write the chosen level of real GDP in the base period $t = -1$ as \bar{Y} . Then the Laspeyres real GDP level for any period t is

$$Y_t = \frac{\bar{Y}}{Y^n} \sum_j p_j y_{j,t}.$$

We define Y_t as the Laspeyres real GDP *level*. Equivalently, we can define a *base-year GDP deflator* as

$$\bar{P} = \frac{Y^n}{\bar{Y}}.$$

Then we have the Laspeyres real GDP level

$$Y_t = \sum_j \frac{p_j}{\bar{P}} y_{j,t}.$$

After a general perturbation, the level change in Laspeyres real GDP is therefore

$$dY_t = \sum_j \frac{p_j}{\bar{P}} (dC_{j,t} + dG_{j,t}).$$

Sequence-space representation. As in the main text, we use the summary statistics for earnings and expenditure heterogeneity

$$\begin{aligned}\zeta_{h,t} &= \frac{W_{h,t}N_{h,t} + d_{h,t}}{Y_t^n} \\ \rho_{h,t} &= \frac{P_{h,t}}{P_t} \\ \pi_{h,t}^\rho &= \frac{\rho_{h,t}}{\rho_{h,t-1}} - 1.\end{aligned}$$

Therefore, we can write

$$\begin{aligned}R_{h,t} &= \frac{P_{h,t-1}}{P_{h,t}} \frac{P_t}{P_{t-1}} R_t = \frac{\rho_{h,t-1}}{\rho_{h,t}} R_t = \frac{1}{1 + \pi_{h,t}^\rho} R_t \\ e_{h,t} &= \frac{1}{P_{h,t}} (1 - \tau_t)(W_{h,t}N_{h,t} + d_{h,t}) = \frac{1}{\rho_{h,t}} (1 - \tau_t) \zeta_{h,t} \frac{Y_t^n}{P_t} \\ T_{h,t} &= \phi_h \frac{P_t}{P_{h,t}} T_t = \phi_h \frac{1}{\rho_{h,t}} T_t\end{aligned}$$

In the absence of Assumption **A1–A3**, the household problem can then be written as

$$\begin{aligned}V_{h,t}(a, z) &= \max_{c, a'} u_h(c) - v_h(N_{h,t}) + \beta_h \mathbb{E}_t \left[V_{h,t+1}(a', z') \right] \\ \text{s.t. } a' &= \frac{1}{1 + \pi_{h,t}^\rho} R_t a + \frac{1}{\rho_{h,t}} (1 - \tau_t) \zeta_{h,t} Y_t z + \phi_h \frac{1}{\rho_{h,t}} T_t z - c\end{aligned}$$

where we directly work with Y_t Laspeyres real GDP, which coincides to first order with writing instead $\frac{Y_t^n}{P_t}$. Therefore, we have the consumption function representation

$$C_{h,t} = \tilde{C}_{h,t}(\mathbf{Y}, \mathbf{R}, \boldsymbol{\zeta}_h, \boldsymbol{\rho}_h, \boldsymbol{\pi}_h^\rho; \boldsymbol{\tau}, \mathbf{T})$$

In the text, we introduced the alternative representation

$$C_{h,t} = \mathcal{C}_{h,t}(\mathbf{R}_h, \mathbf{e}_h, \mathbf{T}_h)$$

and defined the iMPCs

$$M_{h,ts} = \frac{\partial \mathcal{C}_{h,t}}{\partial e_{h,s}} = \frac{\partial \mathcal{C}_{h,t}}{\partial T_{h,s}}.$$

Then notice that we have

$$\tilde{C}_{h,t}(\mathbf{Y}, \mathbf{R}, \boldsymbol{\zeta}_h, \boldsymbol{\rho}_h, \boldsymbol{\pi}_h^\rho; \boldsymbol{\tau}, \mathbf{T}) = \mathcal{C}_{h,t} \left(\left\{ \frac{1}{1 + \pi_{h,s}^\rho} R_s \right\}_{s \geq 0}, \left\{ \frac{1}{\rho_{h,s}} (1 - \tau_s) \zeta_{h,s} Y_s \right\}_{s \geq 0}, \left\{ \phi_h \frac{1}{\rho_{h,s}} T_s \right\}_{s \geq 0} \right)$$

So now we have

$$\frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial Y_s} = \frac{\partial \mathcal{C}_{h,t}}{\partial e_{h,s}} \frac{\partial e_{h,s}}{\partial Y_s} = \frac{\partial \mathcal{C}_{h,t}}{\partial e_{h,s}} \frac{1}{\rho_{h,s}} (1 - \tau_s) \zeta_{h,s} = \frac{1}{\rho_{h,s}} (1 - \tau_s) \zeta_{h,s} \cdot M_{h,ts}$$

Likewise we have

$$\frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial \zeta_{h,s}} = \frac{1}{\rho_{h,s}} (1 - \tau_s) Y_s \cdot M_{h,ts}$$

$$\frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial \tau_s} = -\frac{1}{\rho_{h,s}} Y_s \cdot M_{h,ts}$$

$$\frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial T_s} = \phi_h \frac{1}{\rho_{h,s}} \cdot M_{h,ts}$$

and also

$$\frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial \rho_{h,s}} = -\frac{1}{\rho_{h,s}^2} (1 - \tau_s) \zeta_{h,s} Y_s \cdot M_{h,ts} - \frac{1}{\rho_{h,s}^2} \phi_h T_s \cdot M_{h,ts} = -\frac{1}{\rho_{h,s}^2} \left[(1 - \tau_s) \zeta_{h,s} Y_s + \phi_h T_s \right] M_{h,ts}$$

IKC. We have

$$\begin{aligned} dY_t &= \sum_j \frac{p_j}{\bar{P}} (dC_{j,t} + dG_{j,t}) \\ &= \sum_j \frac{p_j}{\bar{P}} \sum_h \mu_h dC_{hj,t} + \sum_j \frac{p_j}{\bar{P}} dG_{j,t} \\ &= \frac{1}{\bar{P}} \sum_h \mu_h \sum_j p_j dC_{hj,t} + \frac{1}{\bar{P}} \sum_j p_j dG_{j,t} \\ &= \sum_h \mu_h \frac{P_h}{\bar{P}} dC_{h,t} + \frac{P_g}{\bar{P}} dG_t \end{aligned}$$

where we define the Laspeyres level changes in real consumption and real government spending as

$$P_h dC_{h,t} = \sum_j p_j dC_{hj,t} \quad \text{and} \quad P_g dG_t = \sum_j p_j dG_{j,t}$$

where P_h and P_g are the appropriate base-year deflators.

Next, we write

$$\begin{aligned} dC_{h,t} &= d\tilde{\mathcal{C}}_{h,t} \\ &= \sum_{s=0}^{\infty} \left[\frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial Y_s} dY_s + \frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial \zeta_{h,s}} d\zeta_{h,s} + \frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial \rho_{h,s}} d\rho_{h,s} + \frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial \tau_s} d\tau_s + \frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial T_s} dT_s \right] + \sum_{s=0}^{\infty} \left[\frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial R_s} dR_s + \frac{\partial \tilde{\mathcal{C}}_{h,t}}{\partial \pi_{h,s}^\rho} d\pi_{h,s}^\rho \right] \end{aligned}$$

We plug in and rearrange to conclude the proof. Notice that there are two cross-sectional terms: $d\tilde{\zeta}_{h,s}$ and $d\rho_{h,s}$. These terms can be written as cross-sectional covariances. We have

$$\begin{aligned}
\sum_h \mu_h \tilde{\zeta}_{h,t} &= \sum_h \mu_h \frac{W_{h,t} N_{h,t} + \zeta_h \Pi_t}{Y_t^n} \\
&= \sum_h \mu_h \frac{W_{h,t} N_{h,t} + \zeta_h Y_t^n - \sum_h \mu_h W_{h,t} N_{h,t}}{Y_t^n} \\
&= \frac{1}{Y_t^n} \sum_h \mu_h \left[W_{h,t} N_{h,t} + \zeta_h Y_t^n - \sum_h \mu_h W_{h,t} N_{h,t} \right] \\
&= \frac{1}{Y_t^n} \left[\sum_h \mu_h W_{h,t} N_{h,t} + Y_t^n - \sum_h \mu_h W_{h,t} N_{h,t} \right] \\
&= 1
\end{aligned}$$

where we use $\sum_h \mu_h \zeta_h = 1$, and therefore

$$\sum_h \mu_h d\tilde{\zeta}_{h,t} = 0.$$

Similarly, we have

$$\sum_h \mu_h b_h d \log \rho_{h,t} = \sum_h \mu_h b_h d \pi_{h,t}^p = 0$$

Relative price levels and inflation rates aggregate to 0 only when *expenditure-weighted*. This is why Proposition 2 features expenditure weights for these terms.

A.5 Proof of Proposition 3

We maintain Assumptions A1–A3.

Step 1: labor supply aggregation. From earlier results, we have an as-if-RA labor supply curve

$$W_t = P_t \frac{\epsilon_w}{\epsilon_w - 1} \frac{\kappa N_t^\phi}{u'(C_t)}$$

with

$$\begin{aligned}\phi &= \frac{\xi}{\gamma} = \frac{\sum_h v_h \frac{\phi_h}{1+\phi_h}}{\sum_h v_h \frac{1}{1+\phi_h}} \\ \kappa &= \exp \left\{ \frac{1}{\gamma} \sum_h \left[\frac{v_h}{1+\phi_h} \log \kappa_h + \frac{v_h}{1+\phi_h} \phi_h \log \frac{v_h}{\mu_h} - v_h \log v_h \right] \right\} \\ \frac{\epsilon_w}{\epsilon_w - 1} &= \exp \left\{ \frac{1}{\gamma} \sum_h \frac{v_h}{1+\phi_h} \log \frac{\epsilon_h^w}{\epsilon_h^w - 1} \right\}\end{aligned}$$

where $N_t = \sum_j N_{j,t}$.

Therefore, we have

$$d \log W_t = d \log P_t + \phi d \log N_t - d \log u'(C_t).$$

Finally, we have

$$d \log u'(C_t) = \frac{1}{u'(C_t)} du'(C_t) = \frac{u''(C_t)}{u'(C_t)} dC_t = \frac{C_t u''(C_t)}{u'(C_t)} d \log C_t \equiv -\gamma_t d \log C_t,$$

where we define γ_t as the constant of relative risk aversion.

Step 2: firms' cost structure. Next, we define the time-varying markup $\mu_{j,t}$ to represent the sectoral Phillips curve in reduced form,

$$p_{j,t} = \mu_{j,t} MC_{j,t}.$$

This yields $d \log p_{j,t} = d \log \mu_{j,t} + d \log MC_{j,t}$. From cost minimization (and Shephard's lemma) it follows that marginal cost can be unpacked as

$$d \log p_{j,t} = d \log \mu_{j,t} + \sum_{k=1}^N \tilde{\Omega}_{jk} d \log p_{k,t} + \tilde{\Omega}_{jL} d \log W_t - d \log A_{j,t}$$

where we denote by $\tilde{\Omega}_{jL}$ the entry $(j, J+1)$ of the $(J+1) \times (J+1)$ matrix $\tilde{\Omega}$, corresponding to the cost share of the effective labor factor. This equation is identical to the main proof in [Baqee and Farhi \(2020\)](#).

Next, denote sectoral (column) vectors by $\mathbf{p}_t = (p_{1,t}, \dots, p_{J,t})$. Let's also denote by $\tilde{\Omega}^p$ the first $J \times J$ block of the matrix $\tilde{\Omega}$, which corresponds only to intermediate inputs. We have

$$d \log p_{j,t} = d \log \mu_{j,t} + \tilde{\Omega}_{[j,\cdot]}^p d \log \mathbf{p}_t + \tilde{\Omega}_{jL} d \log W_t - d \log A_{j,t}$$

Now we stack and arrive at

$$d \log \mathbf{p}_t = d \log \boldsymbol{\mu}_t + \tilde{\Omega}^p d \log \mathbf{p}_t + \tilde{\Omega}_{[,L]} d \log W_t - d \log \mathbf{A}_t$$

Denoting by $\tilde{\Psi} = (\mathbf{I} - \tilde{\Omega}^p)^{-1}$ the cost-based Leontief inverse as in [Baqee and Farhi \(2020\)](#), and noting that

$$\tilde{\Psi}_{jL} = \sum_k \tilde{\Psi}_{jk} \tilde{\Omega}_{kL} = 1$$

because $\tilde{\Omega}_{jL} + \sum_k \tilde{\Omega}_{jk} = 1$, we have

$$d \log \mathbf{p}_t = \mathbf{1} d \log W_t + \tilde{\Psi} d \log \boldsymbol{\mu}_t - \tilde{\Psi} d \log \mathbf{A}_t.$$

Finally, notice that the Divisia definition of log changes in the GDP deflator is

$$d \log P_t = \mathbf{b}' d \log \mathbf{p}_t = d \log W_t + \tilde{\lambda}(d \log \boldsymbol{\mu}_t - d \log \mathbf{A}_t)$$

because $\mathbf{b}' \mathbf{1} = 1$ and $\mathbf{b}' \tilde{\Psi} = \tilde{\lambda}$.

We have therefore related changes in real wages to changes in markups and technologies,

$$d \log w_t = d \log W_t - d \log P_t = -\tilde{\lambda}(d \log \boldsymbol{\mu}_t - d \log \mathbf{A}_t).$$

Step 3: the aggregate labor income share. Define

$$\Lambda_t^L = \frac{W_t N_t}{Y_t^n} = \frac{W_t N_t}{P_t Y_t}.$$

Then we can express changes in labor supply in terms of changes in the labor income share as

$$d \log N_t = d \log \Lambda_t^L - d \log w_t + d \log Y_t.$$

Step 4: putting it together. We have the three equations

$$d \log w_t = \phi d \log N_t + \gamma_t d \log C_t$$

$$d \log w_t = -\tilde{\lambda}(d \log \boldsymbol{\mu}_t - d \log \mathbf{A}_t)$$

$$d \log N_t = d \log \Lambda_t^L - d \log w_t + d \log Y_t.$$

Together, they imply

$$\phi d \log N_t + \gamma_t d \log C_t = -\tilde{\lambda}(d \log \boldsymbol{\mu}_t - d \log \mathbf{A}_t)$$

$$d \log N_t = d \log \Lambda_t^L + \tilde{\lambda}(d \log \boldsymbol{\mu}_t - d \log \mathbf{A}_t) + d \log Y_t.$$

And therefore

$$\phi(d \log \Lambda_t^L + \tilde{\lambda}(d \log \mu_t - d \log A_t) + d \log Y_t) + \gamma_t d \log C_t = -\tilde{\lambda}(d \log \mu_t - d \log A_t)$$

Finally, holding fixed government spending, we have $dY_t = dC_t$ and $C d \log C_t = Y d \log Y_t$,

$$d \log C_t = \frac{1}{b_C} d \log Y_t.$$

Rearranging, we arrive at

$$\left(\phi + \frac{\gamma}{b_C}\right) d \log Y_t = (1 + \phi)(\tilde{\lambda} d \log A_t - \tilde{\lambda} d \log \mu_t) - \phi d \log \Lambda_t^L,$$

which concludes the proof.

