Welfare Assessments with Heterogeneous Individuals*

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Abstract

This paper introduces an exact decomposition of welfare assessments for general dynamic stochastic economies with heterogeneous individuals into four distinct components: i) aggregate efficiency, ii) intertemporal-sharing, iii) risk-sharing, and iv) redistribution. For welfarist planners, the decomposition is based on constructing individual, dynamic, and stochastic weights that characterize how a planner makes tradeoffs across individuals, dates, and histories. For DS-planners, such weights are defined as a primitive of the welfare assessment, which allows for a systematic formalization of new welfare criteria based on the decomposition. Three applications illustrate the value of the decomposition to address substantive issues in welfare analysis.

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1 Introduction

Assessing the aggregate welfare impact of policies or shocks in dynamic stochastic economies with heterogeneous individuals and imperfect financial markets is far from trivial. In particular, it is challenging to identify the specific normative considerations that underlie a particular welfare assessment. This paper tackles this challenge by developing a decomposition of welfare assessments that is based on individual, dynamic, and stochastic weights and satisfies desirable properties.

We introduce our results in a canonical dynamic stochastic environment in which heterogeneous individuals consume a single good and supply a single factor (labor) at each history. We initially consider welfare assessments for welfarist planners — those who use a Social Welfare Function. Since comparisons in utils are meaningless — due to the ordinal nature of individual utilities — we first express welfare assessments in terms of normalized individual, dynamic, and stochastic weights, which allow us to interpret how welfarist planners make tradeoffs across individuals, dates, and histories in common units. To define such units, we select welfare numeraires.

After expressing welfare assessments in comparable units, we show how to decompose a welfare assessment into i) an efficiency component and ii) a redistribution component, as illustrated by Figure 1. The efficiency component exactly corresponds to Kaldor-Hicks efficiency: it is the sum of individual willingness-to-pay for the perturbation in units of the lifetime welfare numeraire. Kaldor-Hicks efficiency is typically justified on the grounds of the compensation principle (Boadway and Bruce, 1984). That is, whenever the efficiency component is positive, the winners of the perturbation could hypothetically compensate the losers. For a given lifetime welfare numeraire, this is the unique decomposition in which a normalized welfare assessment can be expressed as the (unweighted) sum of individual willingness-to-pay (efficiency) and its complement (redistribution). The redistribution component, which captures the equity concerns embedded in a particular Social Welfare Function, is positive when those individuals relatively favored in a perturbation are those relatively preferred by the planner, i.e., have higher normalized individual weights.

We establish three properties of the efficiency/redistribution decomposition. First, the efficiency component is identical for all welfarist planners, which implies that all differences in the assessments of welfarist planners are due to redistribution considerations. Second, the efficiency component is invariant to preference-preserving utility transformations, which implies that the impact of preference-preserving utility transformations on welfare assessments is exclusively confined to the redistribution component. Third, the efficiency component is strictly positive for (strict or weak) Pareto-improving perturbations, which guarantees that the efficiency component of the welfare assessment of any feasible perturbation must be negative at Pareto efficient allocations.

By appealing once again to the compensation principle — now at each date and history — we decompose efficiency gains into a component that captures changes in aggregate instantaneous

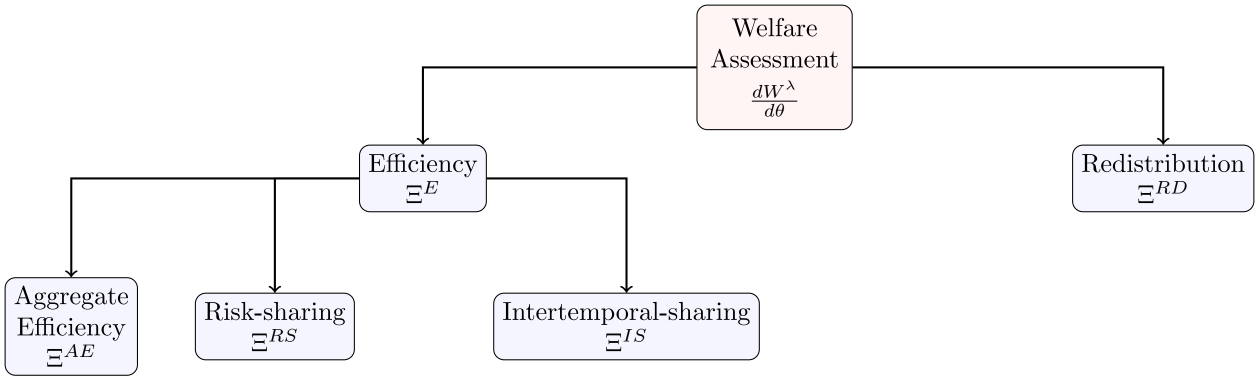

Figure 1: Welfare Assessment Decomposition

Note: This figure illustrates the decomposition of welfare assessments introduced in this paper. See Propositions 1 and 3 for formal definitions of each of the components and Propositions 2, 3, 5, and 6 for properties.

welfare gains (aggregate efficiency) and two components that capture the differential impact of a perturbation towards individuals with different valuations across dates (intertemporal-sharing) and histories (risk-sharing). For given welfare numeraires, this is the unique decomposition in which the efficiency component can be expressed as the discounted sum — using aggregate time and stochastic discount factors — of aggregate instantaneous welfare gains, and its complement.

The differences in individual valuations across dates and histories that govern the risk-sharing and intertemporal-sharing components depend on the extent to which individuals can freely smooth consumption (in general, the instantaneous welfare numeraire) intertemporally and across histories. We hence show that i) the risk-sharing and intertemporal-sharing components are zero when marginal rates of substitution across all dates and histories are equalized across individuals — a condition that complete markets economies satisfy — and ii) the intertemporal-sharing component is zero when marginal rates of substitution across dates are equalized across individuals — a condition satisfied when all individuals can frictionlessly borrow and save.

More generally, we identify conditions on i) normalized weights and ii) welfare gains that guarantee that the risk-sharing, intertemporal-sharing, or redistribution components are zero. Intuitively, normalized weights and welfare gains must vary cross-sectionally along the relevant dimensions for these three components to be non-zero. We also identify particular economies of practical relevance in which specific components of the welfare decomposition are zero. We show that i) single individual economies exclusively feature aggregate efficiency, ii) risk-sharing is zero in perfect foresight economies, iii) intertemporal-sharing and redistribution are zero in economies with ex-ante (but not necessarily ex-post) identical individuals, iv) risk- and intertemporal-sharing are zero in static economies, v) aggregate efficiency is zero in single good endowment economies in which the aggregate endowment is fixed. We also characterize which particular components of the welfare decomposition are zero when planners can costlessly transfer resources among individuals along particular dimensions.

Next, we leverage the welfare decomposition to systematically construct non-welfarist welfare criteria based on individual, dynamic, and stochastic weights. Unlike the welfarist approach — which takes a Social Welfare Function as primitive — welfare assessments by DS-planners are defined in marginal form, which allows us to define normative criteria that the welfarist approach cannot capture. For example, this approach allows us to formalize welfare objectives that isolate specific normative considerations while disregarding others. A central result of this section is a characterization of the properties of the three pseudo-welfarist planners we define: aggregate efficiency (AE), aggregate efficiency/risk-sharing (AR), and no-redistribution (NR) pseudo-welfarist DS-planners. These pseudo-welfarist planners are constructed so that specific (sums of) components of the welfare decomposition for a given welfarist planner can be interpreted as welfare assessments for particular DS-planners.

Section 5 briefly summarizes extensions and additional results covered in the Online Appendix. There, we describe how to extend our results to more general environments, and how to further decompose the components of the welfare decomposition, among other results.

At last, we illustrate how the welfare decomposition introduced in this paper can be used to draw normative conclusions in three applications of practical relevance. Our first application analyzes the welfare effects of a transfer policy that smooths consumption across individuals who face idiosyncratic consumption risk. The central takeaway is that the persistence of the endowment process determines whether welfare gains are attributed to risk-sharing, intertemporal-sharing, or redistribution, even when welfare assessments may be invariant to such persistence. Our results also highlight that a flat term structure of welfare assessments may mask substantial variation on each of its components, with backloaded risk-sharing gains and frontloaded intertemporal-sharing and redistribution gains that turn into losses in the long run.

Our second application contrasts the welfare effects of (linear) labor income taxes in two settings: i) a deterministic environment in which individuals differ in their productivity at the time of the welfare assessment, and ii) a stochastic environment in which individuals are identical at the time of the welfare assessment, but experience different shocks. In both environments, increasing tax rates causes aggregate efficiency losses by distorting labor supply. While both environments can be parameterized to yield a quantitatively identical optimal tax, a utilitarian planner attributes the welfare gains from the tax to redistribution in the deterministic environment and to risk-sharing in the stochastic environment. Moreover, in the stochastic environment *all* welfarist planners agree on the magnitude of the optimal tax, which is Pareto-improving in that case, while in the deterministic environment the optimal tax is sensitive to the choice of welfare function. This application also illustrates that perturbations may yield efficiency gains even though aggregate consumption falls at all times.

Our third application studies the welfare implications of a change in credit conditions in an economy in which borrowing-constrained individuals make an investment decision. Considering changes in the borrowing limit in this economy is a tractable perturbation that parameterizes changes in the degree of market completeness. This application illustrates how relaxing a borrowing constraint can feature at the same time i) positive aggregate efficiency and intertemporal-sharing components, by allowing investors to invest more and by reallocating resources towards borrowing-constrained individuals, and ii) a negative risk-sharing component, since investors end up bearing higher risk by virtue of their increased investment. This application also illustrates how the redistributive implications of a change in credit conditions can i) be traced back to pecuniary effects in competitive economies and ii) vary depending on the level of the borrowing limit.

Related Literature. This paper contributes to several literatures, including those on i) welfare decompositions, ii) welfare evaluation of policies in dynamic stochastic environments, iii) interpersonal welfare comparisons, and iv) institutional mandates.

The welfare decomposition introduced in this paper is most related to the work that seeks to decompose welfare changes in models with heterogeneous agents. The most recent contribution to this literature is the work by Bhandari et al. (2021), who propose a decomposition of welfare changes when switching from a given policy to another that can be applied to a larger set of economies than the earlier contributions of Benabou (2002), Floden (2001), and Seshadri and Yuki (2004), among others.[1] We explain how our decomposition differs from these in Section G.3 of the Online Appendix.

Our results are also related to the consumption-equivalent approach introduced by Lucas (1987), in particular to its marginal formulation in Alvarez and Jermann (2004). We show that our results nest the marginal approach to making welfare assessments in representative agent economies of Alvarez and Jermann (2004) in Section G.2 of the Online Appendix.

The question of how to make interpersonal welfare comparisons to form aggregate welfare assessments has a long history in economics — see, among many others, Kaldor (1939), Hicks (1939), Bergson (1938), Samuelson (1947), Harsanyi (1955), Sen (1970) or, more recently, Kaplow and Shavell (2001), Saez and Stantcheva (2016), Hendren (2020), Schulz, Tsyvinski and Werquin (2023), and Hendren and Sprung-Keyser (2020). However, perhaps surprisingly, dynamic stochastic considerations have not been central to this literature. By introducing normalized weights, our results provide a new characterization of how welfarist planners make tradeoff across individuals,

[1] Benabou (2002) states that:

> *"standard social welfare functions (...) cannot distinguish between the effects of policy that operate through its role as a substitute for missing markets, and those that reflect an implicit equity concern."*

Our results show that it is actually possible to distinguish between the welfare effects of policy that capture the contribution for missing markets and those that reflect equity concerns when using standard Social Welfare Functions.

dates, and histories. The introduction of DS-planners in Section 4 generalizes the work of Saez and Stantcheva (2016) by allowing for welfare criteria based not only on individual generalized weights, but also dynamic and stochastic generalized weights — see also Section G.1 of the Online Appendix.

Finally, we hope that understanding how DS-planners — introduced in Section 4 — make welfare assessments opens the door to future disciplined discussions on policy-making mandates. For instance, while Rogoff (1985) shows that a particular institutional mandate (a conservative central banker) may be at times desirable in a representative agent framework, our results allow to define institutional mandates that incorporate or disregard specific cross-sectional considerations, such as risk-sharing, intertemporal-sharing, or redistribution.

2 Environment

Our notation closely follows that of Ljungqvist and Sargent (2018). We consider an economy populated by a finite number $I \geq 1$ of individuals, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$. At each date $t \in \{0, \ldots, T\}$, where $T \leq \infty$, there is a realization of a stochastic event $s_t \in S$. We denote the history of events up to date t by $s^t = (s_0, s_1, \ldots, s_t)$, and the probability of observing a particular sequence of events s^t by $\pi_t \left(s^t \middle| s_0 \right)$. The initial value of s_0 is predetermined, so $\pi_0 \left(s^0 | s_0 \right) = 1$. At all dates and histories, individuals consume a single good and supply a single factor, e.g. labor.

Preferences. An individual i derives utility from consumption and (dis)utility from factor supply, with a lifetime utility representation, starting from s_0, given by

$$V^i = \sum_t \left(\beta^i \right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0 \right) u_t^i \left(c_t^i \left(s^t \right), n_t^i \left(s^t \right); s^t \right), \qquad \text{(Preferences)} \tag{1}$$

where $c_t^i \left(s^t \right)$ and $n_t^i \left(s^t \right)$ respectively denote the consumption and factor supply of individual i at history s^t. $u_t^i \left(\cdot ; s^t \right)$ corresponds to individual i's instantaneous utility at history s^t, with $\frac{\partial u_t^i(s^t)}{\partial c_t^i} = \frac{\partial u_t^i(c_t^i(s^t), n_t^i(s^t); s^t)}{\partial c_t^i(s^t)} > 0$ and $\frac{\partial u_t^i(s^t)}{\partial n_t^i} = \frac{\partial u_t^i(c_t^i(s^t), n_t^i(s^t); s^t)}{\partial n_t^i(s^t)} < 0$, and $\beta^i \in [0, 1)$ denotes individual i's discount factor. We refer to the unit of V^i as individual i utils.

Equation (1) corresponds to the time-separable expected utility preferences with exponential discounting and homogeneous beliefs widely used in macroeconomics and finance, augmented to allow for time- and history-dependent individual-specific preferences. Section D of the Online Appendix considers more general environments.

Perturbation. We assume that $c_t^i \left(s^t \right)$ and $n_t^i \left(s^t \right)$ are smooth functions of a perturbation parameter $\theta \in [0, 1]$, so derivatives such as $\frac{dc_t^i(s^t)}{d\theta}$ and $\frac{dn_t^i(s^t)}{d\theta}$ are well-defined. A perturbation $d\theta$ captures changes in policies or any other primitive in a fully specified model. Typically, the mapping between consumption and factor supply, $c_t^i(s^t)$ and $n_t^i(s^t)$, and θ — which we take as given — emerges endogenously and accounts for general equilibrium effects, as we illustrate in our applications. However, our results do not require to further specify technologies, resource or budget constraints, equilibrium notions, etc.

Social Welfare Function. Until Section 4, we study welfare assessments for welfarist planners, that is, planners with a Social Welfare Function (SWF) given by

$$W = \mathcal{W}\left(V^1, \dots, V^i, \dots, V^I\right), \qquad \text{(Social Welfare Function)} \tag{2}$$

where individual lifetime utilities V^i are defined in (1).[2] We refer to the units of W as social utils. In the body of the paper, we assume that $\frac{\partial \mathcal{W}}{\partial V^i} > 0$, $\forall i$. Section D.5 of the Online Appendix allows for $\frac{\partial \mathcal{W}}{\partial V^i} = 0$ for some individuals.

A welfarist planner finds a perturbation $d\theta$ desirable (undesirable) if

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \frac{dV^i}{d\theta} > (<)\, 0.$$

The welfarist approach is widely used because it is Paretian, that is, it concludes that every Pareto-improving perturbation is desirable.[3] However, because individual utilities are ordinal, understanding how a welfarist planner makes tradeoffs in comparable units is not straightforward, as we show next.

3 Welfare Assessment Decomposition: Welfarist Planners

In this section, we present the central result of the paper: a decomposition of welfare assessments for welfarist planners that satisfies desirable properties.

3.1 Normalized Welfare Assessment

In order to introduce the decomposition, it is necessary to understand how a welfarist planner values welfare gains across individuals, dates, and histories. Since comparisons in utils are meaningless — due to the ordinal nature of individual utilities — we choose comparable units to express welfare

[2] As in Boadway and Bruce (1984) or Kaplow (2011), we refer to the use of SWFs — typically traced back to Bergson (1938) and Samuelson (1947) — as the *welfarist* approach. As explained in Kaplow (2011), the critical restriction implied by the welfarist approach is that the social welfare function $\mathcal{W}(\cdot)$ cannot depend on any model outcomes besides individual utility levels. The utilitarian SWF, which adds up a (weighted) sum of individual utilities, is the most used in practice. See Mas-Colell, Whinston and Green (1995), Kaplow (2011), or Adler and Fleurbaey (2016) for descriptions of alternative SWFs.

[3] A perturbation is strictly (weakly) Pareto-improving if every individual i is strictly (weakly) better off after the policy change. Formally, a policy change is strictly (weakly) Pareto-improving when $\frac{dV^i}{d\theta} > (\geq)\, 0$, $\forall i$. Even though Pareto improvements are unambiguously desirable, they are rare, in particular in economies with many individuals.

gains. We refer to these units as lifetime, date, and instantaneous welfare numeraires. Lemma 1 thus represents welfare assessments in terms of normalized welfare gains and normalized weights. Normalized welfare gains represent lifetime, date, and instantaneous welfare gains for different individuals in welfare numeraire units. Normalized weights capture how welfarist planners make tradeoffs in such common units.

Goods or factors (or bundles of goods or factors) that can be easily transferred across individuals either privately or by a planner, at least hypothetically, are natural welfare numeraires since they justify the use of the compensation principle — see Propositions 1 and 3 for applications of this principle. Therefore, in economies with a single consumption good like the one considered here, it is natural to aggregate and compare welfare gains in consumption-equivalents. That is, it is natural to choose a unit of the consumption good as the instantaneous welfare numeraire at each history (history-s^t consumption), a unit of the consumption good at all histories at a given date as the date welfare numeraire at each date (date-t consumption), and a unit of the consumption good at all dates and histories as the lifetime welfare numeraire (permanent consumption). Hence, to simplify the exposition, we adopt such consumption-based welfare numeraires in the body of the paper. Section F.1 of the Online Appendix considers general welfare numeraires.

Given the choice of consumption-based welfare numeraires, Lemma 1 expresses welfare assessments in terms of the inputs of the components of the welfare decomposition: normalized lifetime, date, and instantaneous welfare gains, and normalized individual, dynamic, and stochastic weights. In terms of notation, variables indexed by λ are expressed in the appropriate numeraire.

Lemma 1. *(Normalized Welfare Gains and Normalized Weights) A normalized welfare assessment for a welfarist planner can be represented as*

$$\frac{dW^{\lambda}}{d\theta}=\frac{\frac{dW}{d\theta}}{\frac{1}{I}\sum_i\frac{\partial\mathcal{W}}{\partial V^i}\lambda^i}=\sum_i\omega^i\frac{dV^{i|\lambda}}{d\theta}, \tag{3}$$

where $\lambda^i=\sum_t\left(\beta^i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_t^i(s^t)}{\partial c_t^i}$. *Here,* $\frac{dV^{i|\lambda}}{d\theta}$, $\frac{dV_t^{i|\lambda}}{d\theta}$, *and* $\frac{dV_t^{i|\lambda}(s^t)}{d\theta}$ *respectively denote lifetime, date, and instantaneous welfare gains, given by*

$$\frac{dV^{i|\lambda}}{d\theta}=\sum_t\omega_t^i\frac{dV_t^{i|\lambda}}{d\theta} \qquad \text{(Normalized Lifetime Welfare Gains)} \tag{4}$$

$$\frac{dV_t^{i|\lambda}}{d\theta}=\sum_{s^t}\omega_t^i\left(s^t\right)\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} \qquad \text{(Normalized Date Welfare Gains)} \tag{5}$$

$$\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}=\frac{dc_t^i\left(s^t\right)}{d\theta}+\frac{\frac{\partial u_t^i(s^t)}{\partial n_t^i}}{\frac{\partial u_t^i(s^t)}{\partial c_t^i}}\frac{dn_t^i\left(s^t\right)}{d\theta}. \qquad \text{(Normalized Instantaneous Welfare Gains)} \tag{6}$$

And ω^i, ω_t^i, and $\omega_t^i(s^t)$ respectively denote normalized individual, dynamic, and stochastic weights, given by

$$\omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i} \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}}{\frac{1}{I} \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}} \qquad \text{(Normalized Individual Weight)} \qquad (7)$$

$$\omega_t^i = \frac{\left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t | s_0\right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}}{\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}} \qquad \text{(Normalized Dynamic Weight)} \qquad (8)$$

$$\omega_t^i\left(s^t\right) = \frac{\pi_t\left(s^t | s_0\right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}}{\sum_{s^t} \pi_t\left(s^t | s_0\right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}}. \qquad \text{(Normalized Stochastic Weight)} \qquad (9)$$

Normalized lifetime welfare gains for individual i, $\frac{dV^{i|\lambda}}{d\theta}$, have the interpretation of i's willingness-to-pay for the perturbation in units of the lifetime welfare numeraire (a unit of permanent consumption). Normalized date welfare gains for individual i at date t, $\frac{dV_t^{i|\lambda}}{d\theta}$, correspond to i's willingness-to-pay for the perturbation at date t in units of the date welfare numeraire (a unit of date-t consumption). Normalized instantaneous welfare gains for individual i at history s^t, $\frac{dV_t^{i|\lambda}(s^t)}{d\theta}$, correspond to i's willingness-to-pay for the perturbation at history s^t in units of the instantaneous welfare numeraire (a unit of history-s^t consumption).

Normalized instantaneous welfare gains, $\frac{dV_t^{i|\lambda}(s^t)}{d\theta}$, define a consumption-equivalent at a particular history, while date and lifetime gains can be interpreted as time- and risk-discounted sums of instantaneous consumption-equivalents. In fact, Lemma 1 shows that every welfare assessment can be expressed as a triple weighted sum of instantaneous welfare gains, since

$$\frac{dW^\lambda}{d\theta} = \sum_i \omega^i \sum_t \omega_t^i \sum_{s^t} \omega_t^i\left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}.$$

Dividing $\frac{dW}{d\theta}$ by $\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i$ ensures that the normalized welfare assessment $\frac{dW^\lambda}{d\theta}$ is expressed in units of the lifetime welfare numeraire, and that it can be interpreted in terms of a perturbation that distributes permanent consumption equally. That is, a normalized welfare assessment of, for instance, $\frac{dW^\lambda}{d\theta} = 3$ is equivalent to a perturbation in which 3 units of permanent consumption are equally distributed across all individuals. Unnormalized and normalized assessments, $\frac{dW}{d\theta}$ and $\frac{dW^\lambda}{d\theta}$, agree on whether a perturbation is desirable or not.

The normalized individual weight ω^i defines how a welfarist planner trades off lifetime welfare gains across individuals. For instance, if $\omega^i = 1.3$, a welfarist planner finds the welfare gain associated with distributing 1 unit of permanent consumption to individual i equivalent to distributing 1.3 units of permanent consumption equally across all individuals. Note that normalized individual weights average to one, so $\frac{1}{I}\sum_i \omega^i = 1$.

The normalized dynamic weight ω_t^i defines a marginal rate of substitution between a unit of date t consumption and a unit of permanent consumption for individual i. For instance, if $\omega_t^i = 0.1$, a welfarist planner finds the welfare gain associated with distributing 1 unit of date t consumption-equivalent to individual i equivalent to distributing 0.1 units of permanent consumption to that individual. Since permanent consumption is a bundle of consumption at all dates, normalized dynamic weights add up to one, defining a *normalized discount factor*, so $\sum_i \omega_t^i = 1$, $\forall i$.

The normalized stochastic weight $\omega_t^i(s^t)$ defines a marginal rate of substitution between a unit of history-s^t consumption and a unit of date t consumption for individual i. For instance, if $\omega_t^i(s^t) = 0.4$, a welfarist planner finds the welfare gain associated with distributing 1 unit of history-s^t consumption-equivalent to individual i equivalent to distributing 0.4 units of date-t consumption to that individual. Normalized stochastic weights add up to one, so $\sum_{s^t} \omega_t^i(s^t) = 1$, $\forall t$, $\forall i$, defining *risk-neutral probabilities.*[4]

Through the lens of Asset Pricing, Lemma 1 implies that every welfare assessment corresponds to a weighted sum of the values given by I individuals to claims to the instantaneous normalized welfare gains, $\frac{dV_t^{i|\lambda}(s^t)}{d\theta}$, which play the role of individual-specific payoffs. This is related to but different from the literature on asset pricing with incomplete markets (Constantinides and Duffie, 1996; Krueger and Lustig, 2010), which focuses on pricing claims in which payoffs are not individual-specific.

3.2 Efficiency vs. Redistribution

After expressing welfare assessments in comparable units, we first decompose welfare assessments into i) an *efficiency* component, which adds up normalized lifetime welfare gains across individuals, and ii) its complement, a *redistribution* component, which captures the differential impact of a perturbation towards those individuals preferred by the welfarist planner. Figure 1 on page 3 illustrates the decomposition.

Proposition 1. *(Efficiency/Redistribution Decomposition) A normalized welfare assessment for a welfarist planner can be decomposed into efficiency and redistribution components, Ξ^E and Ξ^{RD}, as follows:*

$$\frac{dW^\lambda}{d\theta} = \sum_i \omega^i \frac{dV^{i|\lambda}}{d\theta} = \underbrace{\sum_i \frac{dV^{i|\lambda}}{d\theta}}_{\Xi^E \textit{ (Efficiency)}} + \underbrace{\mathbb{C}ov_i^\Sigma\left[\omega^i, \frac{dV^{i|\lambda}}{d\theta}\right]}_{\Xi^{RD} \textit{ (Redistribution)}}, \quad (10)$$

where $\mathbb{C}ov_i^\Sigma[\cdot,\cdot] = I \cdot \mathbb{C}ov_i^\Sigma[\cdot,\cdot]$ denotes a cross-sectional covariance-sum among individuals.

The efficiency component in Proposition 1 exactly corresponds to Kaldor-Hicks efficiency (Kaldor,

[4] Risk-neutral probabilities are widely used in finance (Duffie, 2001; Cochrane, 2005), while normalized discount factors are common in the study of repeated games (Fudenberg and Tirole, 1991; Mailath and Samuelson, 2006).

1939; Hicks, 1939): it is the sum of individual willingness-to-pay for the perturbation in units of the lifetime welfare numeraire.[5] Kaldor-Hicks efficiency is typically justified on the grounds of the compensation principle (Boadway and Bruce, 1984). That is, whenever $\Xi^{E} > 0$, the winners of the perturbation could hypothetically compensate the losers in terms of the lifetime welfare numeraire. For a given lifetime welfare numeraire, Proposition 1 is the *unique* decomposition in which a normalized welfare assessment can be expressed as the (unweighted) sum of individual willingness-to-pay and its complement.

The redistribution component — which can equivalently be expressed as $\Xi^{RD} = \sum_i \left(\omega^i - 1\right) \frac{dV^{i|\lambda}}{d\theta}$ — captures the equity concerns embedded in a particular Social Welfare Function. Ξ^{RD} is positive when the individuals relatively favored in a perturbation are those relatively preferred by the planner, i.e., have higher normalized individual weights ω^i.

In Proposition 2, we state three properties of the efficiency/redistribution decomposition just introduced that further justify the choice of labels for each component.

Proposition 2. *(Properties of Efficiency/Redistribution Decomposition)*

a) (Invariance of efficiency component to SWF) The efficiency component is identical for all welfarist planners. Differences in welfare assessments among welfarist planners are exclusively due to the redistribution component.

b) (Invariance of efficiency component to preference-preserving utility transformations) The efficiency component is invariant to i) monotonically increasing transformations of individuals' lifetime utilities and ii) positive affine (increasing linear) transformations of individuals' instantaneous utilities.

c) (Pareto improvements increase efficiency) The efficiency component is strictly positive for (strict or weak) Pareto-improving perturbations.

Proposition 2a) follows from the fact that normalized lifetime utilities $\frac{dV^{i|\lambda}}{d\theta}$ do not depend on the choice of SWF; only the normalized individual weights ω^i do. This property implies that welfarist planners cannot disagree about the efficiency consequences of a perturbation. The reason why different welfarist planners make different welfare assessments is simply because they use different normalized individual weights, implying different social preferences for redistribution.

Proposition 2b) follows from the fact that normalized lifetime utilities $\frac{dV^{i|\lambda}}{d\theta}$ — with units $\frac{\text{lifetime welfare numeraire}}{\text{units of } \theta}$ — do not depend on the choice of individual utility units. This property implies that even though welfarist planners mechanically overweight welfare gains by individuals whose lifetime (instantaneous) utility experiences a monotonically increasing (positive affine)

[5] Proposition 1 implies that welfare assessments based on a SWF are equivalent to relying on Kaldor-Hicks efficiency with a correction for redistribution.

transformation (Campbell, 2018) — even though this has no impact on allocations — this only impacts the redistribution component.[6] Hence, the impact of preference-preserving utility transformations on welfare assessments is exclusively confined to the redistribution component.

Proposition 2c) follows from the fact that the sum of willingness-to-pay for a Pareto-improving perturbation must be strictly positive, since there are no losers. This property guarantees that Pareto efficient allocations must satisfy $\Xi^E \leq 0$ for any feasible perturbation. By contrast, Ξ^{RD} can be negative for Pareto-improving perturbations, even though $\Xi^E + \Xi^{RD} > 0$ in that case since welfarist planners are Paretian.

3.3 Aggregate Efficiency vs. Risk-Sharing vs. Intertemporal-Sharing

Proposition 1 implies that the efficiency component of a normalized welfare assessment is simply the sum of discounted individual welfare gains using individual discount factors — captured by normalized dynamic and stochastic weights. Hence, every decomposition of the efficiency component necessarily corresponds to a particular grouping of the weighted sum

$$\Xi^E = \sum_i \sum_t \sum_{s^t} \omega_t^i \omega_t^i \left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}.$$

If all individuals value instantaneous welfare gains over time and across histories equally, then efficiency simply corresponds to the discounted value — using the common discount factor — of aggregate instantaneous welfare gains $\sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}$. In general, when individuals have different valuations, Proposition 3 decomposes efficiency gains into an *aggregate efficiency* component, which corresponds to the discounted value of aggregate instantaneous welfare gains $\sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}$ using a common discount factor, and two components, *intertemporal-sharing* and *risk-sharing,* which respectively capture the differential impact of a perturbation towards individuals with different valuations over dates or histories.[7]

Proposition 3. *(Aggregate Efficiency/Risk-Sharing/Intertemporal-Sharing Decomposition) The efficiency component of a normalized welfare assessment can be decomposed into aggregate efficiency,*

[6]The invariance to positive affine transformations is only meaningful with expected utility.

[7]By choosing the term risk-sharing and the (less conventional) term intertemporal-sharing, we seek to highlight the cross-sectional nature of both components. Terms such as insurance, consumption smoothing, or intertemporal smoothing do not have such connotation since they could be applied to a single individual.

risk-sharing, and intertemporal-sharing components, Ξ^{AE}, Ξ^{RS}, and Ξ^{IS}, as follows:

$$\Xi^E = \underbrace{\sum_t \omega_t \sum_{s^t} \omega_t\left(s^t\right) \sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}}_{\Xi^{AE}\ \textit{(Aggregate Efficiency)}} + \underbrace{\sum_t \omega_t \sum_{s^t} \omega_t\left(s^t\right) \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]}_{\Xi^{RS}\ \textit{(Risk-Sharing)}} \tag{11}$$

$$+ \underbrace{\sum_t \omega_t \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}}{d\theta}\right]}_{\Xi^{IS}\ \textit{(Intertemporal-Sharing)}},$$

where the averages of normalized weights $\omega_t = \frac{1}{I}\sum_i \omega_t^i$ and $\omega_t\left(s^t\right) = \frac{1}{I}\sum_i \omega_t^i\left(s^t\right)$ define aggregate time and stochastic discount factors, and where $\mathbb{C}ov_i^{\Sigma}[\cdot,\cdot] = I \cdot \mathbb{C}ov_i^{\Sigma}[\cdot,\cdot]$ denotes a cross-sectional covariance-sum among individuals.

The justification for this decomposition is once again based on the compensation principle, now applied over dates and histories. The sum of normalized date welfare gains at date t, $\sum_i \frac{dV_t^{i|\lambda}}{d\theta}$, corresponds to the aggregate willingness-to-pay for the impact of the perturbation at that date in units of the date welfare numeraire (a unit of date-t consumption). Hence, when $\sum_i \frac{dV_t^{i|\lambda}}{d\theta} > 0$, the winners of the perturbation at date t could hypothetically compensate the losers in terms of the date welfare numeraire at that date. The aggregate time discount factor that makes it possible to add up aggregate gains across different dates, by expressing them in units of the lifetime welfare numeraire, is $\omega_t = \frac{1}{I}\sum_i \omega_t^i$.