A.6 Proof of Proposition 4

Step 1: firm marginal cost and GDP deflator. Following very similar steps as in Appendix A.5 and Baqaee and Farhi (2020), we have

$$d \log p_{j,t} = d \log \mu_{j,t} + \sum_{k=1}^J \tilde{\Omega}_{jk} d \log p_{k,t} + \sum_{h=1}^H \tilde{\Omega}_{j,J+h} d \log W_{h,t} - d \log A_{j,t}.$$

Using the same definitions of cost-based Domar weights and Leontief inverses as above, we have

$$d \log p_{j,t} = \sum_{h=1}^H \tilde{\Psi}_{j,J+h} d \log W_{h,t} + \sum_{k=1}^J \tilde{\Psi}_{jk} (d \log \mu_{k,t} - d \log A_{k,t}).$$

Pre-multiplying by expenditure weights, we have

$$d \log P_t = \mathbf{b}' d \log \mathbf{p}_t = \sum_h \tilde{\Lambda}_{h,t}^L d \log W_{h,t} + \sum_k \tilde{\lambda}_k (d \log \mu_{k,t} - d \log A_{k,t})$$

or simply

$$d \log P_t = \tilde{\Lambda}' d \log \mathbf{W}_t + \tilde{\lambda}' d \log \mu_t - \tilde{\lambda}' d \log A_t.$$

Step 2: labor income shares. Define

$$\Lambda_{h,t}^L = \frac{W_{h,t} N_{h,t}}{Y_t^n},$$

so that

$$d \log \Lambda_{h,t}^L = d \log W_{h,t} + d \log N_{h,t} - d \log P_t - d \log Y_t.$$

Step 3: labor supply schedules. From the labor supply schedules

$$W_{h,t} = P_{h,t} \frac{\epsilon_h^w}{\epsilon_h^w - 1} \frac{v'_h(N_{h,t})}{u'_h(C_{h,t})}$$

we have

$$d \log w_{h,t} = d \log W_{h,t} - d \log P_{h,t} = \phi_h d \log N_{h,t} + \tilde{\gamma}_h d \log C_{h,t}$$

where

$$\phi_h = \frac{N_h v''_h(N_h)}{v'_h(N_h)} \quad \text{and} \quad \tilde{\gamma}_h = -\frac{C_h u''_h(C_h)}{u'_h(C_h)}.$$

Step 4: putting it all together. Combining the labor supply schedule with the definition of labor income shares yields

$$d \log W_{h,t} = \frac{1}{1 + \phi_h} d \log \rho_{h,t} + d \log P_t + \frac{\phi_h}{1 + \phi_h} d \log Y_t + \frac{\phi_h}{1 + \phi_h} d \log \Lambda_{h,t}^L + \frac{\tilde{\gamma}_h}{1 + \phi_h} d \log C_{h,t}$$

Plugging into the pricing equation and noting that $\sum_h \tilde{\Lambda}_{h,t}^L = 1$, we have

$$d \log P_t = \sum_h \tilde{\Lambda}_{h,t}^L \left(\frac{1}{1 + \phi_h} d \log \rho_{h,t} + d \log P_t + \frac{\phi_h}{1 + \phi_h} d \log Y_t + \frac{\phi_h}{1 + \phi_h} d \log \Lambda_{h,t}^L + \frac{\tilde{\gamma}_h}{1 + \phi_h} d \log C_{h,t} \right) \\ + \sum_k \lambda_{k,t} \left(d \log \mu_{k,t} - d \log A_{k,t} \right)$$

and simplifying, we arrive at

$$\sum_h \tilde{\Lambda}_{h,t} \frac{\phi_h}{1 + \phi_h} d \log Y_t = - \sum_h \tilde{\Lambda}_{h,t} \frac{1}{1 + \phi_h} d \log \rho_{h,t} - \sum_h \tilde{\Lambda}_{h,t} \frac{\phi_h}{1 + \phi_h} d \log \Lambda_{h,t} - \sum_h \tilde{\Lambda}_{h,t} \frac{\tilde{\gamma}_h}{1 + \phi_h} d \log C_{h,t} \\ + \sum_k \lambda_{k,t} \left(d \log A_{k,t} - d \log \mu_{k,t} \right)$$

Introducing the cross-sectional consumption dispersion measure $d \log \delta_{h,t}$, defining $\gamma_h = \tilde{\gamma}_h / b_h$, and rearranging concludes the proof.

B Data

This appendix provides details on the data we use to construct our empirical results.

B.1 Factor Shares

We obtain the factor shares for sectors from the BEA's I-O GDP by Industry dataset from 1997 after industry classifications are based on NAICS. We then crosswalk sectors based on 2-digit NAICS level to 22 sectors in total. All concordances are weighted by gross output levels from BEA's Gross Output by Industry Table. First, we calculate the labor share in the production of primary factors $1 - \alpha_j$ for each sector j . Given the Cobb-Douglas structure of our primary factor production function, the parameters are calculated as the ratio of compensation of employees to the value added adjusted for taxes and subsidies. We obtain this ratio for each year from 1997-2015, then take the average value. Second, we obtain the share of intermediate inputs in the production function $\mu_{x,j}$ for each sector j . The parameters are calculated as the intermediate input expenditures as a percentage of gross output, averaged over our sample period. We obtain this ratio for each year from 1997-2015, then take the average value. Finally, we calculate the labor share in the over production function $\mu_{l,j} = (1 - \mu_{x,j})(1 - \alpha_j)$, and the capital share in the over production function $\mu_{k,j} = (1 - \mu_{x,j})(\alpha_j)$. Figure 6 plots the factor shares in the production for each sector j . Sectors with the highest labor share are service-related, such as Education Services, Healthcare, Professional and Technical Services. Manufacturing is particularly intermediates dependent. Housing and leasing industries are the most capital-intensive.

B.2 Capital Investment

In our baseline, we use the Investment Flows Data for 41 Sector Partition from [Vom Lehn and Winberry \(2022\)](#) from 1997-2015 to calculate the share of capital investment inputs from each sector Γ_j^{inv} . First, for each year from 1997 to 2015, we calculate the share of total capital investment across all purchasing sectors (by summing the row in the Investment Flows tables) and calculating the shares. We then crosswalk the 41 sectors based on 2-digit NAICS level to 22 sectors in total. Finally, we take the average of each year's share to get Γ_j^{inv} . Figure 7 plots sectoral input shares of total capital investment. The top investment hubs are Construction, Durable Manufacturing, and Professional and Technical Services. These three investment hub industries together account for nearly 80% of inputs used in the production of capital investment.

For robustness checks in Appendix C, we use BEA's Capital Flows Table in 1997 to derive the share of capital investment inputs from each of the 66 BEA sectors. The Capital Flows table includes 180 commodities with corresponding NAICS codes. Our first step is to match the commodities used to the sector categories. Most of the matching are straightforward given the NAICS codes in both the Capital Flows table and the Input-Output table. Special attention should be paid to the following: (1) "Manufactured homes, mobile homes" in commodities is categorized under the "Housing"

sector; (2) “Retail trade” in commodities include both 44 and 45 by NAICS codes, we divide and assign it to “Motor vehicle and parts dealers”, “Food and beverage stores”, “General merchandise stores”, “Other retail” according to these four sectors’ sizes measured by the gross output in 1997; (3) “Offices of real estate agents and brokers” in commodities is categorized under the “Rental and leasing services and lessors of intangible assets” sector; (4) “Noncomparable imports” is excluded. The top investment hubs are “Construction”, “Machinery”, “Motor vehicles, bodies and trailers, and parts”, and “Computer and electronic products”. These four investment hub industries together account for nearly 70% of inputs used in the production of capital investment. The sparseness of the investment network is collaborated in [Vom Lehn and Winberry \(2022\)](#).

B.3 Government Spending

We use the BEA industry input-output “Use” table to compute the share of government spending on goods from different sectors Γ_j^g . The parameters are calculated as the government expenditures in sector j as a percentage of total government spending. “Government” includes federal government, federal government enterprises, state and local government, and state and local government enterprises. We average the annual share across 1997-2015. Figure 8 plots the government spending share, averaged across the sample period.

B.4 Mark-ups

Steady state markups across sectors are given by $\mu_j = \frac{\epsilon_j}{\epsilon_j - 1}$. We calibrate ϵ_j directly to match sectoral markups using data from [Baqae and Farhi \(2020\)](#). They use three alternative approaches to estimate sectoral markups from 1997 to 2015. The average markup for each sector in any particular year is computed as the harmonic sales-weighted average of firm markups, which are taken from Compustat and assigned to BEA sectors. In our baseline model, we use the average of their benchmark estimates following the accounting profits approach because the average markup is then around 10% and thus closer to the standard markup assumed in the HANK literature. Figure 9 plots the accounting-profits markups for each of the 22 sectors. We have also run robustness checks of the empirical regularities using the other two approaches (user-cost and production function) in Appendix C. To ensure $\epsilon_j > 0$, for each approach, we replace markups below 1 with the lowest markup in all sectors above 1 using that particular approach.

B.5 Monthly Price Adjustment Frequency

To measure monthly price adjustment frequency across industries, we compare two data sources. First, in our baseline model, we map the sector-specific monthly price adjustment frequency, $1 - \Theta$, from [Pasten et al. \(2017\)](#) to the 22 sector categories. They use the data underlying the Producer Price Index (PPI) for 754 industries (defined by 6-digits NAICS codes) from the U.S. Bureau of Labor Statistics, from 2005 to 2011. The PPI measures covers both final consumption prices and

intermediates prices. The mapping from NAICS to BEA sectors are not one-to-one. Therefore, we need to make certain inferences based on industry similarities.

Second, we estimate the frequency of price changes using the confidential micro-level price data underlying the the Consumer Price Index (CPI) from 1998 to 2005, made available by [Nakamura and Steinsson \(2008\)](#). The estimates are available for 272 “Entry-Level Item” (ELI) categories, in two measures, with and without sales. They cover most of consumer expenditures, but not intermediate industries. The mapping from ELI to BEA sectors are not one-to-one. Therefore, we need to make certain inferences based on industry similarities.

We use measures from [Pasten et al. \(2017\)](#) as our baseline because they are more recent than earlier estimates by [Nakamura and Steinsson \(2008\)](#) and account for price changes in intermediates industries. Figure 10 plots the price adjustment frequency using data from [Pasten et al. \(2017\)](#). We have also run robustness checks of the empirical regularities using the other two approaches ([Nakamura and Steinsson \(2008\)](#) with sales, and [Nakamura and Steinsson \(2008\)](#) without sales) in Appendix C).

B.6 Intermediates Input-output Share

We calibrate the weights on intermediate inputs α_{jk}^x so that our model’s production network is consistent with the BEA’s input-output table. We use the Industry Input-output “Use” Table. For each year, we calculate the parameters of the intermediates input-output network α_{jk}^x as sector j (columns)’s nominal expenditure on intermediate inputs from sector k (rows) as a share of j ’s total expenditure on intermediate inputs. Then we average the ratios across 1997-2015. Figure 11 plots the heatmap of the input-output network, averaged across the sample period.

B.7 Centrality

A reduced-form measure of a sector’s centrality in the input-output production network is the Katz-Bonacich centrality measure discussed by [Carvalho \(2014\)](#). We compute centrality as $c = \eta(I - \lambda\alpha^x)^{-1}\mathbf{1}$, where we set $\eta = \frac{1-\theta}{N} = \frac{1-0.5}{22}$ and $\lambda = 0.5$.

Figure 12 plots the centrality measure for each of the 22 sectors. The most important suppliers in the production network are Professional and Technical Services, Durable and Non-durable Manufacturing, Finance and Insurance.

We also calculated the outdegree of sectors, defined as $d_k = \sum_j \alpha_{jk}^x$, that is, the sum over all the weights of the network in which sector k appears as an input-supplying sector. The correlation between the outdegree measure and the centrality measure is 0.99, providing us with confidence with the central nodes in the production network. We have run robustness check of our empirical regularities results using both measures in Appendix C.

B.8 ACS-IO: Sectoral Payroll Shares

We build the dataset of sector-specific payroll shares for households in various income quantiles.

Data Source. We obtain cross-sectional household occupation and payroll data from the American Community Survey (ACS), made available by IPUMS (Ruggles et al., 2015) from 2000-2015.

Sample Restriction and Household Types. First, we clean up the ACS dataset by excluding those who are not in the labor force as well as outlier data points (wage income below \$1,000 and beyond \$9,999,998). For each year, we further exclude the extreme values (top 1% and bottom 1%), and then divide the remaining earnings data points into 10 income quantiles.

Matching. Second, we map the cleaned earnings data to the sector classification from the BEA. Finally, we follow Clayton et al. (2018) and use the “many-to-one” method to merge the sectoral earnings data. For the BEA sectors that do not have a corresponding ACS industry identifier, we borrow the variables of interest from industries closest to those.

B.9 CEX-IO: Expenditure Share

Data Source. We use the U.S. Consumer Expenditure Survey (CEX) by the BLS from 1997-2015 to obtain expenditure shares across product categories for households at different quantiles of the income distribution. The CEX is a widely used consumption survey tracking spending in all product categories, including goods, services, housing, and health. It consists of two parts, the Interview and Diary surveys. The Interview surveys collect responses from households annually for up to 4 consecutive quarters of questions, covering a wide range of purchases. The Dairy questionnaire contains more detailed questions about daily purchases, and are collected at weekly frequency. We use both for our data construction.

Sample Restriction. We follow the literature and impose a set of sample restrictions. We restrict the samples to urban households, with heads aged 25-64, have a full-year of interview coverage and complete income responses. We exclude outlier data points (households with the top 1% and bottom 1% of after-tax income, households with the top 1% and bottom 1% of total expenditure, and households whose income is below \$1,000). We also eliminate data points when expenditures are negative (as a result of Medicaid and Medicare reimbursement).

Expenditure adjustments. We follow Comin et al. (2021) and make certain adjustment to the expenditure data. We exclude any taxes and social security payments. Similar to Aguiar and Bils (2015), we exclude alimony, child support payments, support for college students, and occupational expenses. To avoid double counting for expenditures associated with owning a house, we follow

Hubmer (2023) and subtract estimated monthly rental value of owned home, estimated monthly rental value of vacation home, and estimated rental value of timeshare and treat those as part of household income.

Income adjustment and distribution. We add estimated monthly rental value of owned home, estimated monthly rental value of vacation home, and estimated rental value of timeshare to after-tax household income as reported in the CEX (FINCATXM and FINCAEFX) to get household income levels. We divide the households into 100 quantiles according to their adjusted income levels.

Matching. We match the UCC categories in our sample to 22 BEA industries mostly based on a manual concordance assembled by Levinson and O'Brien (2019). Additionally, we match Diary items and missing Interview items by hand based on our best judgment.

Treatment of zeros. For some goods, households expenditure share in those will be zero. This is either because households forgot to record, or households don't spend in some pure intermediate sectors (such as oil and gas extraction). When there are missing values, we assume a minuscule amount of 0.0001% on household expenditure share in order to avoid dropping these households from our sample.

C Robustness Checks of Empirical Regularities

C.1 Expenditure Heterogeneity using CEX-IO Dataset

Expenditure-share weighted sectoral heterogeneity for different income percentiles. In Figure 13, we show the comparison of expenditure-share weighted sectoral heterogeneity: for pre-tax income percentiles in navy and after-tax income percentiles in red. The pre-tax income percentiles uses FINCBTAX in the CEX dataset, and the after-tax income percentiles uses FINCATAX instead. The results illustrates that our empirical regularities are robust to using either income measures.

Expenditure-share weighted sectoral heterogeneity for different levels of aggregation. In Figure 14, we show four sets of expenditure-share weighted sectoral heterogeneity at different levels of aggregation for after-tax income percentiles: at 66-sector level (the full BEA I-O) in red, 22-sector level (2-digit NAICS) in blue, 9-sector level (1-digit NAICS) in green, and 3-sector level (manufacturing, agriculture, and services) in gray. The empirical regularities are robust between the 66-sector level and the 22-sector level. But if we split the consumption categories into agriculture, manufacturing, and services like in Comin et al. (2021), the results are not consistent. Figure 15 shows the correlation of sectoral features between different levels of aggregation versus the 66 sectors. The results start to stabilize after 22-sector aggregation.

Factor shares. Figure 16 plots the expenditure-weighted three factor shares using with different levels of sectoral disaggregation: (1) labor shares, (2) capital shares, and (3) intermediates shares. The relationship between income percentile and factor shares is robust across measures and disaggregation levels.

Monthly frequency of price change. Figure 17 plots the expenditure-weighted monthly frequency of price change using three measures with different levels of sectoral disaggregation: (1) Pasten et al. (2017), (2) Nakamura and Steinsson (2008) with sales, and (3) Nakamura and Steinsson (2008) without sales. The hump-shaped relationship between income percentile and price rigidity is robust across measures and disaggregation levels, where the middle-income households have the least rigid consumption basket. Our results using Nakamura and Steinsson (2008) measures, either with or without sales are consistent with [Cravino 2020], where they use the same dataset for price rigidity.

Markups. Figure 18 plots the expenditure-weighted markups using three markups measures with different levels of sectoral disaggregation: (1) accounting-profits markups (AP), (2) user-cost markups (UC), and (3) production-function markups (PF).