Therefore, the *unique* way to decompose Ξ^E into a component that corresponds to the discounted sum — using an aggregate discount factor — of the aggregate willingness-to-pay for the perturbation at each date and its complement is

$$\Xi^E = \sum_i \sum_t \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta} = \underbrace{\sum_t \omega_t \sum_i \frac{dV_t^{i|\lambda}}{d\theta}}_{\Xi^{AE}+\Xi^{RS}} + \underbrace{\sum_t \omega_t \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}}{d\theta}\right]}_{\Xi^{IS}}. \tag{12}$$

The intertemporal-sharing component — which can equivalently be expressed as $\Xi^{IS} = \sum_t \omega_t \sum_i \left(\frac{\omega_t^i}{\omega_t} - 1\right) \frac{dV_t^{i|\lambda}}{d\theta}$ — captures the contribution to efficiency due to differences in valuation over time across individuals. The date-t element of Ξ^{IS} is positive when a perturbation relatively favors individuals with a higher relative valuation (dynamic weight) for date t, and vice versa.

The same logic applies to decomposing aggregate normalized date welfare gains at date t,

$$\sum_i \frac{dV_t^{i|\lambda}}{d\theta} = \sum_i \sum_{s^t} \omega_t^i\left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} = \sum_{s^t} \omega_t\left(s^t\right) \sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} + \sum_{s^t} \omega_t\left(s^t\right) \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right].$$

The sum of normalized instantaneous welfare gains at history s^t, $\sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}$, corresponds to the aggregate willingness-to-pay for the impact of the perturbation at that history in units of the instantaneous welfare numeraire (a unit of history-s^t consumption). Hence, when $\sum_i \frac{dV_t^{i|\lambda}(s^t)}{d\theta} > 0$, the winners of the perturbation at history s^t could hypothetically compensate the losers in terms of the instantaneous welfare numeraire at that history. The aggregate stochastic discount factor that makes it possible to add up aggregate gains across different histories at a given date, by expressing them in units of the date welfare numeraire, is $\omega_t(s^t) = \frac{1}{I}\sum_i \omega_t^i(s^t)$.

Therefore, Proposition 3 is the *unique* decomposition in which the efficiency component can be expressed as the discounted sum — using aggregate time and stochastic discount factors — of aggregate instantaneous welfare gains, Ξ^{AE}, and its complement, $\Xi^{RS} + \Xi^{IS}$. The risk-sharing component — which can equivalently be expressed as $\Xi^{RS} = \sum_t \omega_t \sum_{s^t} \omega_t(s^t) \sum_i \left(\frac{\omega_t^i(s^t)}{\omega_t(s^t)} - 1\right) \frac{dV_t^{i|\lambda}(s^t)}{d\theta}$ — captures the contribution to efficiency due to differences in valuation over histories across individuals. The history-s^t element of Ξ^{RS} is positive when a perturbation relatively favors individuals with a higher relative valuation (stochastic weight) for history s^t, and vice versa. In Section E of the Appendix, we discuss alternative subdecompositions for the risk-sharing and intertemporal-sharing components.

By construction, the aggregate efficiency component is exclusively a function of aggregate instantaneous welfare gains, while the risk-sharing and intertemporal-sharing components depend on how instantaneous welfare gains accrue to individuals with different valuations for particular dates or histories. Importantly, for Ξ^{AE} to be non-zero, it must be that a perturbation changes aggregate instantaneous welfare gains at particular dates and histories — see Proposition 4c below. In Section E of the Appendix, we show how it is possible to subdecompose aggregate efficiency into a component that captures improved smoothing of aggregate instantaneous welfare gains — this is the single force behind the cost-of-business-cycles computation in Lucas (1987) — and a component that captures changes in expected aggregate instantaneous welfare gains.

The differences in individual valuations across dates and histories that govern the risk-sharing and intertemporal-sharing components depend on the extent to which individuals can freely smooth consumption (in general, the instantaneous welfare numeraire) intertemporally and across histories. In Proposition 4, we show that i) risk-sharing and intertemporal-sharing are zero when individual marginal rates of substitution across all dates and histories are equalized (which occurs when markets are complete) and ii) intertemporal-sharing is zero when marginal rates of substitution across dates are equalized across agents (which occurs when all individuals can frictionlessly borrow and save).[8]

Proposition 4. *(Properties of Aggregate Efficiency/Risk-Sharing/Intertemporal-Sharing Decomposition)*

[8]Similar to Proposition 2, each of Ξ^{AE}, Ξ^{RS}, and Ξ^{IS} are identical for all welfarist planners and invariant to preference-preserving utility transformations.

a) (Complete markets) When marginal rates of substitution across all dates and histories are equalized across individuals — a condition that complete markets economies satisfy — the risk-sharing and intertemporal-sharing components are zero: $\Xi^{RS} = \Xi^{IS} = 0$.

b) (Frictionless borrowing and saving) When marginal rates of substitution across dates are equalized across individuals — a condition satisfied when all individuals can frictionlessly borrow and save — the intertemporal-sharing component is zero: $\Xi^{IS} = 0$.

c) (Zero aggregate normalized welfare instantaneous gains) When a perturbation features zero aggregate normalized instantaneous welfare gains at all dates and histories, the aggregate efficiency component is zero: $\Xi^{AE} = 0$.

Proposition 4a and 4b follow from the fact that complete markets ensure that individual valuations across dates and histories are identical, while frictionless borrowing and saving do the same exclusively across dates. Intuitively, normalized dynamic and stochastic weights can be expressed in terms of state-prices as follows:

$$\omega_t^i\left(s^t\right) = \frac{q_t^i\left(s^t\right)}{\sum_{s^t} q_t^i\left(s^t\right)} \qquad \text{and} \qquad \omega_t^i = \frac{\sum_{s^t} q_t^i\left(s^t\right)}{\sum_t \sum_{s^t} q_t^i\left(s^t\right)}, \tag{13}$$

where $q_t^i\left(s^t\right) = \left(\beta^i\right)^t \pi_t\left(s^t \middle| s_0\right) \frac{\partial u^i\left(s^t\right)}{\partial c_t^i} / \frac{\partial u^i\left(s^0\right)}{\partial c_0^i}$ denotes individual i's (shadow) date-0 state-price over history s^t. When markets are complete, all valuations are equalized, so $q_t^i = q_t$, $\forall i$, which implies that $\omega_t^i = \omega_t$ and $\omega_t^i\left(s^t\right) = \omega_t\left(s^t\right)$. Hence, in this case, welfare assessments are exclusively driven by aggregate efficiency and redistribution. Under frictionless borrowing and saving, the valuation of zero-coupon bonds is equalized, so $\sum_{s^t} q_t^i\left(s^t\right) = \sum_{s^t} q_t\left(s^t\right)$, which implies that $\omega_t^i = \omega_t$. Proposition 4 also implies that the good/factor on which financial claims are written (e.g., the consumption good or, more generally, a nominal numeraire) is the natural instantaneous welfare numeraire. Proposition 4c shows that the aggregate efficiency component can only take non-zero values for perturbations in which aggregate normalized instantaneous welfare gains are non-zero.

Finally, while Proposition 2c) concludes that the efficiency component is strictly positive for every Pareto-improving perturbation, one or two of the three components of the decomposition of the efficiency component introduced in Proposition 3 may be negative. That is, Pareto efficiency exclusively requires that the sum of the aggregate efficiency, risk-sharing and intertemporal-sharing components is positive, so $\Xi^E = \Xi^{AE} + \Xi^{RS} + \Xi^{IS} > 0$, but not each of them. Application 2 in Section 6 illustrates this possibility.

3.4 Additional Properties

In the remainder of this section, we present additional properties of the welfare assessment decomposition introduced in Propositions 1 and 3.[9] Proposition 5 identifies conditions on i) normalized weights and ii) welfare gains that respectively guarantee that Ξ^{RS}, Ξ^{IS}, or Ξ^{RD} are zero.

Proposition 5. *(Properties of Welfare Decomposition: Individual-Invariant Weights or Welfare Gains)*

a) (Individual-invariant normalized weights) If normalized stochastic weights are constant across individuals at all dates and histories, then $\Xi^{RS} = 0$. If normalized dynamic weights are constant across individuals at all dates, then $\Xi^{IS} = 0$. If normalized individual weights are constant across individuals, then $\Xi^{RD} = 0$.

b) (Individual-invariant welfare gains) If instantaneous welfare gains $\frac{dV_t^{i|\lambda}(s^t)}{d\theta}$ are identical across individuals at all histories, then $\Xi^{RS} = 0$. If risk-adjusted instantaneous welfare gains $\frac{dV_t^{i|\lambda}}{d\theta}$ are identical across individual at all dates, then $\Xi^{IS} = 0$. If lifetime welfare gains are identical across individuals, then $\Xi^{RD} = 0$.

Proposition 5a) shows that invariance of normalized weights across specific dimensions — individual, dynamic, stochastic — implies that redistribution, intertemporal-sharing, and risk-sharing components are respectively zero. This result highlights the cross-sectional nature of these three components, in contrast to aggregate efficiency. Proposition 5b) shows that particular components of the welfare assessment decomposition are zero when perturbations impact all individuals identically at each history, date, or on a lifetime basis.

Proposition 6 identifies which components of the welfare decomposition are zero in particular economies of practical relevance. This result further justifies the labels for the different components of the decomposition and is useful to quickly conclude which components of the decomposition are inactive in specific applications.

Proposition 6. *(Properties of Welfare Decomposition: Particular Economies)*

a) (Single individual economies) In single individual ($I = 1$) economies: $\Xi^{RS} = \Xi^{IS} = \Xi^{RD} = 0$.

b) (Perfect foresight economies) In perfect foresight economies: $\Xi^{RS} = 0$.

[9] Lemma 1 already implies that normalized individual, dynamic, and stochastic weights and normalized instantaneous welfare gains are sufficient statistics to make normalized welfare assessments. These are also the components of the welfare assessment decomposition introduced in Propositions 2 and 4, which makes computing the decomposition conceptually straightforward.

c) *(Economies with ex-ante identical individuals) In economies with ex-ante (but not necessarily ex-post) identical individuals:* $\Xi^{IS} = \Xi^{RD} = 0$.

d) *(Static economies) In static* ($T = 0$) *economies:* $\Xi^{RS} = \Xi^{IS} = 0$.

e) *(Single good endowment economies) In single good endowment economies in which the aggregate endowment is invariant to the perturbation:* $\Xi^{AE} = 0$.

Even though the welfare decomposition is based on the compensation principle, which is formulated in terms of hypothetical transfers between winners and losers, no transfers of resources need to take place for the decomposition to be valid. The decomposition is simply a valuation exercise. That said, in economies in which planners can (and do) costlessly transfer resources among individuals along particular dimensions, Proposition 7 characterizes which components of the welfare decomposition are zero.

Proposition 7. *(Properties of Welfare Decomposition: Transfers)*

a) *(Lifetime transfers) If a planner can costlessly transfer the lifetime welfare numeraire across individuals, then* $\Xi^{RD} = 0$.

b) *(Date transfers) If a planner can costlessly transfer the date welfare numeraire across individuals at all dates, then* $\Xi^{IS} = \Xi^{RD} = 0$.

c) *(Instantaneous transfers) If a planner can costlessly transfer the instantaneous welfare numeraire across individuals at all histories, then* $\Xi^{RS} = \Xi^{IS} = \Xi^{RD} = 0$.

We conclude this section with the following remark.

Remark 1. *(Welfare decomposition does not rely on optimality)* It is worth highlighting that the decomposition of welfare assessments introduced in Propositions 1 and 3 does not rely on individual optimality (i.e., the envelope theorem). The decomposition is exclusively a function of preferences and the considered perturbation. In specific applications, exploiting individual optimality conditions, along with budget and resource constraints, can simplify the characterization of components of the decomposition, as the applications in Section 6 illustrate.

4 Welfare Assessment Decomposition: DS-planners

In this section, we leverage the welfare decomposition to systematically construct non-welfarist welfare criteria based on individual, dynamic, and stochastic weights. This approach allows us to formalize normative objectives that isolate specific components of the welfare decomposition. Because this normative approach entails defining weights for each time and history, for each individual, we say it is based on Dynamic Stochastic Generalized Social Marginal Welfare Weights (dynamic–stochastic weights or DS-weights, for short). These results have the potential to allow for disciplined discussions about the mandates of independent technocratic institutions (central banks, financial regulators, other regulatory agencies, etc.).

DS-planners: Definition. We begin by formally defining desirable perturbations for a planner who adopts DS-weights, a DS-planner.

Definition. *(Desirable perturbation for a DS-planner) A DS-planner finds a perturbation desirable (undesirable) when* $\frac{dW^{DS}}{d\theta} > (<)\, 0$*, where*

$$\frac{dW^{DS}}{d\theta} = \sum_i \omega^i \sum_t \omega_t^i \sum_{s^t} \omega_t^i \left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}, \tag{14}$$

where $\frac{dV_t^{i|\lambda}(s^t)}{d\theta}$ *denotes the instantaneous welfare gains at history* s^t *in units of the instantaneous welfare numeraire, defined in equation (6), and where* $\omega^i > 0$*,* $\omega_t^i > 0$*, and* $\omega_t^i\left(s^t\right) > 0$ *define individual, dynamic, and stochastic weights that can potentially be functions of outcomes.*

Since $\frac{dV_t^{i|\lambda}(s^t)}{d\theta}$ is expressed in units of the instantaneous welfare numeraire and because we require that $\sum_{s^t} \omega_t^i\left(s^t\right) = 1$, the stochastic weight $\omega_t^i\left(s^t\right)$ defines a marginal rate of substitution between a unit of instantaneous welfare numeraire at history s^t and a unit of instantaneous welfare numeraire across all date t histories for individual i, as in the welfarist case. The dynamic weight ω_t^i defines a marginal rate of substitution between a unit of instantaneous welfare numeraire across all date t histories and, implicitly, a unit of lifetime welfare numeraire for individual i.[10] The individual weight ω^i defines how a DS-planner trades off lifetime welfare gains across individuals. The product $\tilde{\omega}_t^i\left(s^t\right) = \omega^i \omega_t^i \omega_t^i\left(s^t\right)$ defines a dynamic-stochastic weight for individual i.[11]

Unlike the welfarist approach — which takes a social welfare function as primitive — welfare assessments by DS-planners are defined in marginal form. In that sense, DS-planners extend the generalized weight approach in Saez and Stantcheva (2016) to dynamic stochastic environments. Formally, while that paper considers welfare objectives that directly define the individual weight ω^i, DS-planners also define (potentially non-welfarist) dynamic and stochastic weights for each

[10] Any choice of weights in which $\sum_t \omega_t^i = 1$ and $\sum_t \omega_t^i\left(s^t\right) = 1$ ensures that interpersonal comparisons are made in a common unit (or lifetime welfare numeraire). It is nonetheless possible to make meaningful comparisons when dynamic weights do not add up to 1 over time, as explained in Section F.1 of the Online Appendix.

[11] Earlier versions of this paper defined desirable perturbations directly in terms of DS-weights, as in

$$\frac{dW^{DS}}{d\theta} = \sum_i \sum_t \sum_{s^t} \tilde{\omega}_t^i\left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta},$$

subsequently multiplicatively decomposing such weights. Both formulations are equivalent, as explained in the Online Appendix.

individual. The Online Appendix shows how to equivalently define DS-planners in terms of instantaneous social welfare functions with generalized (endogenous) welfare weights and further relates the results of this section to those in Saez and Stantcheva (2016).

DS-planners can be useful to both provide analytical characterizations and to characterize and compute optimal policies guided by particular components of the welfare decomposition introduced in this paper. Lemma 1 trivially implies that every welfarist planner is a DS-planner, while the converse is not true, as we illustrate next.

AE/AR/NR Pseudo-welfarist DS-planners. Starting from equation (14), it is evident that welfare assessments for DS-planners can be decomposed into Ξ^{AE}, Ξ^{RS}, Ξ^{IS}, and Ξ^{RD} components, using the same definitions introduced in (10) and (11). Moreover, Proposition 5a) implies that it is possible to construct welfare objectives in which $\Xi^{RD}=0$, $\Xi^{IS}=0$, or $\Xi^{RS}=0$ by choosing individual, dynamic, or stochastic weights that are invariant across individuals.

While in Section B of the Online Appendix we discuss other DS-planners, here we focus on pseudo-welfarist DS-planners. These planners justify using particular components of the welfare decomposition of a welfarist planner as the welfare assessment of a particular DS-planner by making individual, dynamic, and/or stochastic weights equal to their cross-sectional welfarist average.

Definition. *(Pseudo-welfarist AE/AR/NR DS-planners) AE (aggregate efficiency), AR (aggregate efficiency/risk-sharing), and NR (no-redistribution) pseudo-welfarist DS-planners are characterized by the normalized weights:*

$$\omega^{i,AE}=1,\ \omega_t^{i,AE}=\omega_t^{\mathcal{W}},\ \text{and}\ \omega_t^{i,AE}\left(s^t\right)=\omega_t^{\mathcal{W}}\left(s^t\right)\qquad \textit{AE Planner}\qquad (15)$$

$$\omega^{i,AR}=1,\ \omega_t^{i,AR}=\omega_t^{\mathcal{W}},\ \text{and}\ \omega_t^{i,AR}\left(s^t\right)=\omega_t^{i,\mathcal{W}}\left(s^t\right)\qquad \textit{AR Planner}\qquad (16)$$

$$\omega^{i,NR}=1,\ \omega_t^{i,NR}=\omega_t^{i,\mathcal{W}},\ \text{and}\ \omega_t^{i,NR}\left(s^t\right)=\omega_t^{i,\mathcal{W}}\left(s^t\right)\qquad \textit{NR Planner}\qquad (17)$$

where $\omega_t^{i,\mathcal{W}}$ and $\omega_t^{i,\mathcal{W}}\left(s^t\right)$ are dynamic and stochastic weights for the welfarist planner with SWF $\mathcal{W}(\cdot)$, and where $\omega_t^{\mathcal{W}}=\frac{1}{I}\sum_i\omega_t^{i,\mathcal{W}}$ and $\omega_t^{\mathcal{W}}\left(s^t\right)=\frac{1}{I}\sum_i\omega_t^{i,\mathcal{W}}\left(s^t\right)$ are their cross-sectional averages.

Pseudo-welfarist planners are constructed so that specific (sums of) components of the welfare decomposition for a given welfarist planner can be interpreted as welfare assessments for particular DS-planners, as we formalize in Proposition 8.[12]

Proposition 8. *(Relation between Welfarist Planners and Pseudo-welfarist AE/AR/NR DS-planners)*

[12] The NR pseudo-welfarist planner is equivalent to using a Kaldor-Hicks criterion. This planner is non-welfarist and non-paternalistic — see Remark 2.

a) [AE] The aggregate efficiency component Ξ^{AE} for a welfarist planner can be interpreted as the welfare assessment of an AE pseudo-welfarist DS-planner, defined in (15), for whom $\Xi^{RS} = \Xi^{IS} = \Xi^{RD} = 0$.

b) [AR] The sum of aggregate efficiency and risk-sharing components $\Xi^{AE} + \Xi^{RS}$ for a welfarist planner can be interpreted as the welfare assessment of an AR pseudo-welfarist DS-planner, defined in (16), for whom $\Xi^{IS} = \Xi^{RD} = 0$.

c) [NR] The efficiency component Ξ^{E} for a welfarist planner can be interpreted as the welfare assessment of a NR pseudo-welfarist DS-planner, defined in (17), for whom $\Xi^{RD} = 0$.

We conclude this section with two remarks.

Remark 2. *(Paternalistic vs. Non-paternalistic DS-planners)* DS-planners with non-welfarist dynamic and stochastic weights are paternalistic since their welfare assessments are not based on individual lifetime welfare gains. For instance, those planners may conclude that a perturbation that individuals find Pareto-improving is undesirable. Similarly, the components of the welfare decomposition are based on the weights used by the DS-planner, not those reflecting individual preferences. Therefore, welfare assessments that do not value intertemporal-sharing or risk-sharing as individuals do will be paternalistic.[13] Importantly, the definition of DS-planners in equation (14) respects individual intratemporal preferences since it uses $\frac{dV_t^{i|\lambda}(s^t)}{d\theta}$ as an input for the welfare assessment, although this could be relaxed.

Remark 3. *(Impossibility of defining specific pseudo-welfarist DS-planners)* It is not possible to define pseudo-welfarist DS-planners for whom exclusively the risk-sharing and intertemporal-sharing components are zero. Ensuring that $\Xi^{RS} = \Xi^{IS} = 0$ requires using dynamic and stochastic weights that are identical across individuals, which would impact Ξ^{RD}. A similar logic applies to other components of the welfare decomposition. It is nonetheless possible to define DS-planners that are not pseudo-welfarist but that, for instance, exclusively value aggregate efficiency and redistribution, as we show in Section B of the Online Appendix.

5 Extensions and Additional Results

Sections D, E, and F of the Online Appendix present several extensions and additional results, which we summarize here. Section G further describes how our results relate to existing work.

Generalized Environments. Section D describes how to extend our results to more general environments. First, we show how to use the welfare decomposition in environments with

[13]Welfare assessments by non-welfarist DS-planners (in fact, by non-utilitarian planners) introduce an independent dimension of time inconsistency. There is scope to further explore the time inconsistency of welfare assessments.

heterogeneous beliefs, both for welfarist and non-welfarist planners. Second, we describe how to allow for recursive preferences, in particular, the widely used Epstein-Zin preferences. We also consider the case of non-time separable non-expected utility preferences. Third, we show that allowing for multiple consumption goods and factors simply requires redefining instantaneous welfare gains. Fourth, we describe how to consider perturbations that entail changes in probabilities. Fifth, we show how to generalize the welfare decomposition to scenarios in which normalized individual, dynamic, or stochastic weights are zero. Finally, we briefly discuss how to implement the decomposition in environments with idiosyncratic and aggregate states, a continuum of individuals, dates, or histories, and non-differentiabilities.

Subdecompositions and Alternative Decompositions. Section E describes how to further decompose the components of the welfare decomposition. First, we show that welfare assessments as well as each of the components of the welfare decomposition have a term structure of the form

$$\frac{dW^{\lambda}}{d\theta} = \sum_{t} \omega_t \frac{dW_t^{\lambda}}{d\theta} \quad \text{where} \quad \frac{dW_t^{\lambda}}{d\theta} = \Xi_t^{AE} + \Xi_t^{RS} + \Xi_t^{IS} + \Xi_t^{RD}. \tag{18}$$

This structure can be used to compute transition and steady-state welfare gains, and can be refined to define a stochastic structure of welfare gains. Second, we show that each individual can be attributed a particular share of each of the components of the welfare decomposition. Third, we show that it is possible to decompose each of the components into a term due to consumption or factor supply growth and a term due to reallocation. Fourth, we show how to construct a stochastic decomposition of aggregate efficiency into expected aggregate efficiency and aggregate smoothing, and of redistribution into expected redistribution and redistributive smoothing. Finally, we provide two alternative cross-sectional decompositions of the risk-sharing and intertemporal-sharing components.

Additional Results. Section F includes additional results. First, we derive the welfare decomposition for general welfare numeraires and discuss the implications of different numeraire choices. Second, we explain how the ability to costlessly transfer resources across individuals by a planner impacts the welfare decomposition by limiting cross-sectional variation in normalized weights. Third, we explain how to translate marginal welfare assessments into global welfare assessments. Finally, we provide bounds based on the dispersion of normalized weights and welfare gains for Ξ^{RS}, Ξ^{IS}, and Ξ^{RD}.

6 Applications

In this section, we illustrate how the welfare decomposition introduced in this paper can be used to draw normative conclusions in three scenarios of practical relevance. Table 1 illustrates which components of the welfare decomposition are non-zero in each application.

Table 1: Summary of Applications

#	Application	Ξ^{AE}	Ξ^{RS}	Ξ^{IS}	Ξ^{RD}
1	Consumption Smoothing	$= 0$	✓	✓	✓
2	Labor Income Taxation (deterministic)	✓	$= 0$	$= 0$	✓
2	Labor Income Taxation (stochastic)	✓	✓	$= 0$	$= 0$
3	Credit Constraint Relaxation	✓	✓	✓	✓

Note: This table illustrates which components of the welfare decomposition are non-zero in each of the applications.

6.1 Application 1: Consumption Smoothing

This application analyzes the welfare effects of a transfer policy that perfectly smooths consumption across individuals who face idiosyncratic consumption risk. The central takeaway is that the persistence of the endowment process determines whether the welfare gains from the transfer policy are attributed to risk-sharing, intertemporal-sharing, or redistribution.

6.1.1 Environment

We consider an infinite-horizon economy with two individuals, $i \in \{1, 2\}$, with identical preferences. We formulate preferences recursively as

$$V^i(s) = u\left(c^i(s)\right) + \beta \sum_{s'} \pi(s'|s) V^i(s'), \quad \text{where} \quad u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where $V^i(s)$ and $c^i(s)$ respectively denote the lifetime utility and the consumption of individual i in a given state s; s and s' denote possible states, and $\pi(s'|s)$ denotes Markov transition probabilities; β is a discount factor, and $u(c)$ denotes the instantaneous utility function.

There is a single nonstorable consumption good. We consider an extreme form of incomplete markets: no financial markets. So individual consumption in state s is given by

$$c^i(s) = y^i(s) + \theta T^i(s), \tag{19}$$

where $y^i(s)$ denotes individual i's endowment of the good and $T^i(s)$ denotes the transfer policy, scaled by a parameter $\theta \in [0, 1]$. Uncertainty follows a two-state Markov chain. We denote states by $s = \{L, H\}$, standing for low (L) and high (H) realizations of individual 1's endowments, $y^1(s)$. The transition matrix is given by

$$\Pi = \begin{pmatrix} \rho & 1-\rho \\ 1-\rho & \rho \end{pmatrix},$$

where $\rho \in [0,1]$ captures the persistence of the endowment process. To ensure risk is idiosyncratic, we assume that $y^1(s) = \overline{y} + \varepsilon(s)$ and $y^2(s) = \overline{y} - \varepsilon(s)$, where $\overline{y} > 0$, and where $\varepsilon(L) = -\varepsilon(H)$. We consider the welfare assessment of a transfer policy that fully smooths consumption. By setting $T^1(s) = -\varepsilon(s)$ and $T^2(s) = \varepsilon(s)$, individual consumption takes the form

$$c^1(s) = \overline{y} + \varepsilon(s)(1-\theta) \quad \text{and} \quad c^2(s) = \overline{y} - \varepsilon(s)(1-\theta).$$

By varying θ between 0 and 1, this economy transitions from autarky to perfect consumption smoothing. We consider an equal-weighted utilitarian SWF, so $\mathcal{W}(V^1, V^2) = V^1 + V^2$, and adopt consumption-based welfare numeraires. Our benchmark parameterization assumes $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\gamma = 2$, and $\rho = 0.975$.

6.1.2 Results

Normalized Weights. Figure 2 shows normalized dynamic and stochastic weights — defined in Lemma 1 — when $\theta = 0.25$. Several insights emerge.

First, individuals with an initially low endowment (and high marginal utility) value welfare gains in early periods relatively more than in later periods. And since dynamic weights add up to 1 over time, dynamic weights for different individuals necessarily intersect. Second, stochastic weights show time-dependence despite the stationarity of the model because shocks are persistent. The persistence of the endowment process explains why individuals value early welfare gains more, although the impact of the initial state eventually dissipates. In the long run, individuals value welfare gains more (less) in states with low (high) consumption, as expected. Finally, a normalized utilitarian planner values lifetime welfare gains for the low endowment individual at the time of the assessment at roughly 46% more than for the high endowment individual, since

$$\frac{\omega^1}{\omega^2} = \frac{1.186}{0.814} \approx 1.46, \quad \text{when} \quad s_0 = L.$$

Welfare Decomposition. Figure 2 shows the welfare decomposition for three different parameterizations: $\rho = \{0.5, 0.975, 0.999\}$ when $s_0 = L$; welfare assessments are identical when $s_0 = H$.[14]

[14] If the model featured a state in which individuals are identical, the welfare decomposition at that state would be significantly different. Proposition 5 implies that $\Xi^{IS} = \Xi^{RD} = 0$ in that case, so every welfare assessment would exclusively be due to risk-sharing. This fact underscores that the decomposition of welfare assessments critically

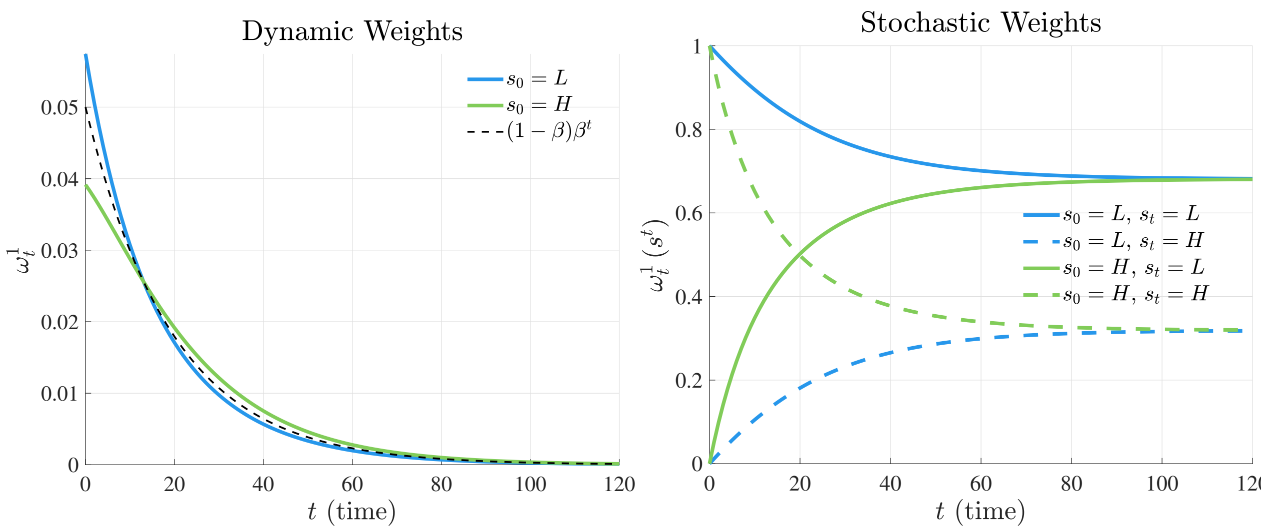

Figure 2: Normalized Weights (Application 1)

Note: This figure shows normalized weights for individual 1 when $\theta = 0.25$. The left panel shows the dynamic weight, ω_t^i, as a function of time when for different initial states $s_0 = \{H, L\}$. For reference, it also shows the dynamic weight for a risk-neutral individual, given by $(1-\beta)\beta^t = \beta^t / \sum_t \beta^t$. The right panel shows the stochastic weights $\omega_t^i(s^t)$ as a function of time for different initial and final states, $s_0 = \{H, L\}$ and $s_t = \{H, L\}$. The individual weights are $\omega^1(s_0 = L) = 1.186$ and $\omega^1(s_0 = H) = 0.814$. Since the model is symmetric, normalized weights for individual 2 can be read off the weights for individual 1 switching the initial state.

This application illustrates how the persistence of endowment shocks changes the relative importance of each of the components of the welfare decomposition. When shocks are transitory ($\rho = 0.5$), risk-sharing dominates, with intertemporal-sharing playing a smaller role and redistribution being virtually zero. When shocks are persistent ($\rho = 0.975$), welfare gains are partly attributed to redistribution, which is larger than intertemporal-sharing, although risk-sharing is still the dominant component. When shocks are virtually permanent ($\rho = 0.999$), redistribution dominates, with risk-sharing and intertemporal-sharing playing a much smaller role. This application is constructed so that the normalized welfare assessment $\frac{dW_t^\lambda}{d\theta}$ is invariant to the level of persistence ρ, underscoring shifts in the relative contribution of each of the components of the decomposition.

We make four additional observations. First, since this is a single good endowment economy in which transfers cancel out in the aggregate, Proposition 6 implies that $\Xi^{AE} = 0$. Second, the optimal policy for a utilitarian planner features perfect consumption smoothing ($\theta^\star = 1$). This application is constructed so that the three non-zero components (risk-sharing, intertemporal-sharing, and redistribution) independently conclude that perfect smoothing is optimal. Hence, in practice, the rationale justifying such a policy can significantly differ depending on primitives. Third, as shown in the bottom right panel of Figure 3, intertemporal-sharing is hump-shaped, peaking at $\rho = 0.96$. Intuitively, the difference in valuations induced by the inability to borrow and save is maximal when

depends on the state in which an assessment takes place.