Centrality. Figure 19 plots the expenditure-weighted markups using two measures with different levels of sectoral disaggregation: (1) Katz-Bonacich centrality and (2) outdegree. The relationship between income percentile and network centrality is robust across measures and disaggregation levels.

Investment and Government Spending Shares. Figure 20 plots the expenditure-weighted investment shares and government spending shares with different levels of sectoral disaggregation. The relationship between income percentile and investment or government spending shares is robust across measures and disaggregation levels.

C.2 Earning Heterogeneity using ACS-IO Dataset

Earnings-share weighted sectoral heterogeneity across different sample periods . In Figure 21 we show three sets of earnings-share weighted sectoral heterogeneity using three sample periods in a 22-sector economy: (1) from 2000-2004 in blue, (2) from 2005-2009 in red, and (3) from 2010-2015 in green. The relationship has been very robust across the time periods.

Earnings-share weighted sectoral heterogeneity for different levels of aggregation . Similar to the robustness check we have done for the CEX-IO Dataset, we show four sets of earnings-share weighted sectoral heterogeneity at different levels of aggregation for total income percentiles in Figure 22: at 66-sector level (the full BEA I-O) in red, 22-sector level (2-digit NAICS) in blue, 9-sector level (1-digit NAICS) in green, and 3-sector level (manufacturing, agriculture, and services) in gray. Figure 23 shows the correlation of sectoral features between different levels of aggregation versus the 66 sectors. The empirical regularities start to converge after a fair amount of disaggregation at 22 sectors.

Factor shares. Figure 24 plots the earnings-weighted three factor shares using with different levels of sectoral disaggregation: (1) labor shares, (2) capital shares, and (3) intermediates shares.

Monthly frequency of price change. Figure 25 plots the earnings-weighted monthly frequency of price change using three measures with different levels of sectoral disaggregation: (1) [Pasten et al. \(2017\)](#), (2) [Nakamura and Steinsson \(2008\)](#) with sales, and (3) [Nakamura and Steinsson \(2008\)](#) without sales.

Markups. Figure 26 plots the earnings-weighted markups using three markups measures with different levels of sectoral disaggregation: (1) accounting-profits markups (AP), (2) user-cost markups (UC), and (3) production-function markups (PF).

Centrality. Figure 27 plots the earnings-weighted markups using two measures with different levels of sectoral disaggregation: (1) Katz-Bonacich centrality and (2) outdegree.

Investment and Government Spending Shares. Figure 28 plots the earnings-weighted investment shares and government spending shares with different levels of sectoral disaggregation.

Sectors	Sectoral Features								1th percentile	4th percentile
	Capital share	Labor share	Intermediates share	Freq. price change	Centrality	Markups	Gov spending share	Investment share		
Manufacturing I	0.139	0.150	0.711	18.429	0.038	0.178	0.060	0.001	15.73%	1
Utilities	0.374	0.188	0.438	38.806	0.040	0.024	0.024	0.000	7.73%	5
Retail trade	0.187	0.468	0.345	21.172	0.023	0.085	0.000	0.022	11.88%	1
Housing	0.648	0.062	0.290	6.983	0.065	0.115	0.044	0.005	16.88%	1
Information	0.330	0.234	0.436	18.675	0.045	0.217	0.076	0.059	6.25%	4
Wholesale trade	0.257	0.397	0.346	8.879	0.026	0.033	0.000	0.054	5.75%	5
Manufacturing II	0.214	0.144	0.641	19.775	0.084	0.139	0.200	0.004	3.47%	3
Warehousing	0.120	0.468	0.413	6.933	0.029	0.115	0.003	0.004	0.05%	0
Agriculture	0.287	0.116	0.597	52.065	0.041	0.130	0.007	0.000	0.07%	0
Mining	0.385	0.183	0.433	38.806	0.048	0.045	0.029	0.035	0.14%	0
Professional and technical	0.174	0.467	0.359	7.133	0.105	0.113	0.148	0.189	1.27%	1
Administrative and waste	0.155	0.458	0.386	13.926	0.059	0.096	0.068	0.000	0.52%	0
Arts and entertainment	0.240	0.385	0.375	5.025	0.029	0.099	0.004	0.002	0.36%	0
Healthcare and social assistance	0.102	0.515	0.384	6.887	0.023	0.124	0.010	0.000	2.46%	3
Transportation	0.177	0.316	0.507	21.376	0.035	0.151	0.030	0.004	2.15%	2
Rental and leasing	0.518	0.119	0.363	18.132	0.032	0.202	0.009	0.000	0.67%	0
Finance and insurance	0.230	0.316	0.454	31.699	0.077	0.115	0.076	0.001	3.89%	5
Manufacturing III	0.155	0.250	0.595	13.870	0.085	0.111	0.098	0.297	7.76%	1
Education services	0.126	0.552	0.322	5.516	0.025	0.099	0.010	0.000	1.79%	1
Construction	0.171	0.336	0.493	20.221	0.030	0.105	0.062	0.325	1.36%	2
Accommodation and food services	0.154	0.395	0.451	27.266	0.032	0.163	0.015	0.000	5.01%	6
Other services	0.164	0.453	0.384	4.643	0.031	0.114	0.026	0.000	4.84%	5
<i>Mean</i>	0.241	0.317	0.442	18.464	0.045	0.117	0.045	0.045		
<i>Max</i>	0.648	0.552	0.711	52.065	0.105	0.217	0.200	0.325		
<i>Min</i>	0.102	0.062	0.290	4.643	0.023	0.024	0.000	0.000		

Table 2. Expenditure Share Differences and Sectoral Features, 22 Sectors

Note. Table 2 reports summary statistics for sectoral features across all 22 production sectors. Sectors are listed in ascending order according to the difference in expenditure shares between the top 1% income percentile and the bottom 1% income percentile. The light blue shade helps visualize each sector's sectoral features relative to the respective range across all sectors.

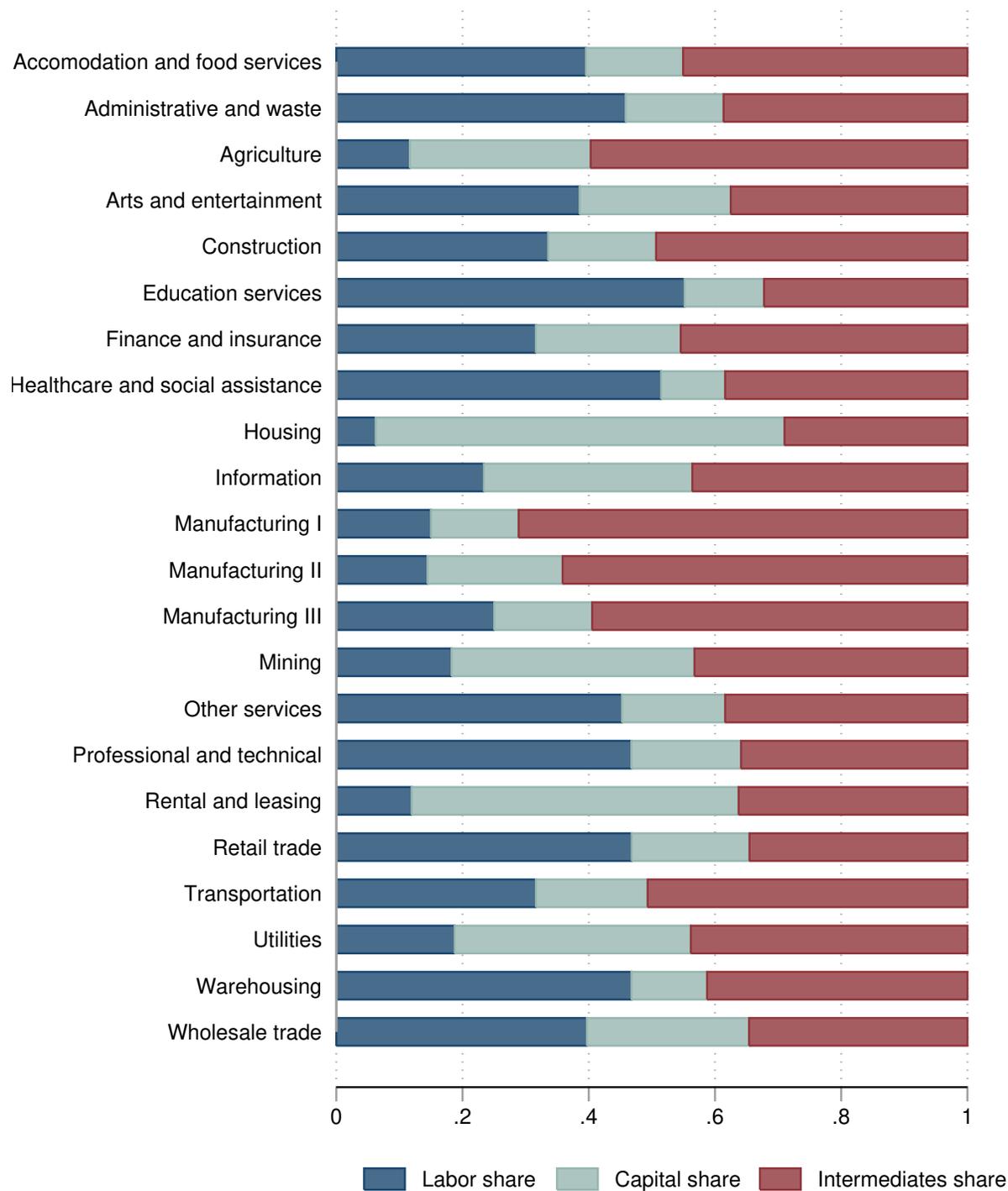


Figure 6. Factor Share in the Total Gross Output

Note. We compute the sector-specific factor shares in the total production from the BEA GDP by Industry dataset. The intermediate input share $\mu_{x,j}$ (in red) is computed as the intermediate input expenditures as a percentage of gross output. The labor share $\mu_{l,j}$ (in blue) is calculated as the product of $1 - \mu_{x,j}$ and the ratio of compensation of employees to the value added adjusted for taxes and subsidies. The remaining is the capital share $\mu_{k,j}$ (in green). We obtain each factor share for each year from 1997-2015, then take the average value.

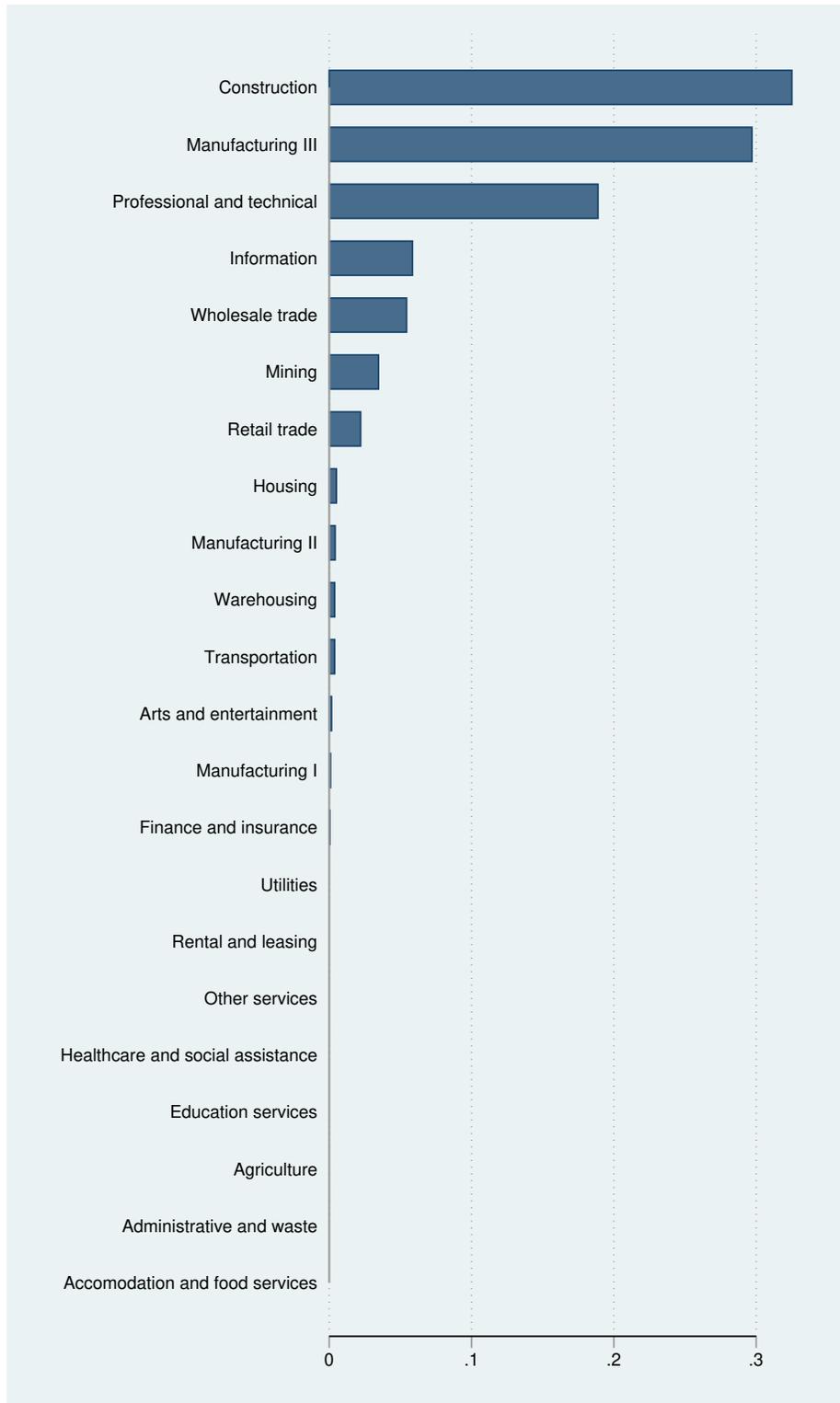


Figure 7. Sectoral Inputs Share in Capital Investment Γ_j^{inv}

Note. We compute the sectoral inputs share in the aggregate capital investment using the Investment Flows Data for 41 Sector Partition from [Vom Lehn and Winberry \(2022\)](#) from 1997-2015.

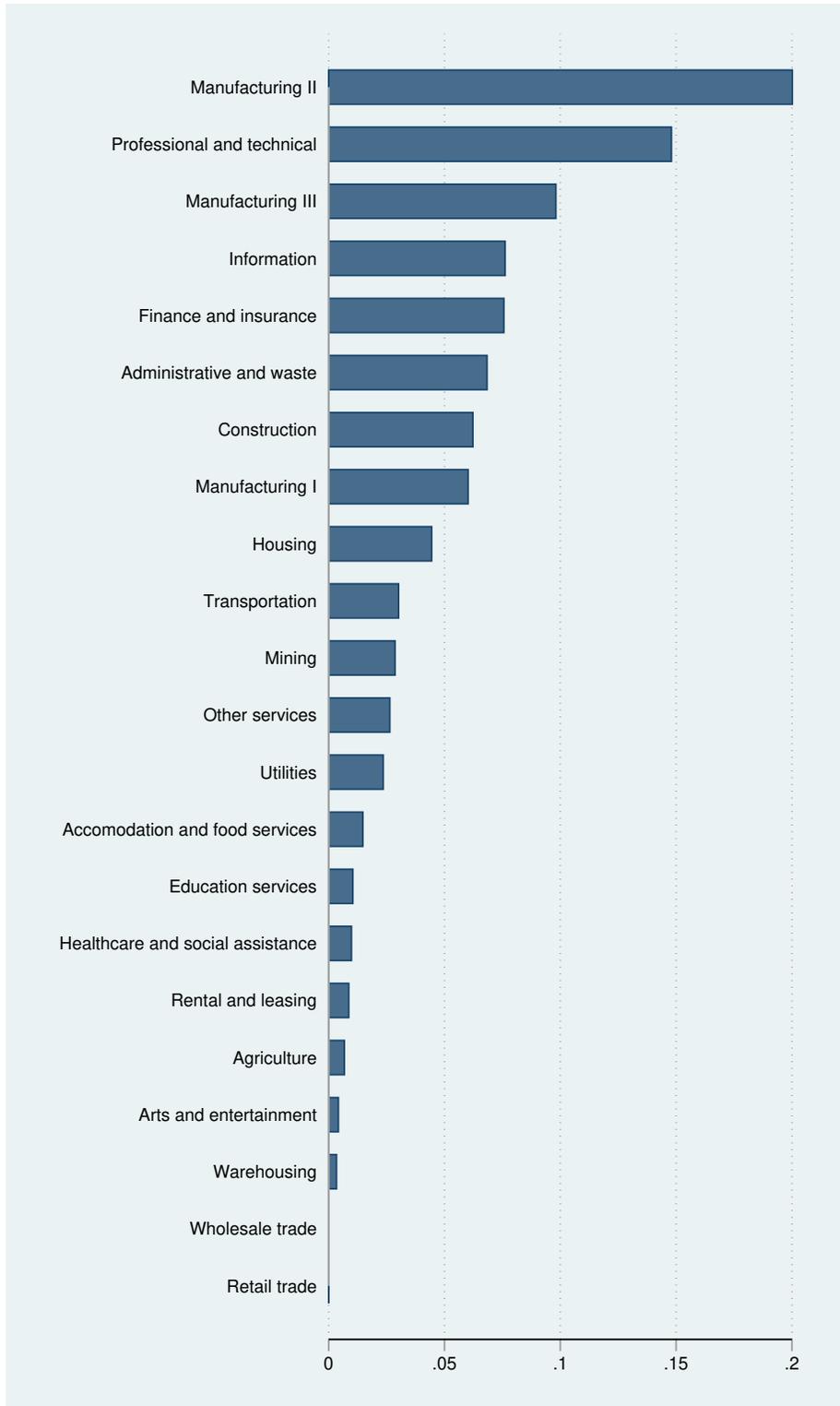


Figure 8. Government Spending Share on Sectoral Goods Γ_j^g

Note. We compute the share of government spending on goods from individual sector j in total government spending every year using the BEA industry input-output “Use” table, then we average the ratio across 1997-2015.