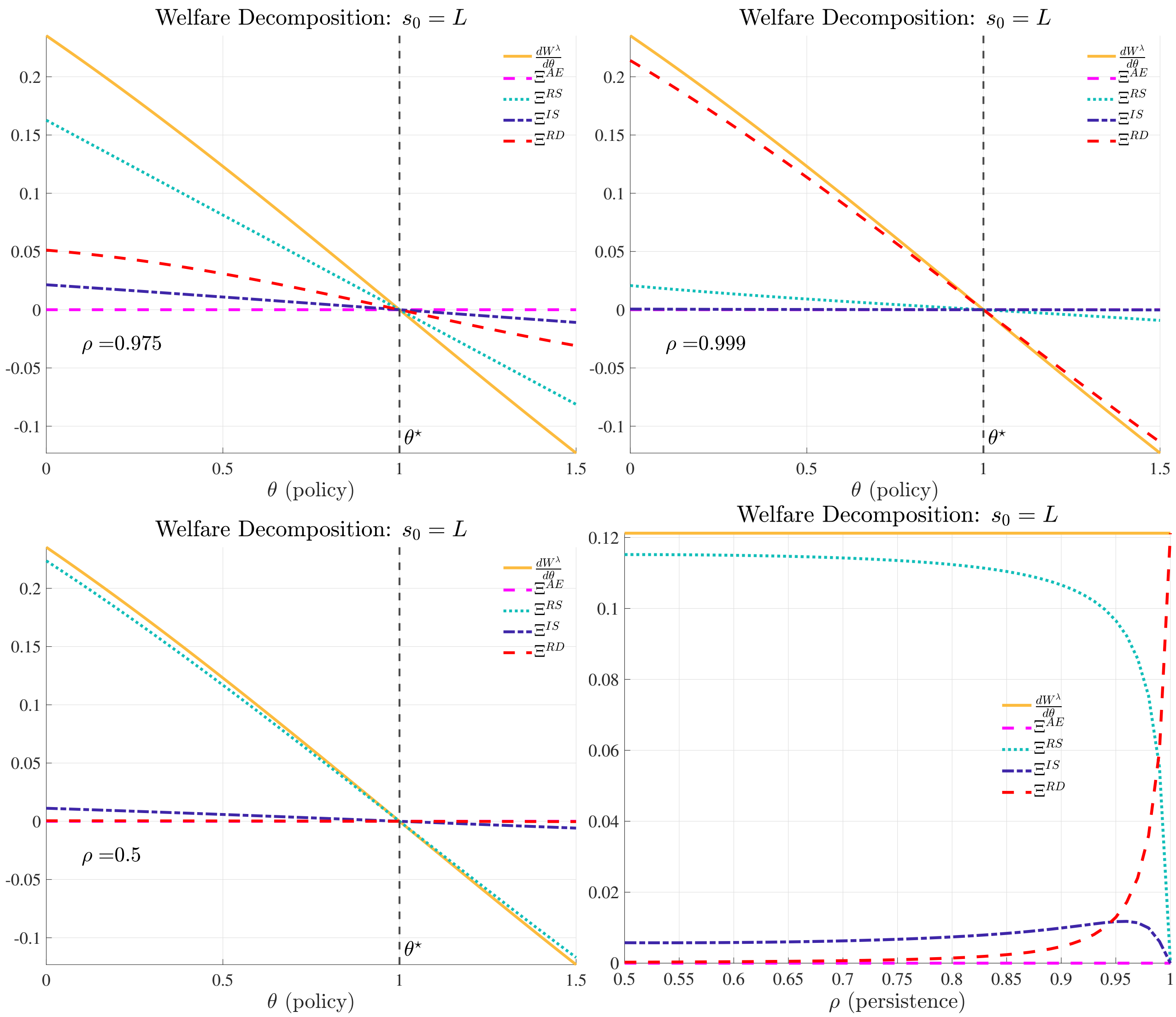

Figure 3: Welfare Decomposition (Application 1)

Note: This figure shows the welfare assessment and its components as a function of the perturbation parameter θ when $s_0 = L$ for three parameterizations: $\rho = 0.975$ (top panel; benchmark), $\rho = 0.5$ (bottom left panel), and $\rho = 0.999$ (top right panel), when $s_0 = L$. The bottom right panel shows the welfare gains from the smoothing policy (integrating marginal welfare gains between $\theta = 0$ and $\theta = 1$) as a function of the persistence parameter. This figure illustrates that the persistence of the endowment process determines whether the welfare gains from the transfer policy are attributed to risk-sharing, intertemporal-sharing, and redistribution

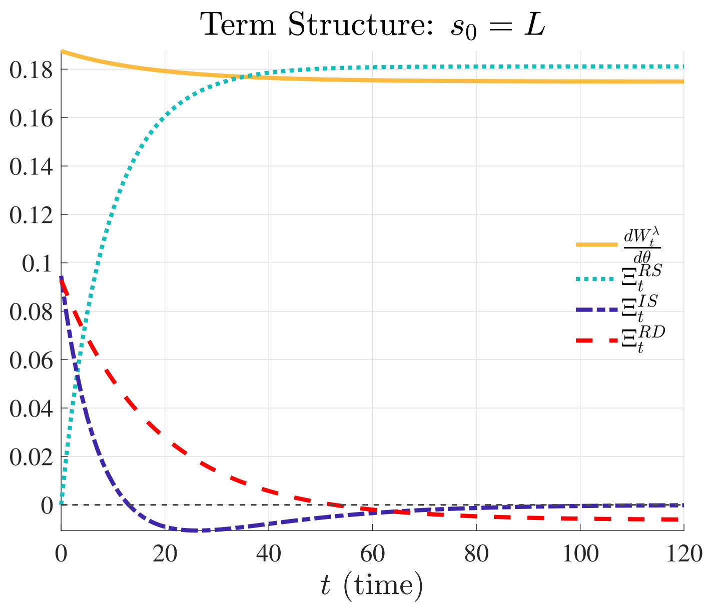

Figure 4: Term Structure of Welfare Decomposition (Application 1)

Note: This figure shows the term structure of welfare assessments, $\frac{dW_t^\lambda}{d\theta}$, and its non-zero components: Ξ_t^{RS}, Ξ_t^{IS}, and Ξ_t^{RD}, as defined in equation (18) and in Section C of the Online Appendix, when $s_0 = L$.

shocks are persistent. Finally, even though a utilitarian planner finds perfect smoothing optimal, $\theta = 1$ is not a Pareto improvement relative to $\theta = 0$: the individual with a higher initial endowment at the time of the assessment is worse off for values of θ near 1, more so when shocks are more persistent.

Term Structure. Figure 4 shows the term structure of welfare assessments, based on equation (18). The normalized date-t welfare assessment, $\frac{dW_t^\lambda}{d\theta}$, is only slightly front-loaded. However, the time-invariance of $\frac{dW_t^\lambda}{d\theta}$ masks significant variation in each of its components. The risk-sharing component, which is zero at $t = 0$ and positive at all times, captures all long-run gains from the policy. This occurs because the smoothing policy has risk-sharing benefits at all dates, since $\mathbb{C}ov_i^\Sigma \left[\frac{\omega_t^i(s^t)}{\omega_t(s^t)}, \frac{dV_t^{i|\lambda}(s^t)}{d\theta}\right] > 0$ at all later times after $t = 0$.

In contrast, both the intertemporal-sharing and redistribution components are positive at $t = 0$ but end up contributing negatively to the welfare assessment of the policy. Since the normalized date welfare gains $\frac{dV_t^{i|\lambda}}{d\theta}$ converge to the same (positive) value for both individuals when $t \to \infty$, then $\lim_{t\to\infty} \Xi_t^{IS} = 0$. The date on which the dynamic components of both individuals intersect determines when Ξ_t^{IS} turns negative. The fact that $\lim_{t\to\infty} \Xi_t^{RD} < 0$ is due to the fact that the individual with the initially low endowment (and high marginal utility) values long-run welfare gains relatively less. The subtle behavior of Ξ_t^{IS} and Ξ_t^{RD} is due to the fact that the dynamic weights intersect, as explained in detail in Section C of the Online Appendix.

6.2 Application 2: Labor Income Taxation

This application contrasts the welfare effects of (linear) labor income taxes in two environments: i) a deterministic environment in which individuals differ in their productivity at the time of the welfare assessment, and ii) a stochastic environment in which individuals are identical at the time of the welfare assessment, but experience different shocks. While both environments can be parameterized to yield a quantitatively identical optimal tax, a utilitarian planner attributes the welfare gains from the tax to redistribution in the deterministic environment and to risk-sharing in the stochastic environment. Moreover, in the stochastic environment *all* welfarist planners agree on the magnitude of the optimal tax, while in the deterministic environment the optimal tax is sensitive to the choice of welfare function.

6.2.1 Deterministic Earnings

Environment. We first consider a single-date environment with two individuals $i \in \{1, 2\}$ who make a consumption-labor decision subject to a linear tax in labor earnings. Formally, individual have identical preferences given by

$$V^i = u\left(c^i, n^i\right),$$

where c^i denotes consumption and n^i hours worked. Individual budget constraints are given by

$$c^i = (1-\tau)\, w^i n^i + g,$$

where τ is the linear tax rate and g is a uniform per-capita grant (demogrant) that must satisfy $g = \frac{1}{I}\tau \sum_i w^i n^i$, and w^i denotes individual i's wage. We consider an equal-weighted utilitarian SWF, so $\mathcal{W}\left(V^1, V^2\right) = V^1 + V^2$, and adopt consumption-based welfare numeraires. To simplify the exposition, we assume that preferences take the form $u\left(c^i, n^i\right) = \frac{1}{1-\gamma}\left(c - \alpha \frac{n^\sigma}{\sigma}\right)^{1-\gamma}$. Our parameterization assumes $w^1 = 1$, $w^2 = 5$, $\gamma = 0.5$, $\sigma = 2$, and $\alpha = 1$.

Welfare Decomposition. We now consider the welfare effects of changing the linear tax rate τ where the demogrant g adjusts to satisfy the government's budget constraint. The lifetime welfare gains for individual i induced by a marginal tax change are given by

$$\frac{dV^{i|\lambda}}{d\tau} = \frac{\frac{dV^i}{d\tau}}{\lambda^i} = -w^i n^i + \frac{dg}{d\tau}, \tag{20}$$

where $\frac{dg}{d\tau} = \frac{1}{I}\left(\sum_i w^i n^i + \tau \sum_i w^i \frac{dn^i}{d\tau}\right)$. Since we are considering a static framework, Proposition 6 implies that $\Xi^{RS} = \Xi^{IS} = 0$, so the welfare decomposition exclusively features aggregate efficiency and redistribution. In fact, the welfare assessment of a change in the tax rate can be decomposed as

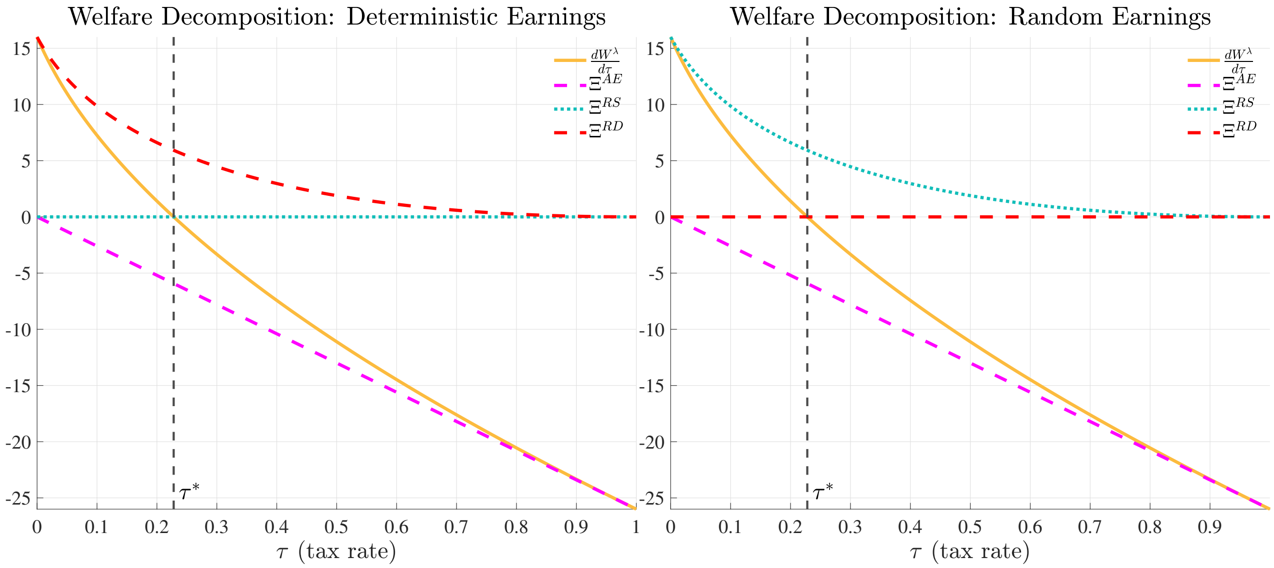

Figure 5: Welfare Decomposition (Application 2)

Note: This figure shows the welfare assessment and the components of the welfare decomposition as a function of the tax rate τ for both the deterministic and random earnings models.

follows:

$$\frac{dW^\lambda}{d\tau} = \underbrace{-\tau \sum_i w^i \left(-\frac{dn^i}{d\tau}\right)}_{\Xi^{AE} \text{ (Aggregate Efficiency)}} + \underbrace{\mathbb{C}ov_i^{\Sigma}\left[\omega^i, -w^i n^i\right]}_{\Xi^{RD} \text{ (Redistribution)}}, \quad \text{where} \quad \omega^i = \frac{\frac{\partial u(c^i,n^i)}{\partial c}}{\frac{1}{I}\sum_i \frac{\partial u(c^i,n^i)}{\partial c}}. \tag{21}$$

As illustrated in the left panel of Figure 5, aggregate efficiency welfare gains are 0 at $\tau = 0$ and become increasingly negative as τ increases. These losses capture how the tax reduces the desire to work by individuals. Redistribution gains are strictly positive but decreasing, so this optimal taxation problem is well-behaved and features an optimal interior tax $\tau^\star$ that optimally trades off aggregate efficiency losses with redistribution gains.[15]

6.2.2 Random Earnings

Environment. We now consider an environment in which two identical individuals $i \in \{1, 2\}$ face uninsured earnings risk. At the time of the welfare assessment, individuals have expected utility of the form

$$V^i = \sum_s \pi(s)\, u\left(c^i(s), n^i(s)\right),$$

[15] The optimal linear income tax problem is first studied by Sheshinski (1972). See Piketty and Saez (2013) and Kaplow (2022) for recent surveys of this area.

where $c^i(s)$ and $n^i(s)$ denote consumption and hours work in state s. For simplicity, we assume that there are two possible states $s = \{H, L\}$, with probability $\pi(s) = \frac{1}{2}$. To ensure that risk is idiosyncratic, we assume that, in state $s = H$, wages are given by $w^1(H) = \overline{w}$ and $w^2(H) = \underline{w}$, while in state L, $w^1(L) = \underline{w}$ and $w^2(H) = \overline{w}$, where $\overline{w} > \underline{w}$. After the state is realized, individuals make a consumption-labor decision facing a linear tax in labor earnings, and face budget constraints given by

$$c^i(s) = (1-\tau)\, w^i(s)\, n^i(s) + g,$$

where τ is the linear tax rate, which we assume to be state-independent — without loss of generality, given the idiosyncratic structure of uncertainty — and $g = \frac{1}{I}\tau \sum_i w^i n^i$ is a demogrant, also state-independent. We again consider an equal-weighted utilitarian SWF, so $\mathcal{W}(V^1, V^2) = V^1 + V^2$, and adopt consumption-based welfare numeraires. We assume again that preferences take the form $u(c^i, n^i) = \frac{1}{1-\gamma}\left(c - \alpha \frac{n^\sigma}{\sigma}\right)^{1-\gamma}$. Our parameterization assumes $\underline{w} = 1$, $\overline{w} = 5$, $\gamma = 0.5$, $\sigma = 2$, and $\alpha = 1$.

Welfare Decomposition. We again consider the welfare effects of changing the linear tax rate τ where the demogrant g adjusts to satisfy the government's budget constraint at each state. The lifetime welfare gains for individual i induced by a marginal tax change are given by

$$\frac{dV^{i|\lambda}}{d\tau} = \frac{\frac{dV^i}{d\tau}}{\lambda^i} = \sum_s \omega_1^i(s)\left[-w^i(s)\, n^i(s) + \frac{dg}{d\tau}\right] \quad \text{where} \quad \omega_1^i(s) = \frac{\pi(s)\frac{\partial u\left(c^i(s), n^i(s)\right)}{\partial c(s)}}{\sum_s \pi(s)\frac{\partial u(c^i(s), n^i(s))}{\partial c(s)}}, \tag{22}$$

and where $\frac{dg}{d\tau} = \frac{1}{I}\left(\sum_i w^i(s)\, n^i(s) + \tau \sum_i w^i(s)\frac{dn^i(s)}{d\tau}\right)$. Since we are considering a framework with ex-ante identical individuals, Proposition 6 implies that $\Xi^{IS} = \Xi^{RD} = 0$, so the welfare decomposition exclusively features aggregate efficiency and risk-sharing. In fact, the welfare assessment of a change in the tax rate can be decomposed as follows:

$$\frac{dW^\lambda}{d\tau} = \underbrace{-\tau \sum_s \omega_1(s) \sum_i w^i(s)\left(-\frac{dn^i(s)}{d\tau}\right)}_{\Xi^{AE}\ \text{(Aggregate Efficiency)}} + \underbrace{\sum_s \omega_1(s)\, \mathbb{C}ov_i^\Sigma\left[\frac{\omega_1^i(s)}{\omega_1(s)}, -w^i(s)\, n^i(s)\right]}_{\Xi_{RS}\ \text{(Risk-Sharing)}}, \tag{23}$$

where $\omega(s) = \frac{1}{I}\sum_i \omega^i(s)$ and where we use the fact that the normalized individual weight ω^i is identical across individuals. As illustrated in the right panel of Figure 5, aggregate efficiency welfare losses are 0 at $\tau = 0$ and become increasingly negative as τ increases, for the same reason as in the deterministic case. Risk-sharing gains are strictly positive but decreasing, as in the deterministic case. Hence, this optimal taxation problem is also well-behaved and features an optimal interior tax $\tau^\star$ that optimally trades off aggregate efficiency losses with risk-sharing gains.

Remark 4. *(Pareto Improvement with $\Xi^E > 0$ and $\Xi^{AE} < 0$)* A tax increase in the random earnings model illustrates how a welfare assessment can concurrently feature $\Xi^E > 0$ and $\Xi^{AE} < 0$. In this economy, an increase in the tax rate below $\tau^\star$ is indeed a Pareto improvement, which necessarily implies that $\Xi^E > 0$, per Proposition 2c). In that region, aggregate consumption and, more importantly, aggregate instantaneous welfare gains decrease as τ increases, which implies that $\Xi^{AE} < 0$. However, the gains from reallocating consumption from individuals with high to low normalized stochastic weights are sufficiently large to make such tax increases desirable and ultimately a positive tax optimal.

6.2.3 Comparison of Deterministic and Random Earnings Models

This application shows that the welfare decomposition can be used to argue that the deterministic and random earnings models feature different equity-efficiency tradeoffs. This contrasts with the following statement by Piketty and Saez (2013):

> *"(...) the random earnings model generates both the same equity-efficiency tradeoff and the same type of optimal tax formula (as the deterministic model)."*

Figure 5 precisely compares these two models. While we have parametrized both models so as to yield the same normalized welfare assessment $\frac{dW^\lambda}{d\tau}$ and optimal tax $\tau^\star$, the welfare decomposition shows that the rationale justifying the optimal tax in both models is substantially different.

In both models, increasing the tax rate has identical distortionary effects reducing labor supply, which leads to a reduction in aggregate instantaneous welfare gains and a negative aggregate efficiency component. However, in the deterministic model the source of welfare gains is redistribution across individuals, while in the random earnings model the source of welfare gains is risk-sharing. Moreover, in the random earnings model *all* welfarist planners agree on the magnitude of the optimal tax, while in the deterministic model the optimal tax is sensitive to the choice of welfare function. This occurs because in the random earnings model there is no equity-efficiency tradeoff: all welfare gains are efficiency gains. In more realistic models in which individuals are both heterogeneous at the assessment and face uninsured risks (see e.g. Heathcote, Storesletten and Violante (2017)), both Ξ^{AE}, Ξ^{RS}, and Ξ^{RD} (and typically Ξ^{IS} in a model with more dates) will interact non-trivially to shape the optimal policy.

6.3 Application 3: Credit Constraint Relaxation

This application studies the welfare implications of a change in credit conditions in an economy in which borrowing-constrained individuals make an investment decision. Varying the borrowing limit in this economy is a tractable perturbation that parameterizes changes in the degree of market completeness. This application, in which all four components of the welfare decomposition are non-zero, illustrates how the welfare decomposition is useful to uncover subtle normative implications of a perturbation.

6.3.1 Environment

We consider a two-date economy populated by two (types of) individuals, $i \in \{1, 2\}$, with identical preferences, given by

$$u\left(c_0^i\right) + \beta \sum_s \pi(s) u\left(c_1^i(s)\right),$$

where c_0^i and $c_1^i(s)$ denote consumption of the single consumption good, $\pi(s)$ denotes the probabilities of different states at date 1; β is a discount factor, and $u(c)$ denotes the instantaneous utility function. Since this application assumes perfect competition, individuals in this model correspond to a continuum of agents in equal measure. We refer to $i = 1$ individuals as investors and to $i = 2$ individuals as lenders.

Investors face budget constraints given by

$$c_0^i = n_0 + q_0 b_0^i - \Upsilon^i\left(k_0^i\right)$$
$$c_1^i(s) = n_1(s) + z(s) k_0^i - b_0^i,$$

where n_0 and $n_1(s)$ denote endowments of the consumption good, b_0^i denotes the face value of the amount borrowed at price q_0^i (the interest rate in this economy is $1/q_0^i$), and $\Upsilon^i(k_0^i)$ denotes the cost of producing k_0^i units of capital at date 0, which yields $z(s)$ units at date 1 in state s. For simplicity, we assume that there are two states $s = \{H, L\}$, with $z(H) > z(L)$, and that $\sum_s \pi(s) z(z)$ is sufficiently large so that investors always find it optimal to invest and borrow. Lenders face identical budget constraints, but cannot operate the capital technology, so $\Upsilon^2(k_0^2) = k_0^2 = 0$.

Investors can borrow up to a predetermined borrowing limit $\bar{b}$:

$$b_0^i \leq \bar{b}.$$

Therefore, this economy features two forms of market incompleteness: i) investors cannot arrange insurance from lenders against the investment risk they bear since they only have access to a non-contingent security, and ii) investors and lenders cannot frictionlessly borrow and save when the borrowing constraint binds.[16]

An equilibrium is characterized by allocations $\{c_0^i, c_1^i(s), b_0^i, k_0^1\}$ and a price of the riskless asset

[16] An alternative exercise that we do not explore here is to understand the welfare impact of changing the aggregate amount of public debt — see, for instance, Woodford (1990), or more recently Azzimonti and Yared (2019), among others.

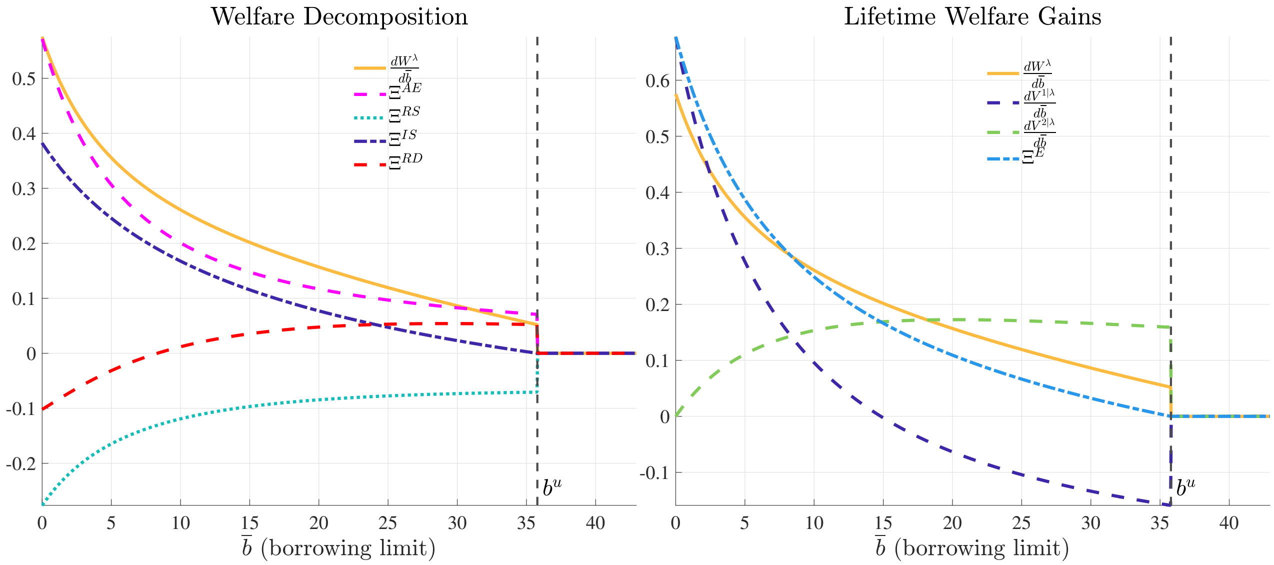

Figure 6: Welfare Decomposition and Lifetime Welfare Gains (Application 3)

Note: The left panel of this figure shows the welfare assessment and the components of the welfare decomposition as a function of the borrowing limit $\bar{b}$. The right panel shows the normalized lifetime welfare gains for investors and lenders, the normalized welfare assessment, and the efficiency component, as a function of the borrowing limit $\bar{b}$.

q_0 that clears the market for borrowing and saving, so that $\sum_i b_0^i = 0$. When solving the model, we assume that $\Upsilon^1\left(k_0^1\right) = \frac{\phi}{2}\left(k_0^1\right)^2$ and $u\left(c\right) = \frac{c^{1-\gamma}}{1-\gamma}$. Our parameterization assumes $\beta = 0.95$, $\gamma = 1.5$, $\phi = 0.1$, $z\left(L\right) = 5$, $z\left(H\right) = 35$, $\pi\left(L\right) = 0.7$, and $n_0^i = n_1^i\left(s\right) = 40$.

6.3.2 Welfare Decomposition

We consider the welfare effects of varying the borrowing limit $\bar{b}$ from 0, which corresponds to an autarky economy, until b^u, the level at which the borrowing constraint ceases to bind. The left panel in Figure 6 shows the normalized welfare assessment and the welfare decomposition associated with this perturbation. The right panel shows normalized lifetime welfare gains for investors and lenders, the normalized welfare assessment, and the efficiency component.

Figure 6 illustrates how changes in the borrowing limit impact welfare through the four components of the welfare decomposition, with each component taking the following sign:

$$\frac{dW^\lambda}{d\bar{b}} = \underbrace{\Xi^{AE}}_{>0} + \underbrace{\Xi^{RS}}_{<0} + \underbrace{\Xi^{IS}}_{>0} + \underbrace{\Xi^{RD}}_{\gtreqless 0}.$$

First, as we relax the borrowing limit, investors are able to invest more, which increases discounted (using an aggregate discount factor) aggregate instantaneous welfare gains, which in turn implies $\Xi^{AE} > 0$. Since relaxing the borrowing limit also increases the resources available to investors at date 0, when their relative valuation is higher, this implies $\Xi^{IS} > 0$. As the borrowing limit approaches the unconstrained level of borrowing b^u, intertemporal-sharing tends towards zero because, at that point, dynamic weights are equalized across investors and lenders. In contrast, Ξ^{AE} is still strictly positive. This is explained by the fact that markets remain incomplete, so the economy remains below first-best.

Second, as we relax the borrowing limit and investors increase their investment, their consumption becomes relatively more exposed to the investment risk, which they are unable to share with lenders: formally $\mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_1^i(s)}{\omega_1(s)}, \frac{dc_1^i(s)}{d\bar{b}}\right] < 0$, where $\frac{dc_1^i(s)}{d\bar{b}} = z(s)\frac{dk_0^i}{d\bar{b}} - \frac{db_0^i}{d\bar{b}}$. This justifies why $\Xi^{RS} < 0$ and illustrates how making markets more complete — in the sense of relaxing a borrowing constraint — may be associated with a negative risk-sharing component, a phenomenon that may seem counterintuitive at first. Similar to Ξ^{AE}, as the borrowing limit approaches b^u, Ξ^{RS} is still strictly negative, as markets remain incomplete for any level of $\bar{b}$.

Third, as we relax the borrowing limit, Ξ^{RD} switches from negative to positive. In this economy, the value of a unit of permanent consumption for lenders is higher than for investors since the former lack access to the profitable investment technology. This difference explains why the normalized individual weights of a utilitarian planner are $\omega^1 \approx 0.82$ and $\omega^2 \approx 1.18$, favoring welfare gains by lenders. As we show in the Appendix, exploiting optimality conditions, it is possible to show that lifetime welfare gains take the form

$$\frac{dV^i}{d\bar{b}} = \underbrace{\left(u'\left(c_0^i\right)q_0 - \beta\sum_s \pi(s)\,u'\left(c_1^i(s)\right)\right)\frac{db_0^i}{d\bar{b}}}_{\text{Direct Borrowing Effect}} + \underbrace{u'\left(c_0^i\right)\frac{dq_0}{d\bar{b}}b_0^i}_{\substack{\text{Distributive}\\ \text{Pecuniary Effect}}}. \tag{24}$$

The direct borrowing effect in (24) is zero for lenders and strictly positive for constrained investors, while the distributive pecuniary effects are zero-sum in units of date-0 consumption, that is $\sum_i \frac{dq_0}{d\bar{b}} b_0^i = 0$ (Dávila and Korinek, 2018). Because relaxing the borrowing limit increases borrowing and interest rates, $\frac{dq_0}{d\bar{b}} < 0$, the distributive pecuniary effect hurts those who borrow (investors) and benefits those who lend (lenders). The right panel in Figure 6 shows that relaxing the borrowing limit for low values of $\bar{b}$ benefits both investors and lenders, with the former benefiting more. As the borrowing limit $\bar{b}$ increases, investors' marginal welfare gains are reduced and eventually turn negative. This occurs because each individual investor fails to internalize how borrowing more increases interest rates in the competitive equilibrium, hurting other investors. The right panel of Figure 6 shows that Ξ^{RD} turns negative when the normalized lifetime welfare gains for investors and lenders intersect.

Finally, it is worth highlighting that the efficiency component as a whole, $\Xi^{E} = \Xi^{AE} + \Xi^{RS} + \Xi^{IS}$, is always positive, becoming zero as the borrowing limit approaches b^u. While it is natural that the efficiency implications of relaxing the constraint approach zero as the constraint ceases to bind, this result implies that $\Xi^{AE} + \Xi^{RS} = 0$ when $\bar{b} = b^u$, further explaining why aggregate efficiency and risk-sharing take opposite signs at that point.

7 Conclusion

This paper introduces a decomposition of welfare assessments into four components: i) aggregate efficiency, ii) intertemporal-sharing, iii) risk-sharing, and iv) redistribution. For welfarist planners, the decomposition — which satisfies desirable properties — is based on constructing individual, dynamic, and stochastic weights that characterize how a planner makes tradeoffs across individuals, dates, and histories. For DS-planners, such weights are defined as a primitive of the welfare assessment, which allows for a systematic formalization of new welfare criteria based on the decomposition.

Retrospectively, the welfare decomposition opens the door to revisiting the exact rationales that have justified welfarist welfare assessments in existing work. Looking forward, we hope that our results inform ongoing and future discussions on i) the desirability of particular policies and the welfare impact of shocks, and ii) the design of policy-making mandates.

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Appendix

A Proofs and Derivations: Section 3

Proof of Lemma 1. (Normalized Welfare Gains and Normalized Weights)

Proof. We can express an (unnormalized) welfare assessment $\frac{dW}{d\theta}$ as

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \frac{dV^i}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \lambda^i \frac{\frac{dV^i}{d\theta}}{\lambda^i},$$

so the normalized welfare assessment takes the form

$$\frac{dW^\lambda}{d\theta} = \frac{\frac{dW}{d\theta}}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i} = \sum_i \omega^i \frac{\frac{dV^i}{d\theta}}{\lambda^i}, \quad \text{where} \quad \omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}.$$

We can then express lifetime welfare gains in units of the lifetime welfare numeraire as

$$\begin{aligned}
\frac{dV^{i|\lambda}}{d\theta} = \frac{\frac{dV^i}{d\theta}}{\lambda^i} &= \sum_t \sum_{s^t} \frac{\left(\beta^i\right)^t \pi_t\left(s^t|s_0\right)}{\lambda^i} \frac{dV_t^i\left(s^t\right)}{d\theta} = \sum_t \sum_{s^t} \frac{\left(\beta^i\right)^t \pi_t\left(s^t|s_0\right)\lambda_t^i\left(s^t\right)}{\lambda^i} \frac{\frac{dV_t^i\left(s^t\right)}{d\theta}}{\lambda_t^i\left(s^t\right)} \\
&= \sum_t \frac{\sum_{s^t}\left(\beta^i\right)^t \pi_t\left(s^t|s_0\right)\lambda_t^i\left(s^t\right)}{\lambda^i} \sum_{s^t} \frac{\left(\beta^i\right)^t \pi_t\left(s^t|s_0\right)\lambda_t^i\left(s^t\right)}{\sum_{s^t}\left(\beta^i\right)^t \pi_t\left(s^t|s_0\right)\lambda_t^i\left(s^t\right)} \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} \\
&= \sum_t \omega_t^i \sum_{s^t} \omega_t^i\left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta},
\end{aligned}$$

where $\omega_t^i = \frac{\sum_{s^t}\left(\beta^i\right)^t \pi_t\left(s^t|s_0\right)\lambda_t^i\left(s^t\right)}{\lambda^i}$, $\omega_t^i\left(s^t\right) = \frac{\left(\beta^i\right)^t \pi_t\left(s^t|s_0\right)\lambda_t^i\left(s^t\right)}{\sum_{s^t}\left(\beta^i\right)^t \pi_t\left(s^t|s_0\right)\lambda_t^i\left(s^t\right)}$, and $\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}$ is defined in equation (6). Equations (7), (8), and (9) follow from the choice of welfare numeraires formalized in equation (OA11). ☐