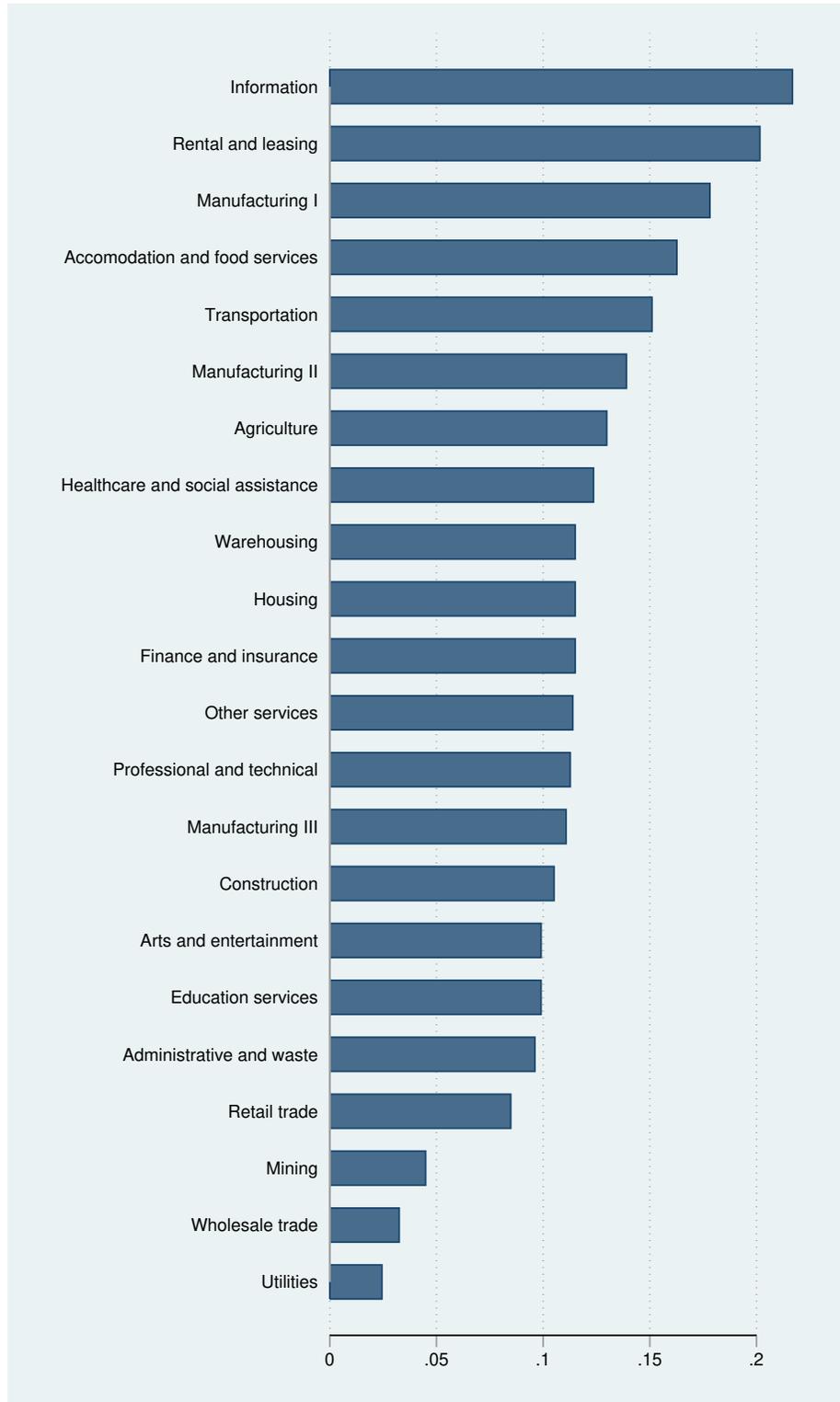


Figure 9. Sectoral Accounting-Profit Markups

Note. We compute the average accounting-profits markups over cost using data from [Baqaei and Farhi \(2020\)](#).

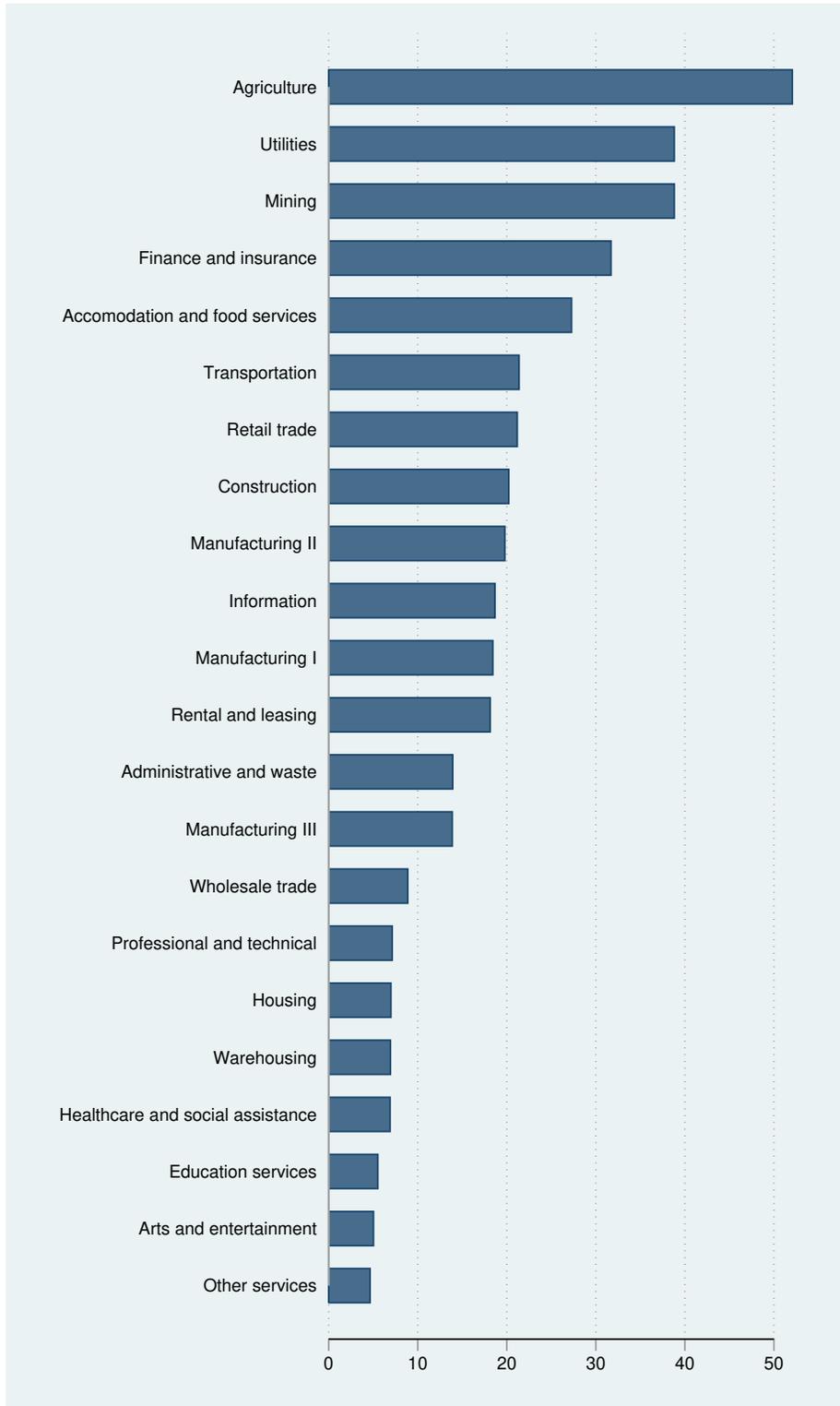


Figure 10. Monthly Price Adjustment Frequency 1 – \ominus From [Pasten et al. \(2017\)](#)

Note. We map the sector-specific monthly price adjustment frequency from [Pasten et al. \(2017\)](#) to the 22 sector categories.

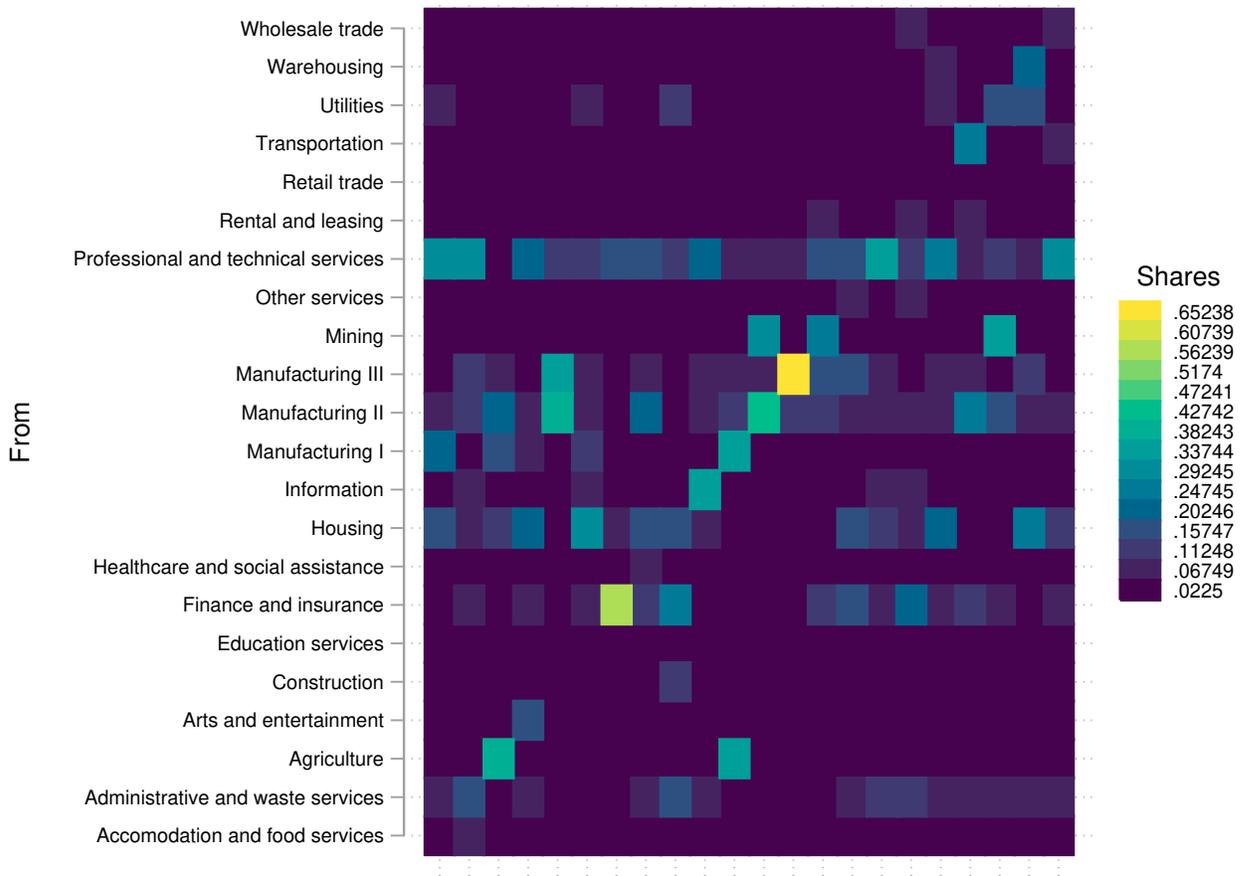


Figure 11. The Input-Output Network α_{jk}^x

Note. We compute the share of nominal expenditure on intermediate inputs from sector k (y-axis) for each sector j (x-axis) every year, then we average the ratio across 1997-2015.

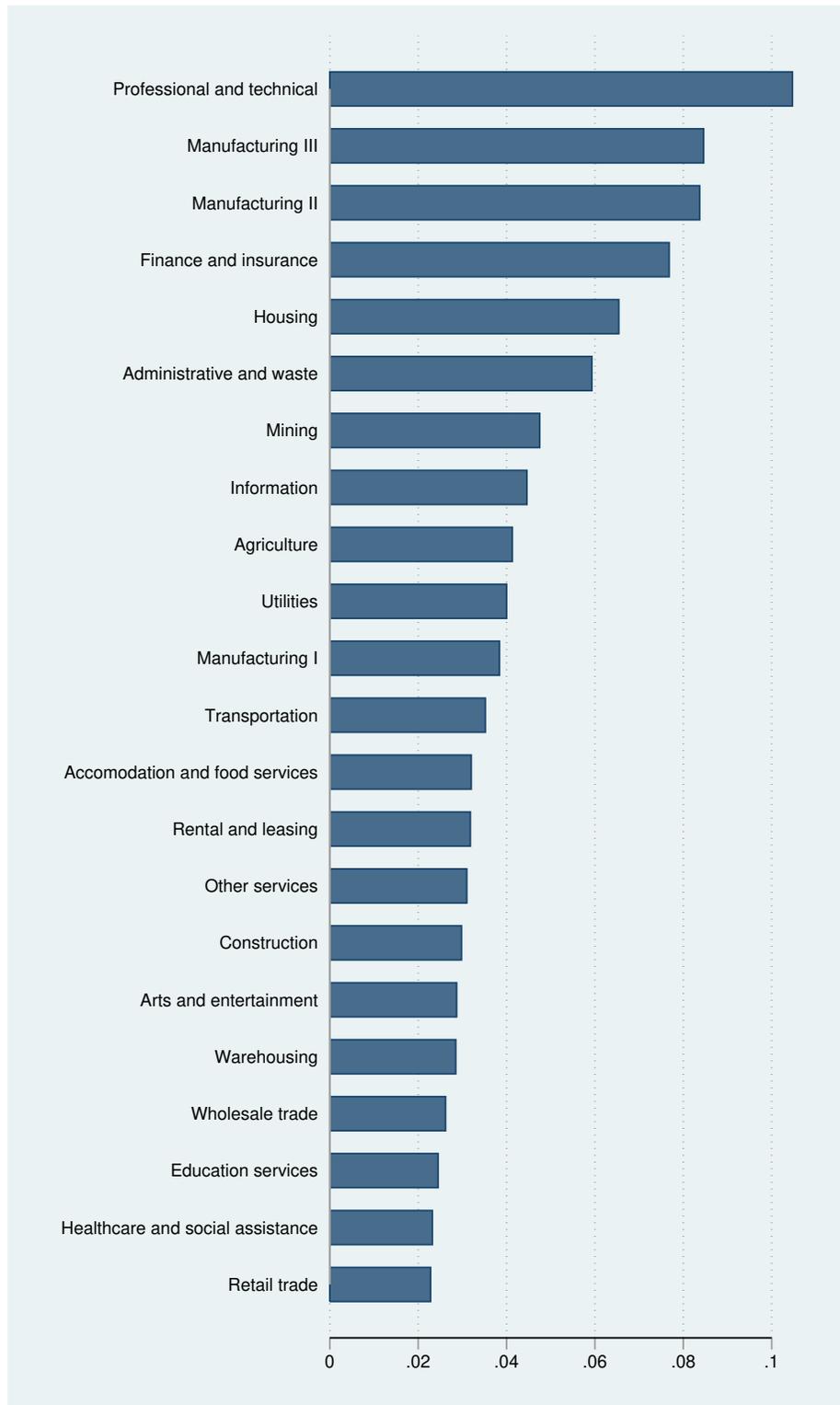


Figure 12. Sectoral Centrality α_{jk}^x

Note. We compute the centrality measure following [Carvalho \(2014\)](#) using the input-output matrix α_{jk}^x .

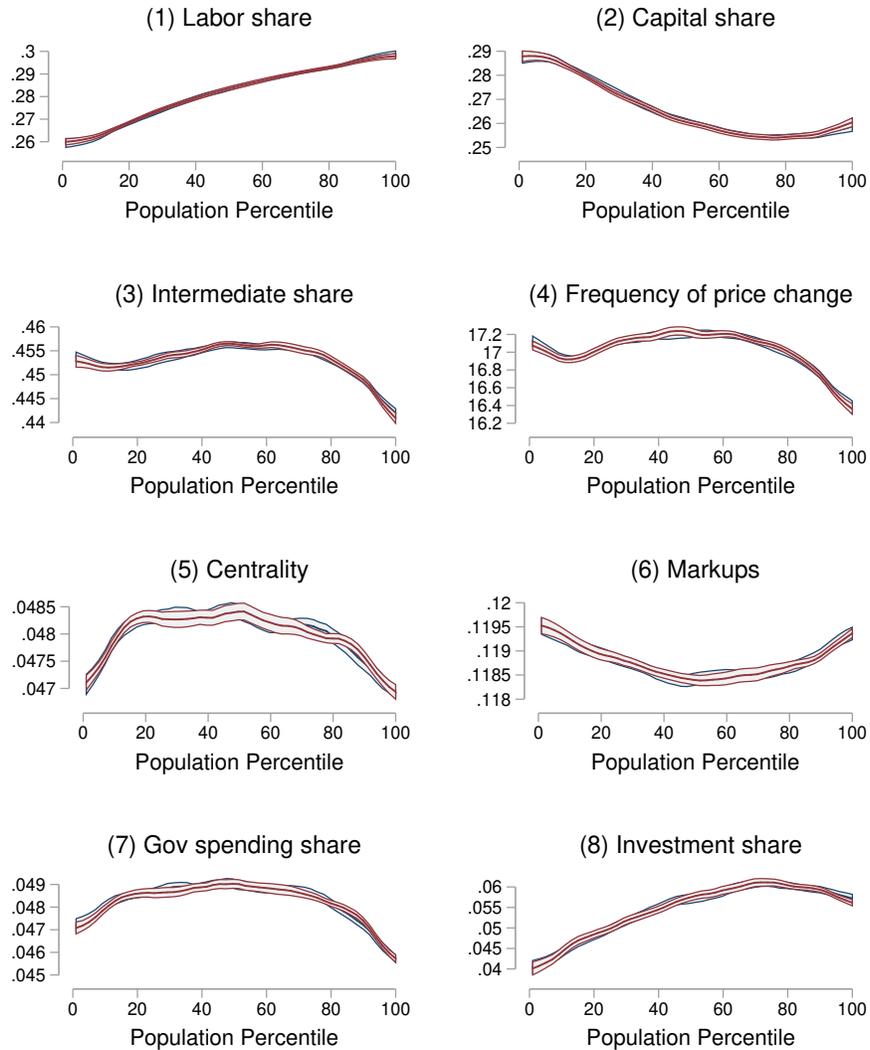


Figure 13. Expenditure-share Weighted Sectoral Heterogeneity for Pre-tax and After-tax Income Percentiles

Note. We plot the expenditure-weighted sectoral features as a function of households' pre-tax income percentile (in blue), and after-tax income percentile (in red), average over the sample period. The horizontal axis for each panel is household percentiles, each representing 1% of the population. The vertical axis reports the average sectoral features, such as labor share, capital share, intermediate share, frequency of price change, centrality, markups, government spending share, and investment share, weighted by the expenditure share across 22 sectors by households of the corresponding percentile.

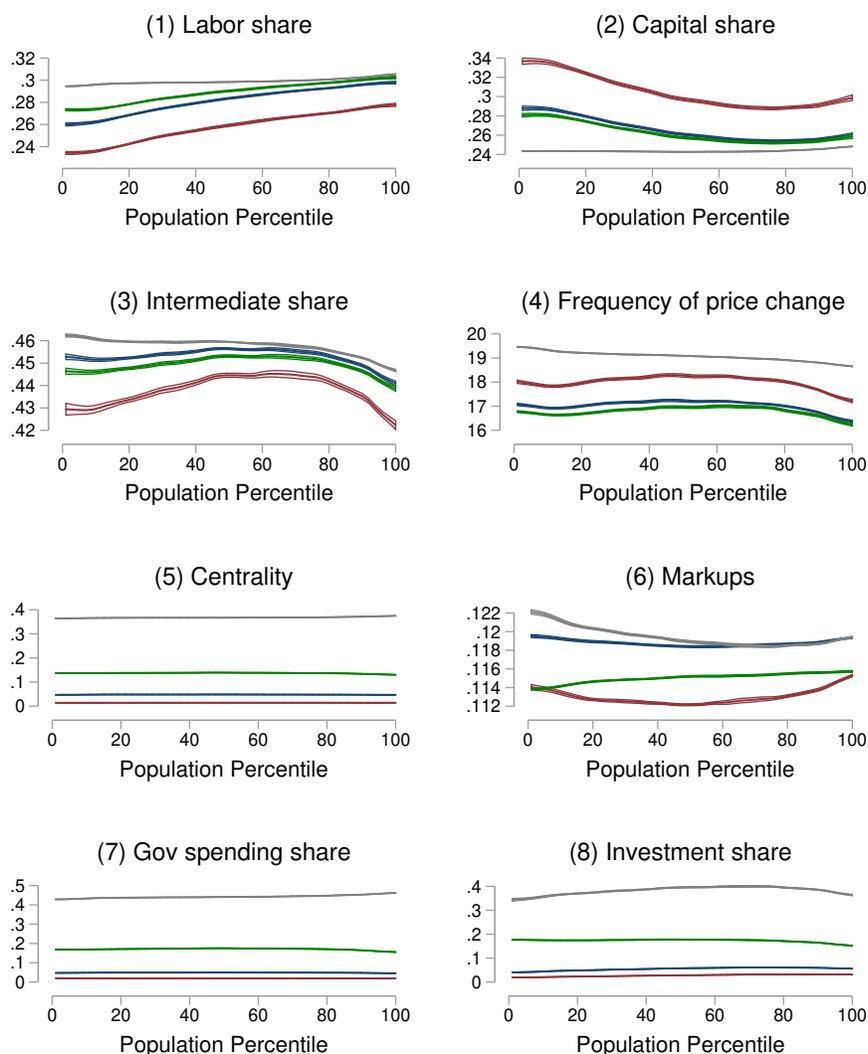


Figure 14. Expenditure-share Weighted Sectoral Heterogeneity for After-tax Income Percentiles with Different Levels of Aggregation

Note. We plot the expenditure-weighted sectoral features as a function of households' after-tax income percentile, average over the sample period. The horizontal axis for each panel is household percentiles, each representing 1% of the population. The vertical axis reports the average sectoral features, such as labor share, capital share, intermediate share, frequency of price change, centrality, markups, government spending share, and investment share, weighted by the expenditure share by households of the corresponding income percentile across 66 sectors in red, 22 sectors in blue, 9 sectors in green, and 3 sectors in gray.

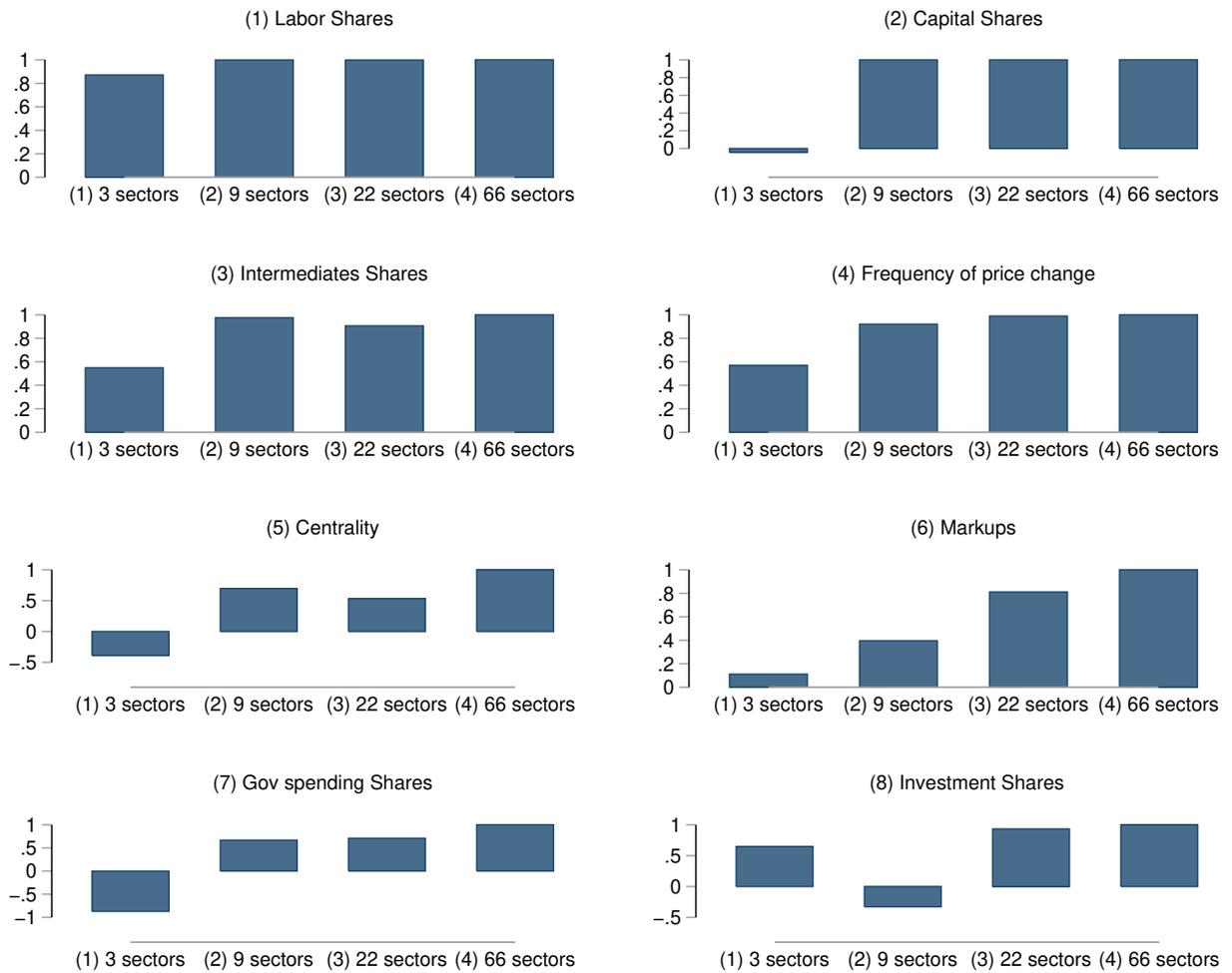


Figure 15. Correlation at Different Levels of Disaggregation Using CEX-IO

Note. We plot the correlation of corresponding expenditure-weighted sectoral features for each households' after-tax income percentiles, aggregating at full I-O sectors (66 sectors), 2-digit NAICS sectors (22 sectors), 1-digit NAICS sectors (9 sectors), and a manufacturing/agriculture/services split (3 sectors). We use the 66 sectors data points as the base.

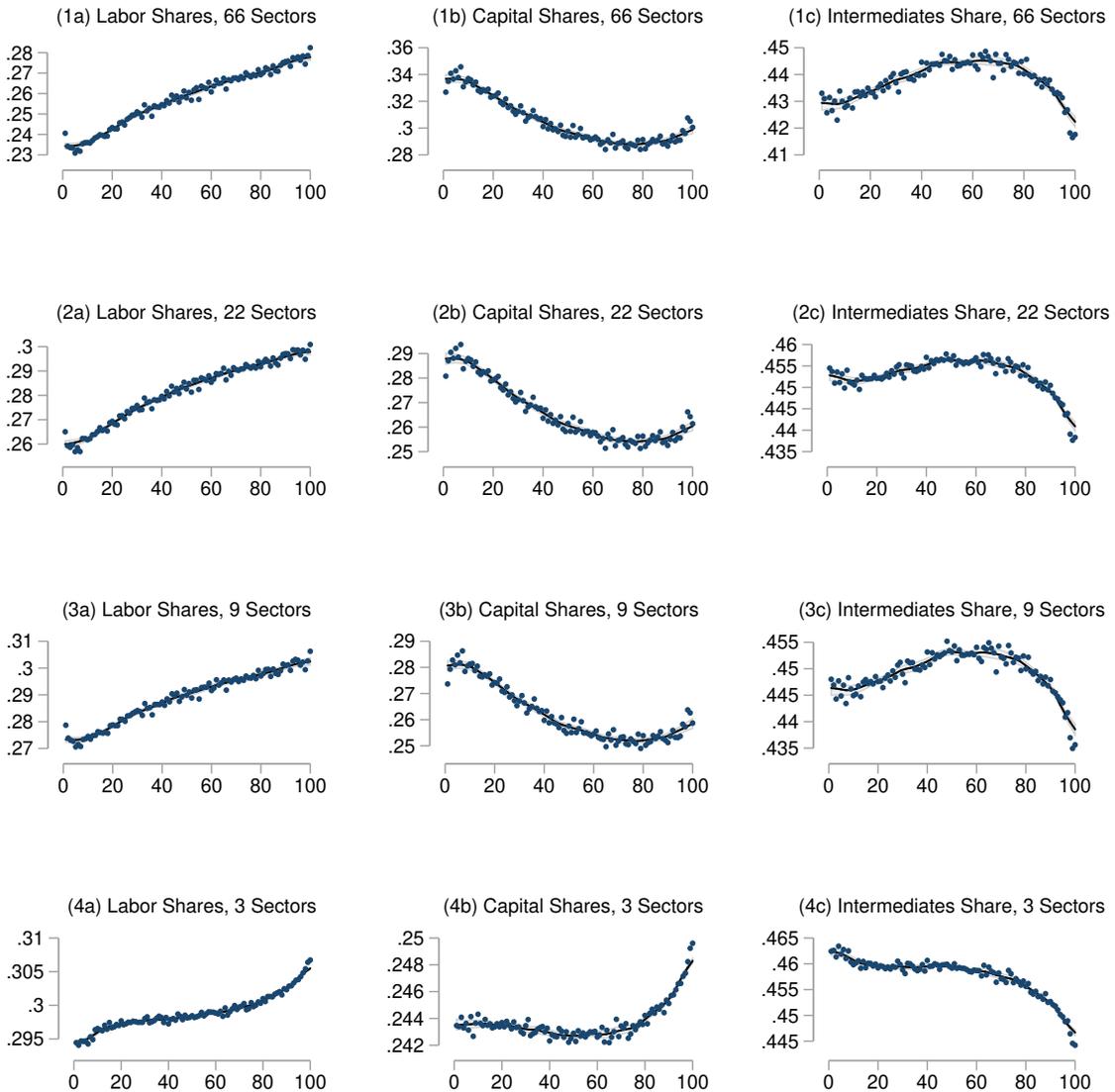


Figure 16. Robustness Checks for Expenditure-weighted Factor Shares Across Income

Note. We plot the expenditure-weighted factor shares (labor shares, capital shares, and intermediates shares) for 100 income percentiles across dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy. The line is the local polynomial fit, and the shaded area is the 95% confidence interval.