Proof of Proposition 1. (Efficiency/Redistribution Decomposition)

Proof. For any two random variables x_i and y_i, it follows that $\sum_i x_i y_i = \frac{1}{I}\sum_i x_i \sum_i y_i + \mathbb{C}ov_i^{\Sigma}\left[x_i, y_i\right]$, where $\mathbb{C}ov_i^{\Sigma}\left[x_i, y_i\right] = I \cdot \mathbb{C}ov_i\left[x_i, y_i\right]$. Equation (10) follows from

$$\frac{dW^\lambda}{d\theta} = \sum_i \omega^i \frac{dV^{i|\lambda}}{d\theta} = \underbrace{\sum_i \frac{dV^{i|\lambda}}{d\theta}}_{\Xi^E} + \underbrace{\mathbb{C}ov_i^{\Sigma}\left[\omega^i, \frac{dV^{i|\lambda}}{d\theta}\right]}_{\Xi^{RD}},$$

where we use the fact that $\frac{1}{I}\sum_i \omega^i = 1$. This is the unique decomposition of the weighted sum $\sum_i \omega^i \frac{dV^{i|\lambda}}{d\theta}$ into an unweighted sum and its complement. ☐

Proof of Proposition 2. (Properties of Efficiency/Redistribution Decomposition)

Proof. a) This result follows from the fact that the SWF exclusively impacts the definition of ω^i, and that $\frac{dV^{i|\lambda}}{d\theta}$ — equivalently, normalized dynamic and stochastic weights — is invariant to the SWF.

b) This result follows from the fact that $\frac{dV^{i|\lambda}}{d\theta}$ — equivalently, normalized dynamic and stochastic weights — is invariant to the preference-preserving transformations considered.

c) In a Pareto-improving perturbation, it must be that $\frac{dV^{i|\lambda}}{d\theta} \geq 0$ for all individuals, with a strict inequality for at least one individual. Therefore, $\sum_i \frac{dV^{i|\lambda}}{d\theta} > 0$. □

Proof of Proposition 3. (Aggregate Efficiency/Risk-Sharing/Intertemporal-Sharing Decomposition)

Proof. Starting from the definition of the efficiency component:

$$\begin{aligned}
\Xi^E &= \sum_i \frac{\frac{dV^i}{d\theta}}{\lambda^i} = \sum_i \sum_t \omega_t^i \sum_{s^t} \omega_t^i \left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} = \sum_i \sum_t \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta} = \sum_t \sum_i \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta} \\
&= \sum_t \omega_t \sum_i \frac{\omega_t^i}{\omega_t} \frac{dV_t^{i|\lambda}}{d\theta} = \sum_t \omega_t \left(\sum_i \frac{dV_t^{i|\lambda}}{d\theta} + \mathbb{C}ov_i^{\Sigma} \left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}}{d\theta} \right] \right) \\
&= \sum_t \omega_t \sum_i \frac{dV_t^{i|\lambda}}{d\theta} + \underbrace{\sum_t \omega_t \mathbb{C}ov_i^{\Sigma} \left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}}{d\theta} \right]}_{\Xi^{IS}},
\end{aligned}$$

where $\omega_t = \frac{1}{I} \sum_i \omega_t^i$ and $\frac{dV_t^{i|\lambda}}{d\theta} = \sum_{s^t} \omega_t^i \left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}$. Note that $\mathbb{C}ov_i^{\Sigma} \left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}}{d\theta} \right] = \sum_i \left(\frac{\omega_t^i}{\omega_t} - 1 \right) \frac{dV_t^{i|\lambda}}{d\theta}$. This is the unique decomposition of the weighted sum $\sum_i \frac{\omega_t^i}{\omega_t} \frac{dV_t^{i|\lambda}}{d\theta}$ into an unweighted sum and its complement.

Moreover

$$\begin{aligned}
\sum_i \frac{dV_t^{i|\lambda}}{d\theta} &= \sum_i \sum_{s^t} \omega_t^i \left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} = \sum_{s^t} \sum_i \omega_t^i \left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} \\
&= \sum_{s^t} \omega_t \left(s^t\right) \sum_i \frac{\omega_t^i \left(s^t\right)}{\omega_t \left(s^t\right)} \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} = \sum_{s^t} \omega_t \left(s^t\right) \left(\sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} + \mathbb{C}ov_i^{\Sigma} \left[\frac{\omega_t^i \left(s^t\right)}{\omega_t \left(s^t\right)}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} \right] \right) \\
&= \underbrace{\sum_{s^t} \omega_t \left(s^t\right) \sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}}_{\Xi_t^{AE}} + \underbrace{\sum_{s^t} \omega_t \left(s^t\right) \mathbb{C}ov_i^{\Sigma} \left[\frac{\omega_t^i \left(s^t\right)}{\omega_t \left(s^t\right)}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} \right]}_{\Xi_t^{RS}},
\end{aligned}$$

where $\omega_t \left(s^t\right) = \frac{1}{I} \sum_i \omega_t^i \left(s^t\right)$, and where $\Xi^{AE} = \sum_t \omega_t \Xi_t^{AE}$ and $\Xi^{RS} = \sum_t \omega_t \Xi_t^{RS}$. Note that $\mathbb{C}ov_i^{\Sigma} \left[\frac{\omega_t^i (s^t)}{\omega_t (s^t)}, \frac{dV_t^{i|\lambda}(s^t)}{d\theta} \right] = \sum_i \left(\frac{\omega_t^i (s^t)}{\omega_t (s^t)} - 1 \right) \frac{dV_t^{i|\lambda}(s^t)}{d\theta}$. This is the unique decomposition of the weighted sum $\sum_i \frac{\omega_t^i (s^t)}{\omega_t (s^t)} \frac{dV_t^{i|\lambda}(s^t)}{d\theta}$ into an unweighted sum and its complement. □

Proof of Proposition 4. (Properties of Aggregate Efficiency/Risk-Sharing/Intertemporal-Sharing Decomposition)

Proof. a) When marginal rates of substitution are equalized across all dates and histories across individuals, $\omega_t^i\left(s^t\right) = \omega_t\left(s^t\right)$ and $\omega_t^i = \omega_t$. Alternatively, from equation (13), when markets are complete there is a unique stochastic discount factor, which implies that $q_t^i\left(s^t\right) = q_t\left(s^t\right)$. Proposition 5a) then implies that $\Xi^{RS} = \Xi^{IS} = 0$.

b) When marginal rates of substitution are equalized across all dates across individuals, $\omega_t^i = \omega_t$. Alternatively, from equation (13), when individuals can frictionlessly borrow and save, $\sum_{s^t} q_t^i\left(s^t\right)$ is identical across individuals. Proposition 5a) then implies that $\Xi^{IS} = 0$.

c) This result follows from the definition of Ξ^{AE} when $\sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} = 0$, $\forall s^t$. □

Proof of Proposition 5. (Properties of Welfare Assessment Decomposition: Individual-Invariant Weights or Welfare Gains)

Proof. a) If $\omega_t^i\left(s^t\right)$ are identical across individuals, $\mathbb{C}ov_i^{\Sigma}\left[\omega_t^i\left(s^t\right), \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right] = 0$, $\forall t$, $\forall s^t$, so $\Xi^{RS} = 0$. If ω_t^i are identical across individuals, $\mathbb{C}ov_i^{\Sigma}\left[\omega_t^i, \frac{dV_t^{i|\lambda}}{d\theta}\right] = 0$, $\forall t$, so $\Xi^{IS} = 0$. If ω^i are identical across individuals, $\mathbb{C}ov_i^{\Sigma}\left[\omega^i, \frac{dV^{i|\lambda}}{d\theta}\right] = 0$, so $\Xi^{RD} = 0$.

b) If $\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}$ are identical across individuals, $\mathbb{C}ov_i^{\Sigma}\left[\omega_t^i\left(s^t\right), \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right] = 0$, $\forall t$, $\forall s^t$, so $\Xi^{RS} = 0$. If $\frac{dV_t^{i|\lambda}}{d\theta}$ are identical across individuals, $\mathbb{C}ov_i^{\Sigma}\left[\omega_t^i, \frac{dV_t^{i|\lambda}}{d\theta}\right] = 0$, $\forall t$, so $\Xi^{IS} = 0$. If $\frac{dV^{i|\lambda}}{d\theta}$ are identical across individuals, $\mathbb{C}ov_i^{\Sigma}\left[\omega^i, \frac{dV^{i|\lambda}}{d\theta}\right] = 0$, so $\Xi^{RD} = 0$. □

Proof of Proposition 6. (Properties of Welfare Decomposition: Particular Economies)

Proof. a) If $I = 1$, all normalized weights are trivially identical across individuals. The result then follows from Proposition 5a).

b) If there is no risk, $\omega_t^i\left(s^t\right) = 1$, $\forall s^t$. The result then follows from Proposition 5a).

c) If individuals are ex-ante identical, ω^i and ω_t^i are identical across individuals. The result then follows from Proposition 5a).

d) Since we have assumed that $\pi_0\left(s^0 | s_0\right) = 1$, $\omega_0^i = \omega_0^i\left(s^t\right) = 1$ when $T = 0$.[17] The result then follows from Proposition 5a).

e) In a single good endowment economy, the single good is the only possible instantaneous welfare numeraire. Hence $\sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} = \sum_i \frac{dc_t^i\left(s^t\right)}{d\theta} = 0$, where the last equality follows from the fact that

[17] Models with a single date but multiple histories — as the random earnings scenario in Application 2 — can be interpreted as a two-date model in which instantaneous utility is zero at the initial date: see Section D.5 of the Online Appendix.

the aggregate consumption equals the aggregate endowment, which is invariant to the perturbation. The result then follows from the definition of Ξ^{AE}. $\square$

Proof of Proposition 7. (Properties of Welfare Decomposition: Transfers)

Proof. In a) a planner sets transfers so that ω^i is identical across individuals. In b), a planner sets transfers so that ω_t^i is identical across individuals. In c), a planner sets transfers so that $\omega_t^i\left(s^t\right)$ is identical across individuals. The results then follow from Proposition 5a). $\square$

Online Appendix

Sections B and C of this Online Appendix include proofs and derivations for Sections 4 and 6 of the paper. Sections D, E, and F include extensions and additional results, and Section G relates our results to existing work.

B Proofs and Derivations: Section 4

Proof of Proposition 8. (Relation between Welfarist and AE/AR/NR Pseudo-welfarist Planners)

Proof. a) The welfare assessment for the AE pseudo-welfarist DS-planner corresponds to

$$\frac{dW^{AE}}{d\theta} = \sum_t \omega_t^{\mathcal{W}} \sum_{s^t} \omega_t^{\mathcal{W}}\left(s^t\right) \sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta},$$

where $\Xi^{RS} = \Xi^{IS} = \Xi^{RD} = 0$, following Proposition 5a).

b) The welfare assessment for the AE pseudo-welfarist DS-planner corresponds to

$$\frac{dW^{AR}}{d\theta} = \sum_t \omega_t^{\mathcal{W}} \sum_{s^t} \omega_t^{\mathcal{W}}\left(s^t\right) \sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} + \sum_t \omega_t^{\mathcal{W}} \sum_{s^t} \omega_t^{\mathcal{W}}\left(s^t\right) \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^{i,\mathcal{W}}\left(s^t\right)}{\omega_t^{\mathcal{W}}\left(s^t\right)}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right],$$

where $\Xi^{IS} = \Xi^{RD} = 0$, following Proposition 5a).

c) The welfare assessment for the AE pseudo-welfarist DS-planner corresponds to

$$\frac{dW^{NR}}{d\theta} = \sum_t \omega_t^{\mathcal{W}} \sum_{s^t} \omega_t^{\mathcal{W}}\left(s^t\right) \sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} + \sum_t \omega_t^{\mathcal{W}} \sum_{s^t} \omega_t^{\mathcal{W}}\left(s^t\right) \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^{i,\mathcal{W}}\left(s^t\right)}{\omega_t^{\mathcal{W}}\left(s^t\right)}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]$$
$$+ \sum_t \omega_t \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^{i,\mathcal{W}}}{\omega_t^{\mathcal{W}}}, \frac{dV_t^{i|\lambda}}{d\theta}\right],$$

where $\Xi^{RD} = 0$, following Proposition 5a). □

General DS-planners. While in the body of the paper we focus on pseudo-welfarist DS-planners, it is straightforward to define DS-planners that are not pseudo-welfarist. In general, Proposition 5a) provides the recipe to define planners for whom specific components of the welfare decomposition are zero. For instance, one could choose the following weights to define an AE DS-planner:

$$\omega^{i,AE}\left(s_0\right) = 1, \quad \omega_t^{i,AE}\left(s_0\right) = \beta^t, \quad \text{and} \quad \omega_t^{i,AE}\left(s^t\right) = \pi_t\left(s^t \middle| s_0\right),$$

Table OA-1: Summary of DS-Planners

DS-Planners	Ξ^{AE}	Ξ^{RS}	Ξ^{IS}	Ξ^{RD}
Aggregate Efficiency (AE)	✓	$=0$	$=0$	$=0$
Aggregate Efficiency/Risk-Sharing (AR)	✓	✓	$=0$	$=0$
No-Redistribution (NR)	✓	✓	✓	$=0$
Welfarist ($\mathcal{W}$)	✓	✓	✓	✓

Note: This table illustrates the non-zero components of the welfare decomposition for particular DS-planners.

for some β, plausibly $\beta = \frac{1}{I}\sum_i \beta^i$. A similar logic can be used to define general (non-pseudo-welfarist) AR and NR planners. Table OA-1 summarizes which components of the welfare decomposition are zero general DS-planners. Non-pseudo-welfarist DS-planners may be helpful in particular applications, partly because they may be easier to operationalize.

DS-weights vs. Normalized Weights. Earlier versions of this paper defined DS-planners in terms of DS-weights, $\tilde{\omega}_t^i\left(s^t\right)$, given by

$$\underbrace{\tilde{\omega}_t^i\left(s^t\right)}_{\text{DS-weight}} = \underbrace{\omega^i\left(s_0\right)}_{\text{individual}}\underbrace{\omega_t^i\left(s_0\right)}_{\text{dynamic}}\underbrace{\omega(s^t)}_{\text{stochastic}},$$

as the primitive to define DS-planners. Up to a choice of units for the (aggregate) welfare assessment, there is a one-to-one relation between both approaches.

Note that these formulations respect intratemporal tradeoffs, by taking $\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}$, as defined in (6), as a primitive of the welfare assessment. By redefining $\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}$ as a weighted sum — based on generalized weights, chosen by a planner — of changes in consumption and factor supply it is possible to define a welfare objective based on generalized weights at the good/factor-history level.

Institutional Design. A central objective of this paper is to provide a framework to systematically formalize new welfare criteria to assess and conduct policy. This has the potential to guide the design of independent technocratic institutions. In practice, such institutions must be given a "mandate", much like defining a set of normalized weights.

Therefore, a society may want to consider designing independent technocratic institutions that have some normative considerations in their mandate but not others, along the lines of the logic developed in this paper. For instance, the current "dual mandate" (stable prices and maximum employment) of the Federal Reserve (as defined by the 1977 Federal Reserve Act) seems to be better described by an aggregate efficiency DS-planner, rather than a welfarist planner, which would care about cross-sectional considerations. Alternatively, an institution like the Federal Emergency Management Agency (FEMA) has as part of its mandate to "support the Nation in a risk-based, comprehensive emergency management system", which unavoidably involves risk-sharing considerations.

Instantaneous SWF Formulation. Section 4 shows that an approach based on generalized marginal DS-weights defined over instantaneous welfare gains allows us to systematically define non-welfarist normative objectives. Here, we show that it is possible to interpret $\frac{dW^{DS}}{d\theta}$, defined in Equation (14), as the welfare assessment of a planner with a (instantaneous) Social Welfare Function that i) takes as arguments individuals' instantaneous utilities, not lifetime utilities, and ii) features generalized (endogenous) welfare weights.