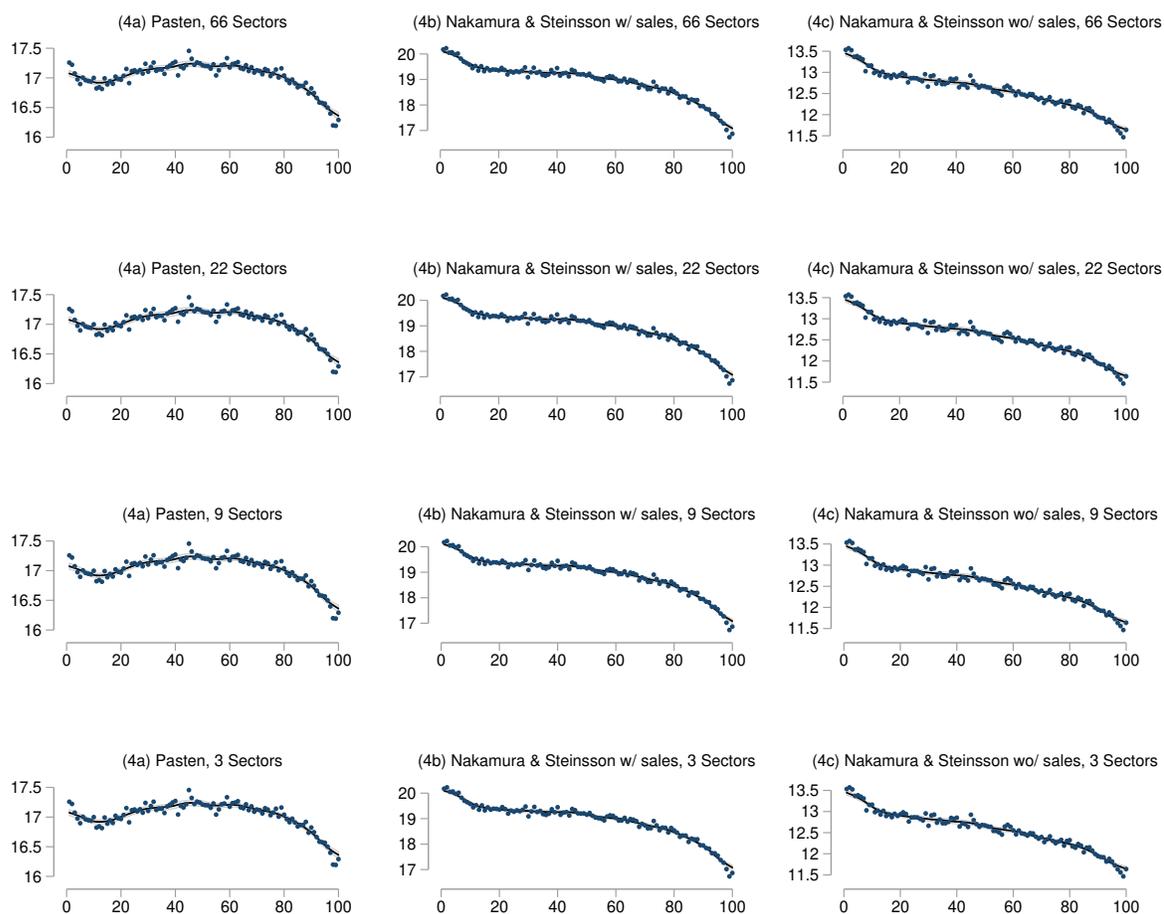


Figure 17. Robustness Checks for Expenditure-weighted Frequency of Price Change Across Income

Note. We plot the expenditure-weighted monthly frequency of price change for 100 income percentiles across two dimensions: different price rigidity measures and dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy. The line is the local polynomial fit, and the shaded area is the 95% confidence interval.

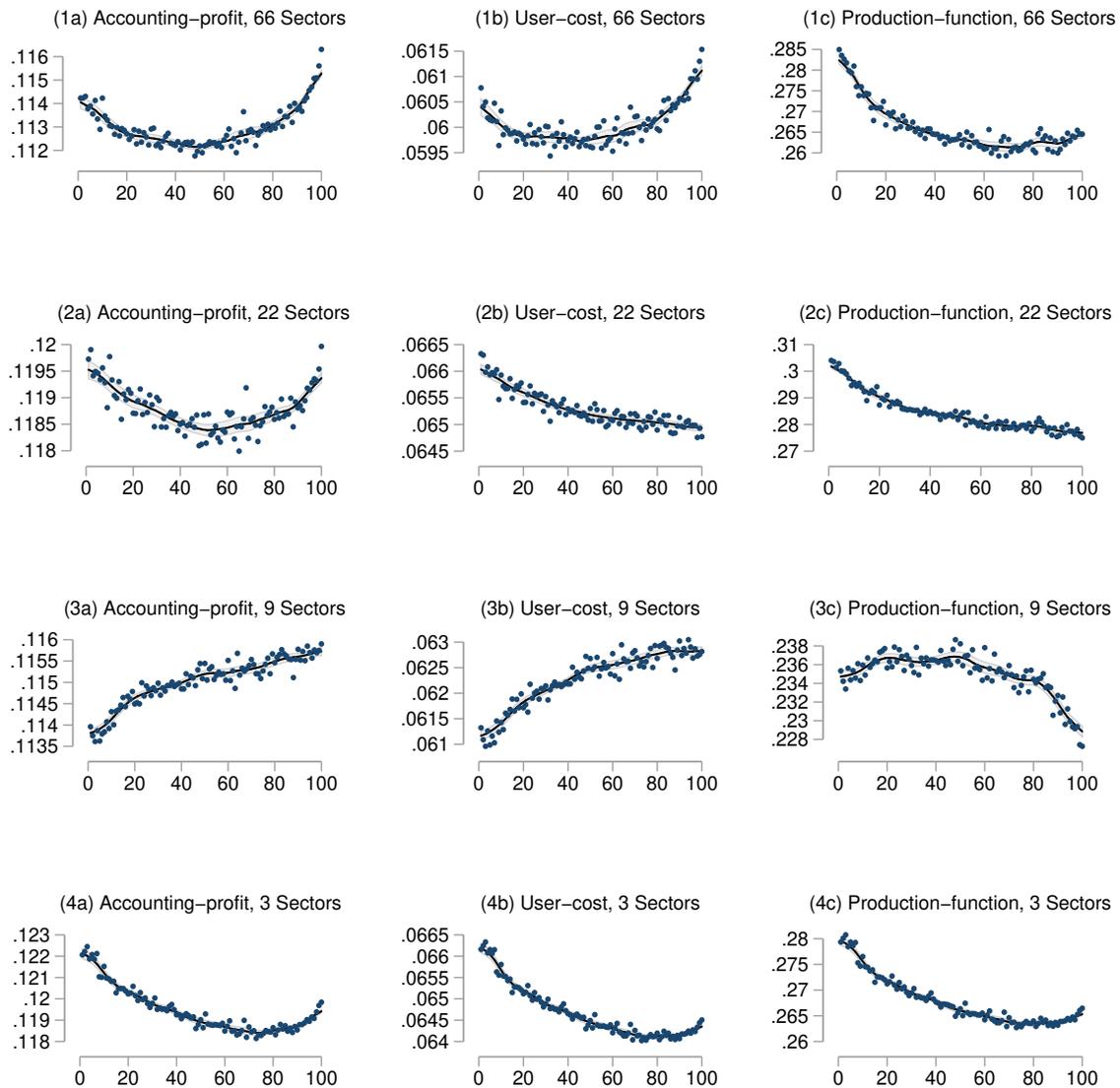


Figure 18. Robustness Checks for Expenditure-weighted Markups Across Income

Note. We plot the expenditure-weighted markups for 100 income percentiles across two dimensions: different markups measures and dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy. The line is the local polynomial fit, and the shaded area is the 95% confidence interval.

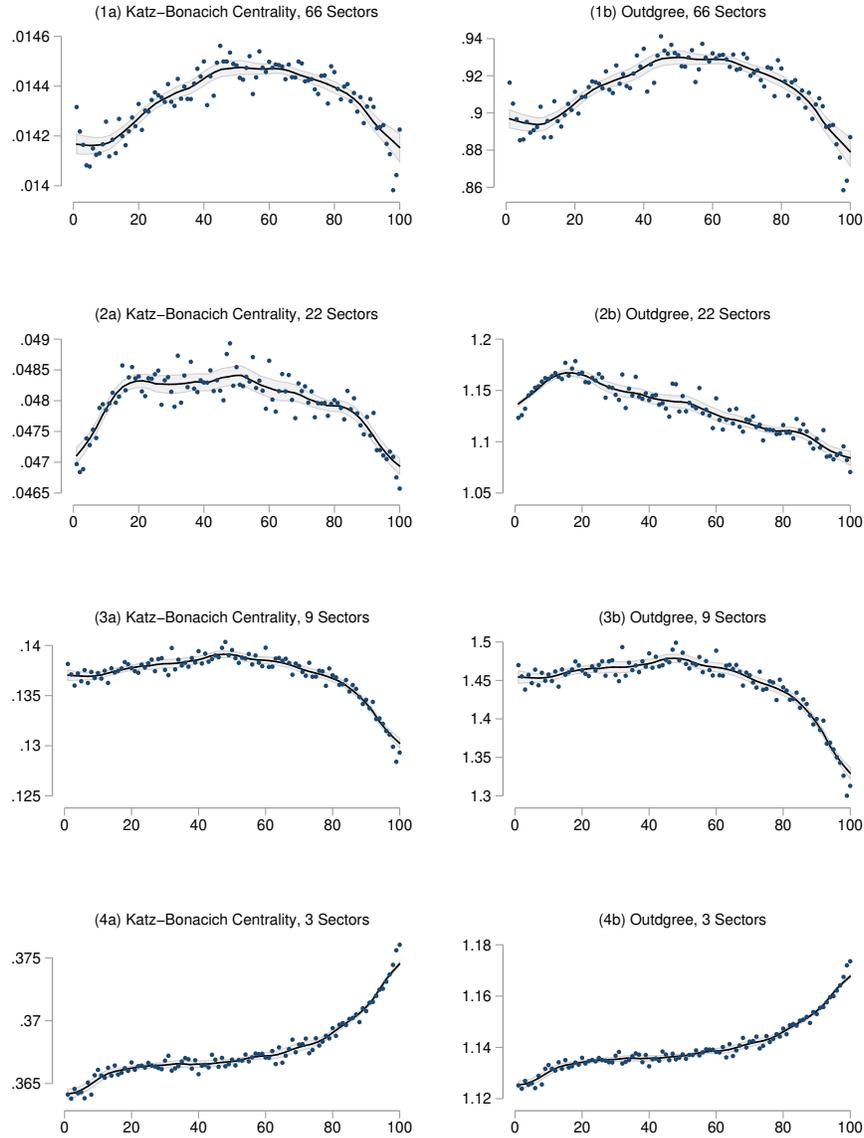


Figure 19. Robustness Checks for Expenditure-weighted Centrality Across Income

Note. We plot the expenditure-weighted centrality for 100 income percentiles across two dimensions: different production network centrality measures and dis-aggregation levels of the economy. Panel (1a) - (2a) are for 66-sector economy with two measures of centrality; panel (1b) - (2b) are for a 22-sector economy; panel (1c) - (2c) are for a 3-sector economy. The line is the local polynomial fit, and the shaded area is the 95% confidence interval.

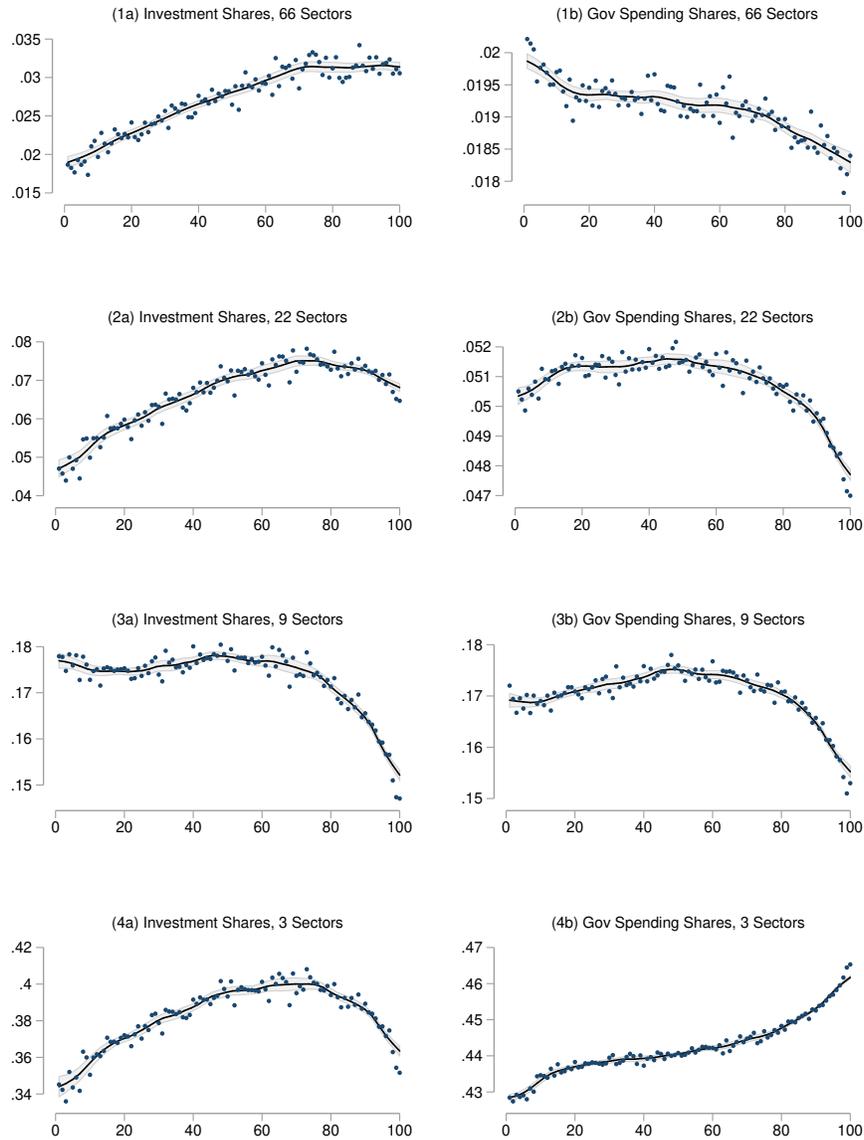


Figure 20. Robustness Checks for Expenditure-weighted Investment and Gov Spending Shares Across Income

Note. We plot the expenditure-weighted investment shares and government spending shares for 100 income percentiles across three dis-aggregation levels of the economy. Panel (1a) - (2a) are for 66-sector economy with investment and government spending shares respectively; panel (1b) - (2b) are for a 22-sector economy; panel (1c) - (2c) are for a 3-sector economy. The line is the local polynomial fit, and the shaded area is the 95% confidence interval.

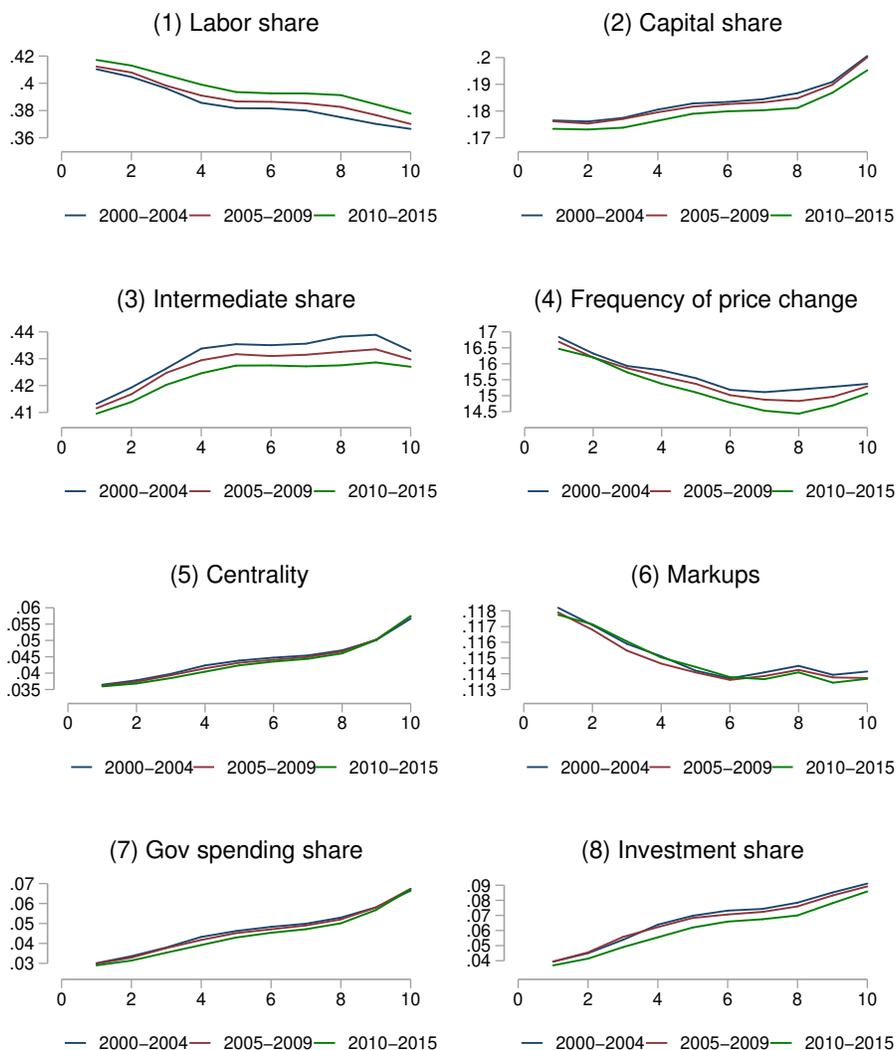


Figure 21. Earnings-share Weighted Sectoral Heterogeneity for Income Percentiles with Different Sample Periods in the 22-sector Economy

Note. We plot the earnings-weighted sectoral features as a function of households' total income percentile, average over the sample period (1) from 2000-2004 in blue, (2) from 2005-2009 in red, and (3) from 2010-2015 in green. The horizontal axis for each panel is household percentiles, each representing 10% of the population. The vertical axis reports the average sectoral features, such as labor share, capital share, intermediate share, frequency of price change, centrality, markups, government spending share, and investment share, weighted by the earnings share from 22 sectors by households of the corresponding income percentile.

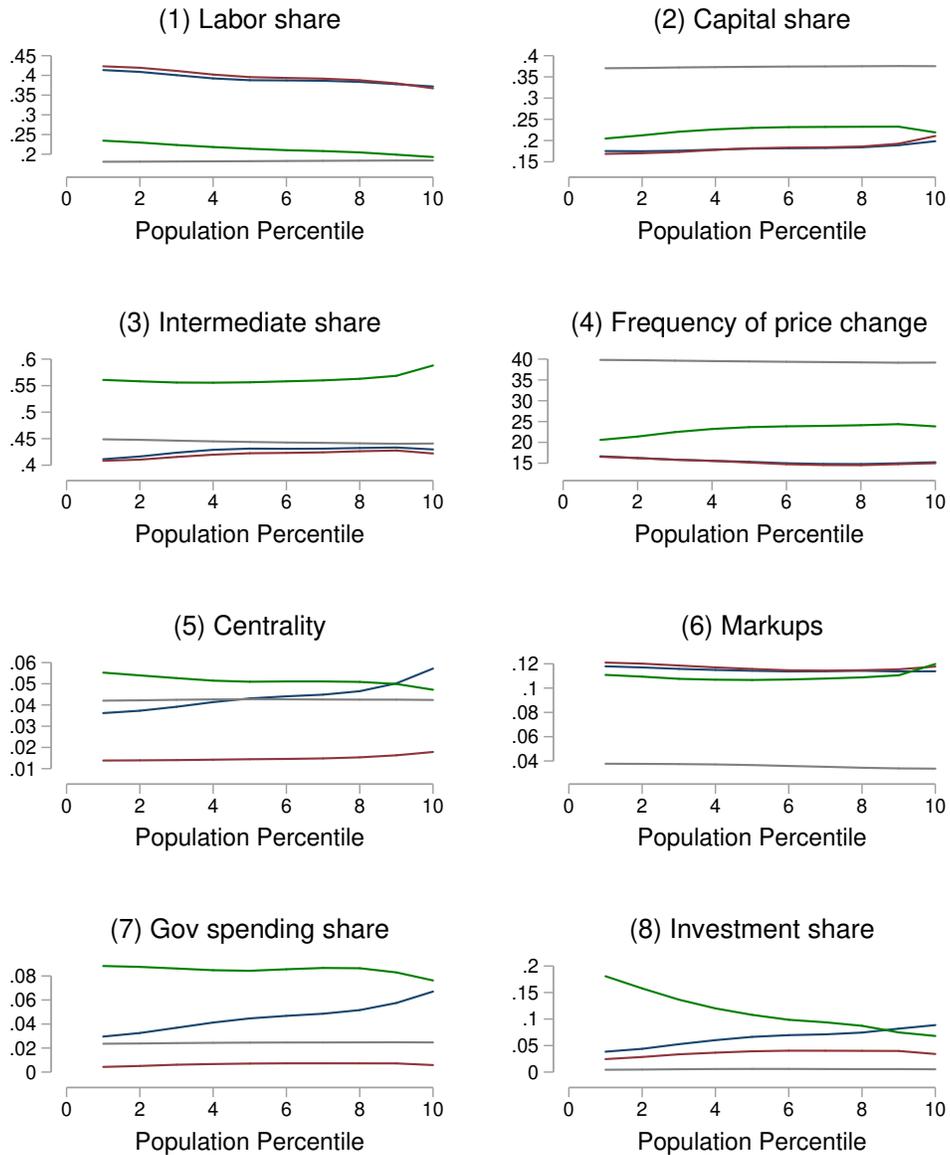


Figure 22. Earnings-share Weighted Sectoral Heterogeneity for Total Income Percentiles with Different Levels of Aggregation

Note. We plot the expenditure-weighted sectoral features as a function of households' after-tax income percentile, average over the sample period. The horizontal axis for each panel is household percentiles, each representing 1% of the population. The vertical axis reports the average sectoral features, such as labor share, capital share, intermediate share, frequency of price change, centrality, markups, government spending share, and investment share, weighted by the expenditure share by households of the corresponding income percentile across 66 sectors in red, 22 sectors in blue, 9 sectors in green, and 3 sectors in gray.

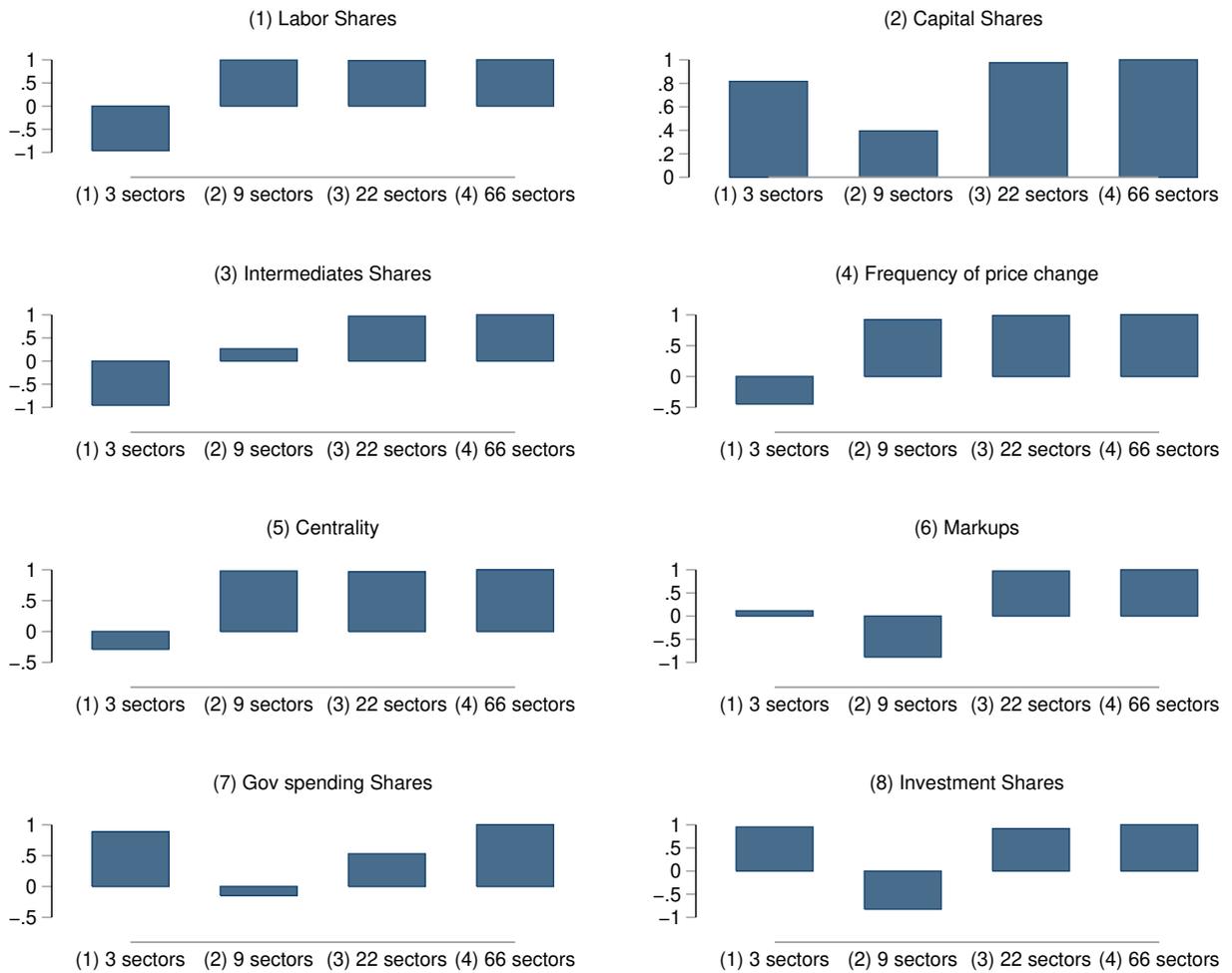


Figure 23. Correlation at Different Levels of Disaggregation Using ACS-IO

Note. We plot the correlation of corresponding earnings-weighted sectoral features for each households' total income percentiles, aggregating at full I-O sectors (66 sectors), 2-digit NAICS sectors (22 sectors), 1-digit NAICS sectors (9 sectors), and a manufacturing/agriculture/services split (3 sectors). We use the 66 sectors data points as the base.

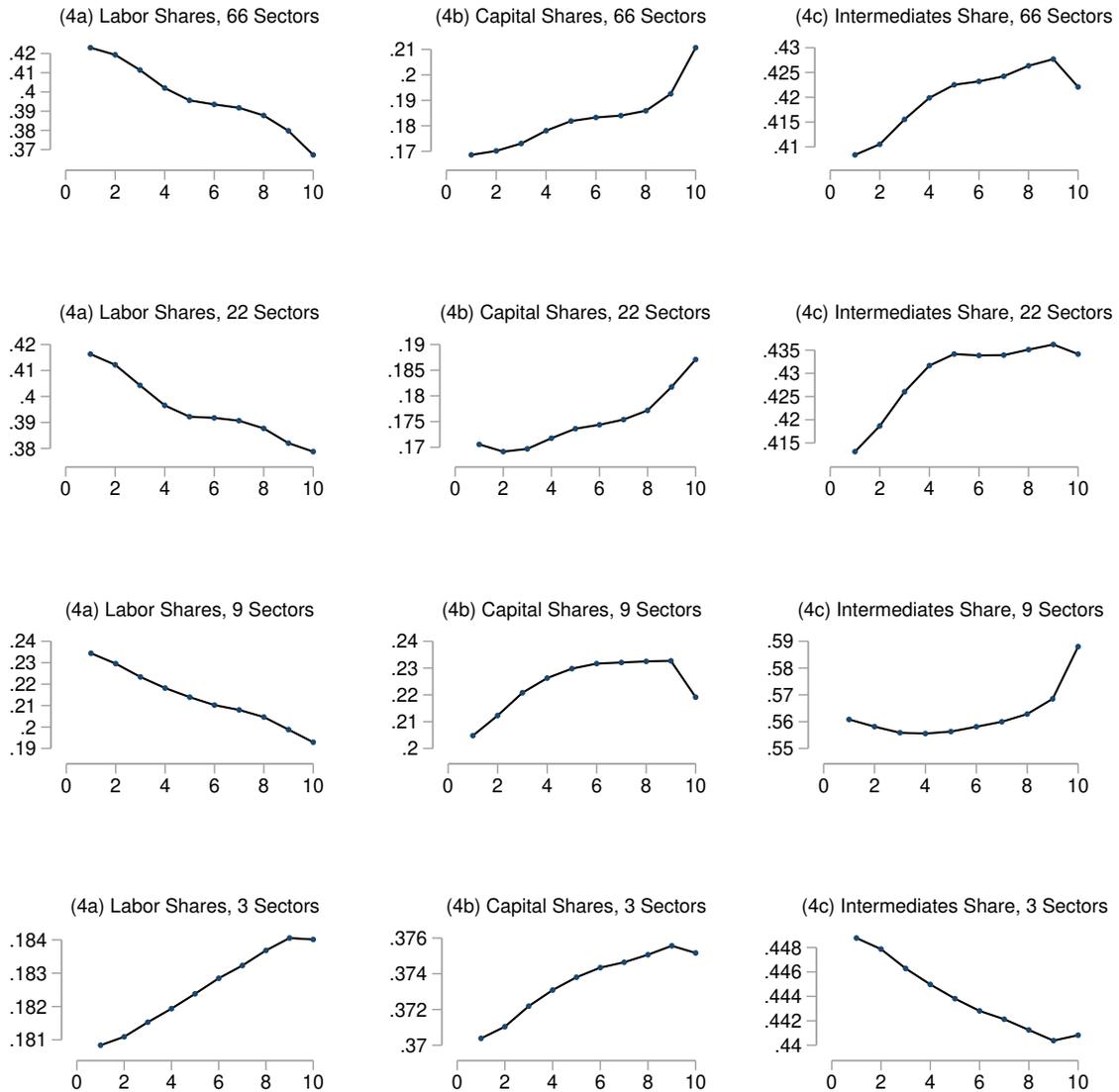


Figure 24. Robustness Checks for Earnings-weighted Factor Shares Across Income

Note. We plot the earnings-weighted factor shares (labor shares, capital shares, and intermediates shares) for 10 income percentiles across dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy.

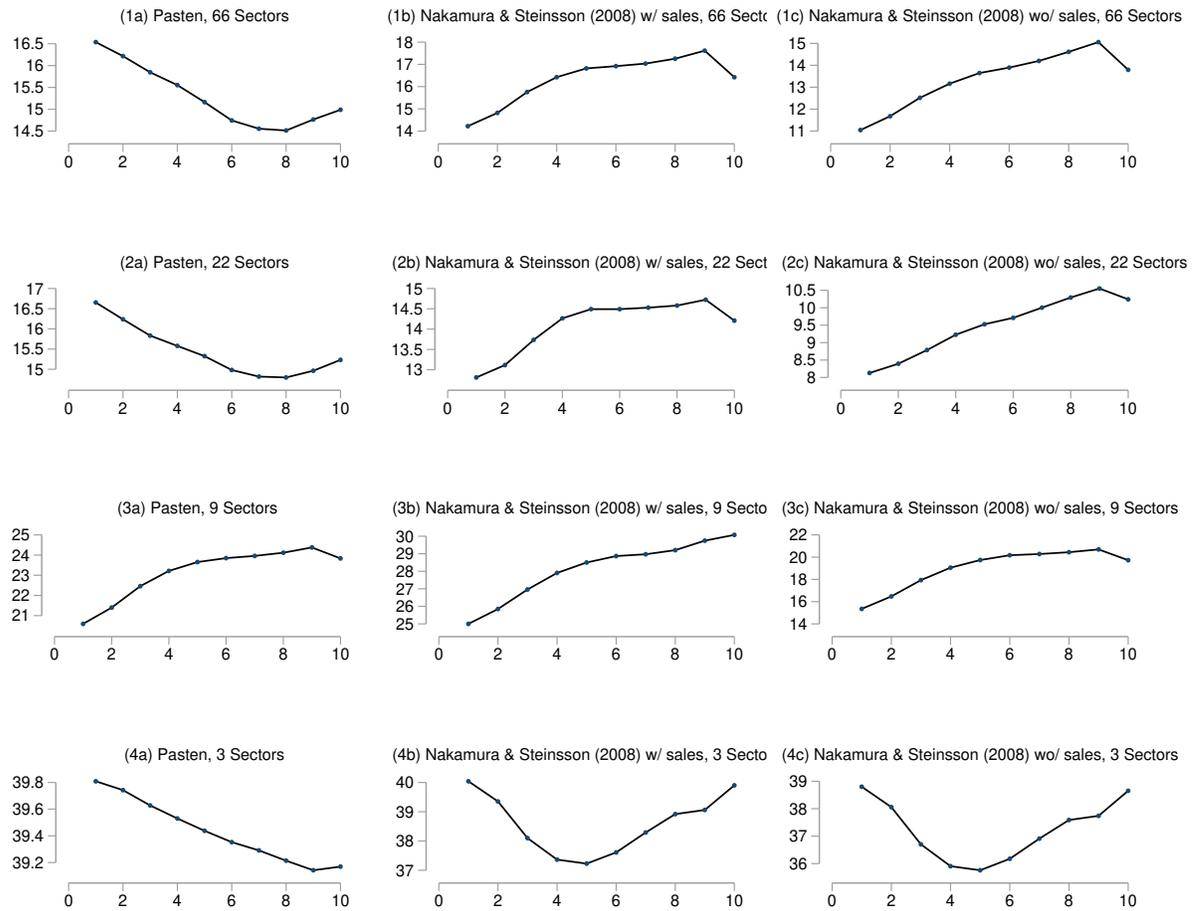


Figure 25. Robustness Checks for Earnings-weighted Frequency of Price Change Across Income

Note. We plot the earnings-weighted monthly frequency of price change for 10 income percentiles across two dimensions: different price rigidity measures and dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy.

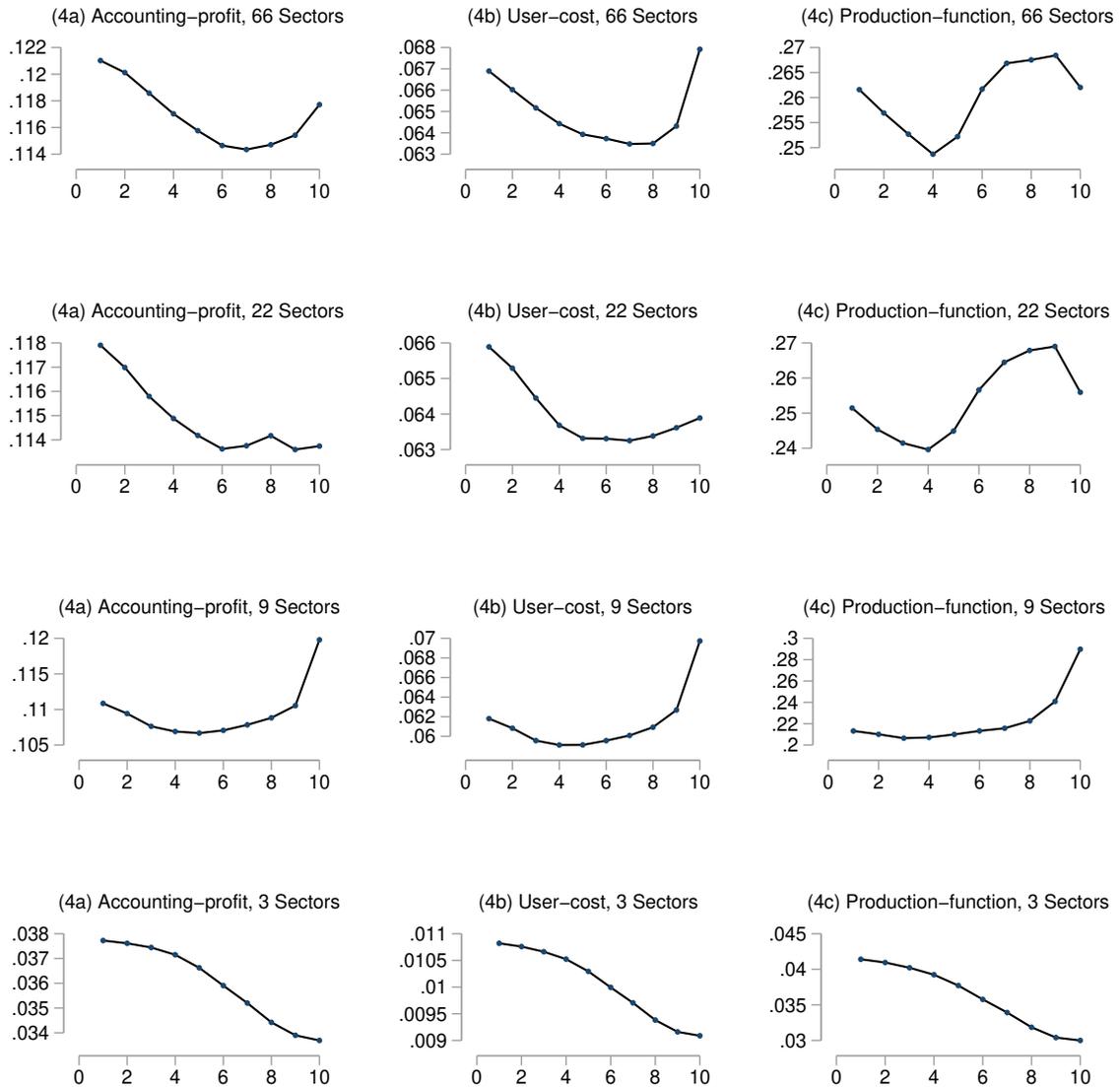


Figure 26. Robustness Checks for Earnings-weighted Markups Across Income

Note. We plot the earnings-weighted markups for 10 income percentiles across two dimensions: different markup measures and dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy.

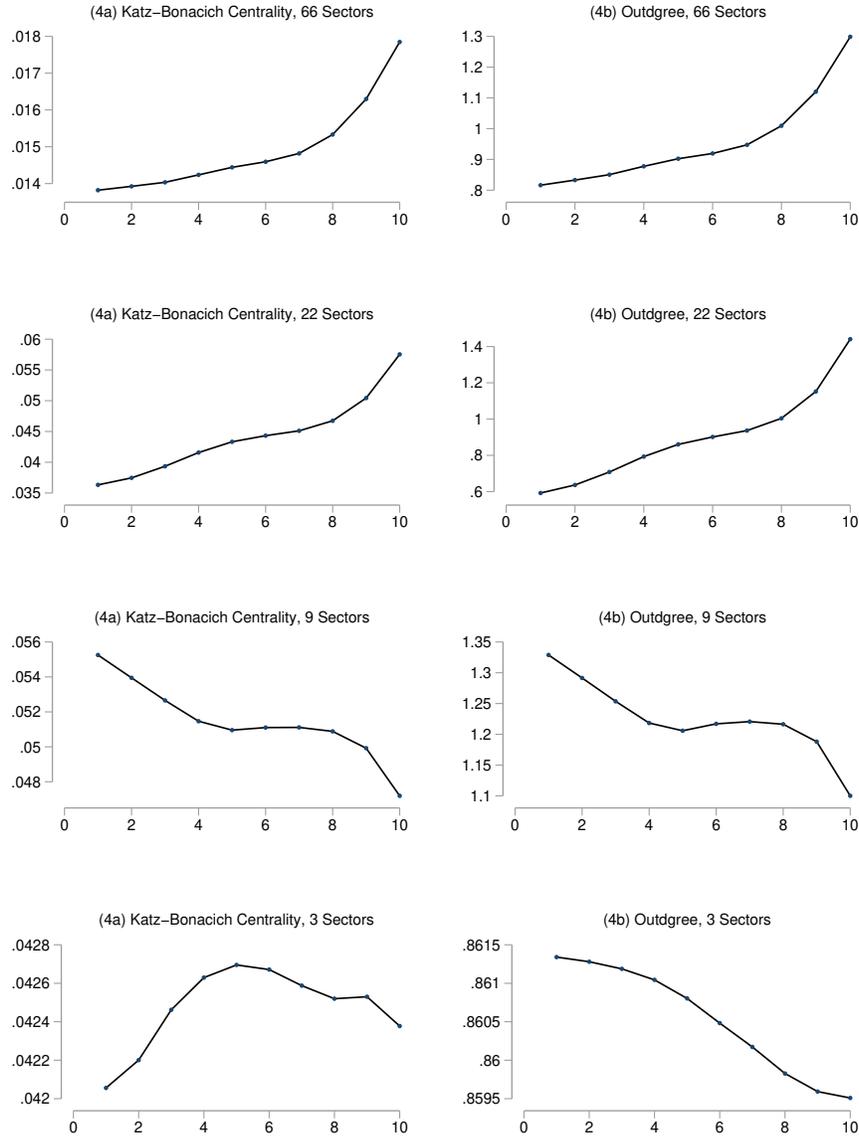


Figure 27. Robustness Checks for Earnings-weighted Centrality Across Income

Note. We plot the earnings-weighted centrality for 10 income percentiles across two dimensions: different production network centrality measures and dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy.

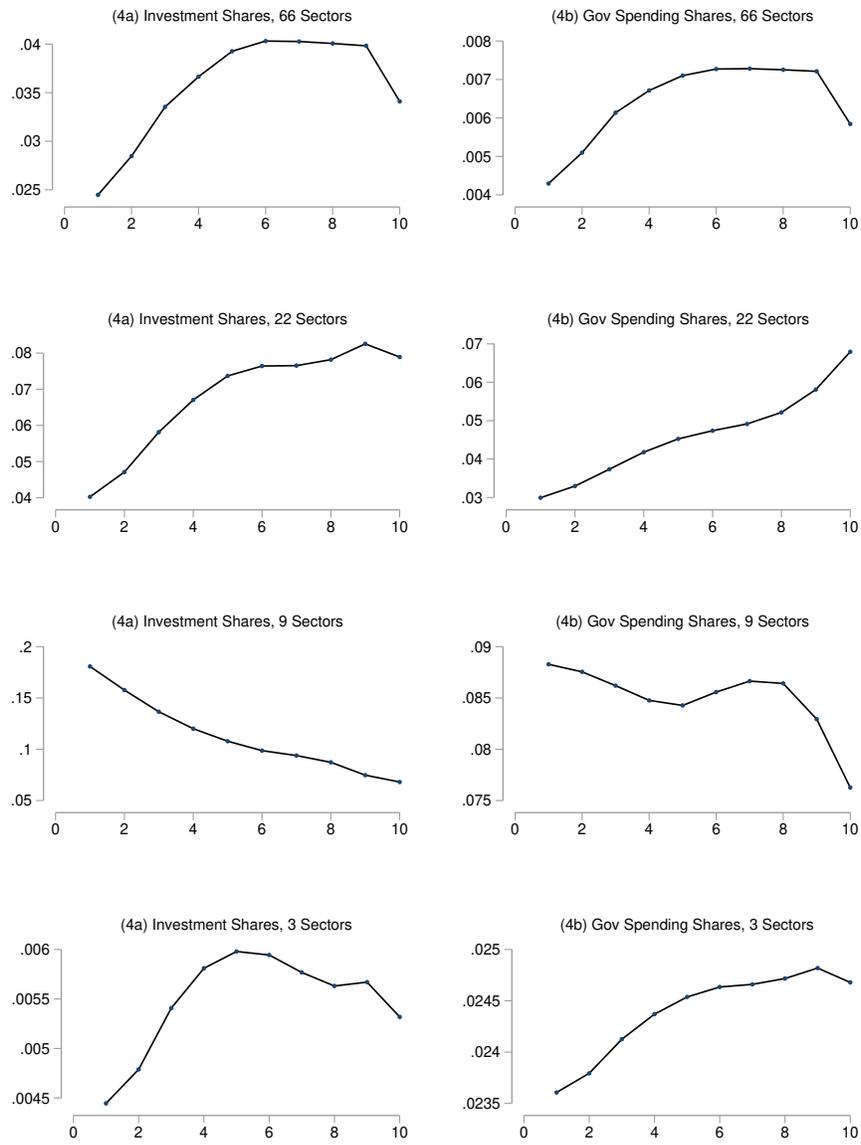


Figure 28. Robustness Checks for Earnings-weighted Investment and Gov Spending Shares Across Income

Note. We plot the earnings-weighted investment shares and government spending shares for 10 income percentiles across three dis-aggregation levels of the economy. Panel (1a) - (1c) are for 66-sector economy with three factor shares; panel (2a) - (2c) are for a 22-sector economy; panel (3a) - (3c) are for a 9-sector economy; panel (4a) - (4c) are for a 3-sector economy.

Sectors	Sectoral Features								1-10th percentile	4th percentile
	Capital share	Labor share	Intermediates share	Freq. price change	Centrality	Markups	Gov spending share	Investment share		
Accommodation and food services	0.154	0.395	0.451	27.266	0.032	0.163	0.015	0.000	17.18%	3
Retail trade	0.187	0.468	0.345	21.172	0.023	0.085	0.000	0.022	18.35%	1
Other services	0.164	0.453	0.384	4.643	0.031	0.114	0.026	0.000	7.14%	5
Administrative and waste	0.155	0.458	0.386	13.926	0.059	0.096	0.068	0.000	5.46%	3
Education services	0.126	0.552	0.322	5.516	0.025	0.099	0.010	0.000	9.54%	1
Arts and entertainment	0.240	0.385	0.375	5.025	0.029	0.099	0.004	0.002	3.74%	1
Agriculture	0.287	0.116	0.597	52.065	0.041	0.130	0.007	0.000	2.32%	1
Manufacturing I	0.139	0.150	0.711	18.429	0.038	0.178	0.060	0.001	1.56%	2
Rental and leasing	0.518	0.119	0.363	18.132	0.032	0.202	0.009	0.000	0.39%	0
Warehousing	0.120	0.468	0.413	6.933	0.029	0.115	0.003	0.004	0.21%	0
Construction	0.171	0.336	0.493	20.221	0.030	0.105	0.062	0.325	5.29%	8
Transportation	0.177	0.316	0.507	21.376	0.035	0.151	0.030	0.004	3.25%	4
Healthcare and social assistance	0.102	0.515	0.384	6.887	0.023	0.124	0.010	0.000	11.25%	1
Mining	0.385	0.183	0.433	38.806	0.048	0.045	0.029	0.035	0.17%	0
Utilities	0.374	0.188	0.438	38.806	0.040	0.024	0.024	0.000	0.12%	0
Housing	0.648	0.062	0.290	6.983	0.065	0.115	0.044	0.005	1.12%	1
Information	0.330	0.234	0.436	18.675	0.045	0.217	0.076	0.059	1.77%	2
Wholesale trade	0.257	0.397	0.346	8.879	0.026	0.033	0.000	0.054	1.75%	3
Manufacturing II	0.214	0.144	0.641	19.775	0.084	0.139	0.200	0.004	1.34%	3
Manufacturing III	0.155	0.250	0.595	13.870	0.085	0.111	0.098	0.297	2.88%	8
Finance and insurance	0.230	0.316	0.454	31.699	0.077	0.115	0.076	0.001	1.82%	9
Professional and technical	0.174	0.467	0.359	7.133	0.105	0.113	0.148	0.189	3.34%	9
<i>Mean</i>	0.241	0.317	0.442	18.464	0.045	0.117	0.045	0.045		
<i>Max</i>	0.648	0.552	0.711	52.065	0.105	0.217	0.200	0.325		
<i>Min</i>	0.102	0.062	0.290	4.643	0.023	0.024	0.000	0.000		

Table 3. Earnings Share Differences and Sectoral Features, 22 Sectors

Note. Table 3 reports summary statistics for sectoral features across all 22 production sectors. Sectors are listed in ascending order according to the difference in payroll shares between the top 10% income percentile and the bottom 10% income percentile. The light blue shade helps visualize each sector's sectoral features relative to the respective range across all sectors.