Formally, a linear instantaneous Social Welfare Function, which we denote by $\mathcal{I}(\cdot)$, is a linear function of individuals' instantaneous utilities, given by

$$\mathcal{I}\left(\left\{u_t^i\left(c_t^i\left(s^t\right),n_t^i\left(s^t\right)\right);s^t\right\}_{i,t,s^t}\right)=\sum_i\sum_t\sum_{s^t}\alpha_t^i\left(s^t\right)u_t^i\left(c_t^i\left(s^t\right),n_t^i\left(s^t\right);s^t\right), \tag{OA1}$$

where the instantaneous Pareto weights $\alpha_t^i\left(s^t\right)$ define scalars that are individual-, date-, and history-specific. For any set of DS-weights, there exist instantaneous Pareto weights $\left\{\alpha_t^i\left(s^t\right)\right\}_{i,t,s^t}$ such that $\frac{dW^{DS}}{d\theta}$, defined in Equation (14), corresponds to the first-order condition of a planner who maximizes a linear instantaneous SWF $\mathcal{I}(\cdot)$ with instantaneous Pareto weights $\alpha_t^i\left(s^t\right)=\omega_t^i\left(s^t;\theta\right)/\frac{\partial u^i\left(s^t;\theta\right)}{\partial c_t^i}$, since

$$\frac{d\mathcal{I}(\cdot)}{d\theta}=\sum_i\sum_t\sum_{s^t}\lambda_t^i\left(s^t\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\frac{du_t^i\left(s^t\right)}{d\theta}. \tag{OA2}$$

Moreover, at a local optimum, in which $\frac{dW^{DS}}{d\theta}=0$, there exist instantaneous Pareto weights $\left\{\alpha_t^i\left(s^t\right)\right\}_{i,t,s^t}$ such that the optimal policy satisfies the first-order condition formula of a linear instantaneous SWF $\mathcal{I}(\cdot)$, defined in Equation (OA1). The instantaneous Pareto weights in that case are evaluated at the optimum, so $\lambda_t^i\left(s^t\right)=\omega_t^i\left(s^t;\theta^\star\right)/\frac{\partial u_i\left(s^t;\theta^\star\right)}{\partial c_t^i}$, where $\theta^\star$ denotes the value of θ at the local optimum.

These results are helpful because they show how to reverse-engineer Pareto weights of a linear instantaneous SWF from DS-weights, while guaranteeing that any local optimum can be interpreted as the solution to the maximization of a particular linear instantaneous SWF. Because the instantaneous Pareto weights $\lambda_t^i\left(s^t\right)$ are evaluated at the optimum $\theta^\star$, they are taken as fixed in the maximization of a linear instantaneous SWF. In practice, it is impossible to define the instantaneous Pareto weights $\lambda_t^i\left(s^t\right)$ without first having solved for the optimum using our approach that starts with DS-weights as primitives. Relatedly, it is typically impossible to translate DS-weights into instantaneous Pareto weights that are invariant to θ and the rest of the environment. As mentioned above, there is scope to explore further the welfare implications of using SWFs directly defined over consumption or factor supply at histories.

DS-PLANNERS

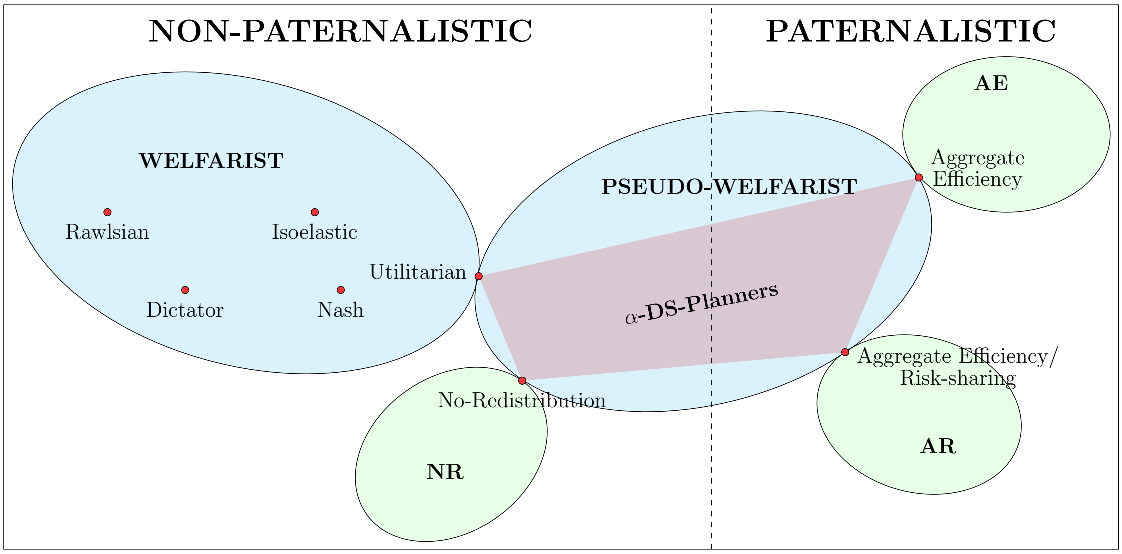

Figure OA-1: DS-planners: Summary

Note: This figure summarizes the relations between the different DS-planners. The vertical dashed line separates non-paternalistic planners from paternalistic planners. All welfarist planners, as well as no-redistribution (NR) planners, are non-paternalistic. Aggregate efficiency (AE) and aggregate efficiency/risk-sharing (AR) planners are paternalistic. Some pseudo-welfarist planners are non-paternalistic (welfarist, NR), while others are paternalistic (AE, AR). In this figure, the $\boldsymbol{\alpha}$-DS-planners are pseudo-welfarist with respect to the utilitarian planner.

$\boldsymbol{\alpha}$-DS-planners. The planners introduced in Section 4 and here by no means exhaust the set of new planners that can be defined using DS-weights. In fact, it is possible to define a new planner that spans i) AE, ii) AR, and iii) NR pseudo-welfarist planners, as well as iv) the associated normalized welfarist planner. We refer to this planner as an $\boldsymbol{\alpha}$-DS-planner.

Definition. *($\boldsymbol{\alpha}$-DS-planner: definition) An $\boldsymbol{\alpha}$-DS-planner is a DS-planner for whom the individual, dynamic, and stochastic weights are linear combinations of the weights of a normalized welfarist planner and the weights of an AE pseudo-welfarist planner. An $\boldsymbol{\alpha}$-DS-planner has DS-weights $\omega_t^{i,\mathcal{W},\boldsymbol{\alpha}}\left(s^t\middle|\, s_0\right)$ defined by*

$$\begin{aligned}
\omega_t^{i,\mathcal{W},\boldsymbol{\alpha}}\left(s^t\right) &= \left(1-\alpha_2\right)\omega_t^{i,\mathcal{W},AE}\left(s^t\right) + \alpha_2\omega_t^{i,\mathcal{W}}\left(s^t\right)\\
\omega_t^{i,\mathcal{W},\boldsymbol{\alpha}} &= \left(1-\alpha_3\right)\omega_t^{i,\mathcal{W},AE} + \alpha_3\omega_t^{i,\mathcal{W}}\\
\omega^{i,\mathcal{W},\boldsymbol{\alpha}} &= \left(1-\alpha_4\right)\omega^{i,\mathcal{W},AE} + \alpha_4\omega^{i,\mathcal{W}},
\end{aligned}$$

where $\boldsymbol{\alpha} = (\alpha_2, \alpha_3, \alpha_4)$, and where $\alpha_2 \in [0,1]$, $\alpha_3 \in [0,1]$, $\alpha_4 \in [0,1]$.

Depending on the value of $\boldsymbol{\alpha}$, an $\boldsymbol{\alpha}$-DS-planner behaves as a particular pseudo-welfarist planner or as a combination of pseudo-welfarist planners. When $\boldsymbol{\alpha} = (0,0,0)$, we have a pseudo-welfarist

AE DS-planner; when $\boldsymbol{\alpha} = (1, 0, 0)$, we have an pseudo-welfarist AR DS-planner; when $\boldsymbol{\alpha} = (1, 1, 0)$, we have a pseudo-welfarist NR DS-planner; and when $\boldsymbol{\alpha} = (1, 1, 1)$, we have a welfarist planner. By varying $\boldsymbol{\alpha}$, it is possible to model planners who care about the different components to different degrees. Moreover, estimating $\boldsymbol{\alpha}$ from actual policies in the context of a particular policy problem has the potential to uncover the weights that a particular policymaker attaches in practice to the different components of the welfare decomposition.

C Proofs and Derivations: Section 6

C.1 Application 1

Figure OA-2 here explains the behavior of Ξ_t^{IS} and Ξ_t^{RD} in Figure 4 in the text. Note that

$$\begin{aligned}
\Xi_t^{IS} &= \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}}{d\theta}\right] \\
\Xi_t^{RD} &= \mathbb{C}ov_i^{\Sigma}\left[\omega^i, \frac{\omega_t^i}{\omega_t}\frac{dV_t^{i|\lambda}}{d\theta}\right].
\end{aligned}$$

The left panel in Figure OA-2 shows that the risk-discounted welfare gains for individual i at date t, $\frac{dV_t^{i|\lambda}}{d\theta}$, are initially positive for the individual with the lower endowment at $s_0 = L$ (individual 1) and negative for the individual with the higher endowment, although they both converge to a positive common value. This captures the fact that the policy initially hurts the individual who starts with a high endowment and benefits the individual who starts with a low endowment, but as time goes by, the identity of the favored individual is uncertain. In the long run, both individuals are favored by the policy by eliminating consumption risk.

The right panel in Figure OA-2 shows that the risk-discounted welfare gains for individual i at date t relative to the average, $\frac{\omega_t^i}{\omega_t}\frac{dV_t^{i|\lambda}}{d\theta}$, converge to positive values for both individuals, but initially negative and higher in the long run for individual 2 (with the higher endowment at $s_0 = L$). Because $\frac{\omega_t^i}{\omega_t}\frac{dV_t^{i|\lambda}}{d\theta}$ does not converge to the same value for both individuals, Ξ_t^{RD} is non-zero (negative) in the long-run. Intuitively, while the long-run welfare gains of the policy at date t are positive and equal for both individuals at date t, such gains are valued more by the individual with a higher endowment at the time of the assessment, since this individual values future consumption in the future relatively more — see the dynamic weights in the left panel of Figure 2. And since the individual with the higher endowment at the time of the assessment also features a lower individual weight ω^i, this logic makes Ξ_t^{RD} negative in the long run.

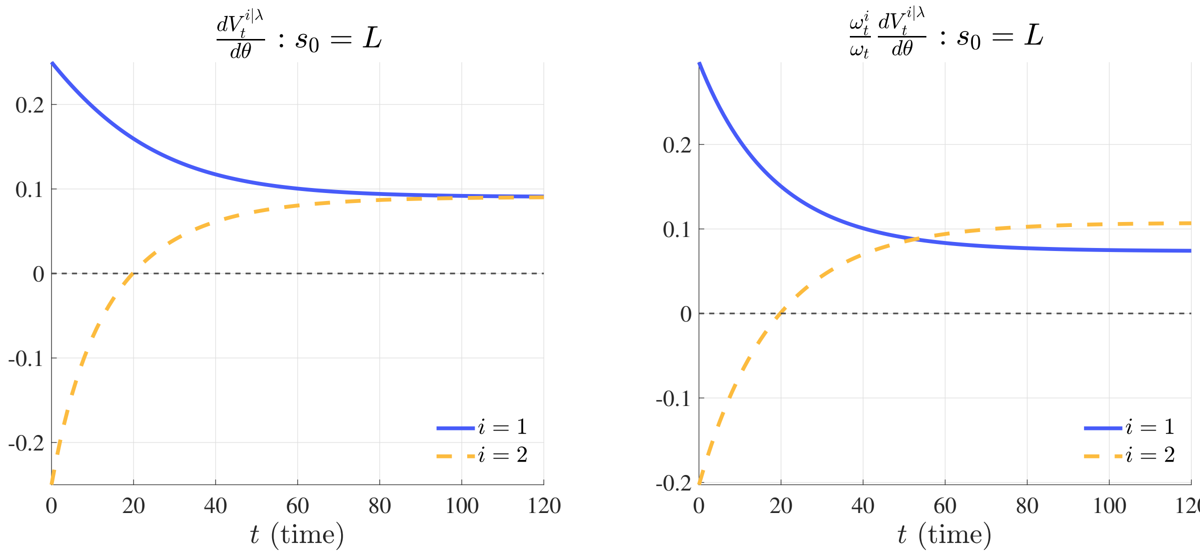

Figure OA-2: Welfare Decomposition and Lifetime Welfare Gains (Application 1)

Note: The left panel of this figure shows the risk-discounted welfare gains for individual i at date t, $\frac{dV_t^{i|\lambda}}{d\theta}$. The right panel of this figure shows the risk-discounted welfare gains for individual i at date t relative to the average, $\frac{\omega_t^i}{\omega_t}\frac{dV_t^{i|\lambda}}{d\theta}$

C.2 Application 2

C.2.1 Deterministic Earnings

The optimal consumption-labor decision for each individual i is given by

$$(1-\tau)\, w^i \frac{\partial u\left(c^i, n^i\right)}{\partial c^i} + \frac{\partial u\left(c^i, n^i\right)}{\partial n^i} = 0. \tag{OA3}$$

Given the assumed preferences $u\left(c^i, n^i\right) = \frac{1}{1-\gamma}\left(c - \alpha \frac{n^\sigma}{\sigma}\right)^{1-\gamma}$, equation (OA3) defines a labor supply function

$$n^i\left(\tau\right) = \left(\frac{\left(1-\tau\right) w^i}{\alpha}\right)^{\frac{1}{\sigma-1}},$$

which allows us to express the demogrant as $g\left(\tau\right) = \tau \frac{1}{I}\sum_i w^i n^i\left(\tau\right)$, which in turn implies that

$$\frac{dg}{d\tau} = \frac{1}{I}\left(\sum_i w^i n^i + \tau \sum_i w^i \frac{dn^i}{d\tau}\right).$$

We can express individual lifetime welfare gains $\frac{dV^i}{d\tau}$ as

$$\begin{aligned}\frac{dV^i}{d\tau} &= \frac{\partial u\left(c^i, n^i\right)}{\partial c^i}\frac{dc^i}{d\tau} + \frac{\partial u\left(c^i, n^i\right)}{\partial n^i}\frac{dn^i}{d\tau} \\ &= \frac{\partial u\left(c^i, n^i\right)}{\partial c^i}\left(-w^i n^i + (1-\tau)\, w^i \frac{dn^i}{d\tau} + \frac{dg}{d\tau}\right) + \frac{\partial u\left(c^i, n^i\right)}{\partial n^i}\frac{dn^i}{d\tau} \\ &= \frac{\partial u\left(c^i, n^i\right)}{\partial c^i}\left(-w^i n^i + \frac{dg}{d\tau}\right),\end{aligned}$$

which corresponds to equation (20) in the text, using consumption as lifetime welfare numeraire: $\lambda^i = \frac{\partial u\left(c^i, n^i\right)}{\partial c^i}$.

Hence, in this economy

$$\frac{dW^\lambda}{d\tau} = \sum_i \omega^i \frac{dV^{i|\lambda}}{d\tau} = \underbrace{\sum_i \frac{dV^{i|\lambda}}{d\theta}}_{\Xi^E} + \underbrace{\mathbb{C}ov_i^\Sigma\left[\omega^i, \frac{dV^{i|\lambda}}{d\theta}\right]}_{\Xi^{RD}},$$

where $\omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i}\frac{\partial u\left(c^i, n^i\right)}{\partial c^i}}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\frac{\partial u\left(c^i, n^i\right)}{\partial c^i}}$ (with $\frac{\partial \mathcal{W}}{\partial V^i} = 1$), and where

$$\begin{aligned}\Xi^E &= \sum_i \left(-w^i n^i + \frac{dg}{d\tau}\right) = \tau \sum_i w^i \frac{dn^i}{d\tau} \\ \Xi^{RD} &= \mathbb{C}ov_i^\Sigma\left[\omega^i, -w^i n^i + \frac{dg}{d\tau}\right] = \mathbb{C}ov_i^\Sigma\left[\omega^i, -w^i n^i\right],\end{aligned}$$

which corresponds to equation (21) in the text.

C.2.2 Random Earnings

The optimal consumption-labor decision for each individual i is identical to the deterministic case for a given realization of s. Hence,

$$(1-\tau)\, w^i(s) \frac{\partial u\left(c^i(s), n^i(s)\right)}{\partial c^i(s)} + \frac{\partial u\left(c^i(s), n^i(s)\right)}{\partial n^i(s)} = 0. \tag{OA4}$$

Hence, the labor supply function in state s is given by

$$n^i(s, \tau) = \left(\frac{(1-\tau)\, w^i(s)}{\alpha}\right)^{\frac{1}{\sigma - 1}},$$

where the demogrant, which the same regardless of s by virtue of the symmetry assumptions, is $g(\tau) = \tau \frac{1}{I} \sum_i w^i(s) n^i(s)$, which in turn implies that

$$\frac{dg}{d\tau} = \frac{1}{I} \left(\sum_i w^i(s) n^i(s) + \tau \sum_i w^i(s) \frac{dn^i(s)}{d\tau} \right).$$

We can express individual lifetime welfare gains $\frac{dV^i}{d\tau}$ as

$$\begin{aligned}
\frac{dV^i}{d\tau} &= \sum_s \pi(s) \left(\frac{\partial u(c^i(s), n^i(s))}{\partial c^i(s)} \frac{dc^i(s)}{d\tau} + \frac{\partial u(c^i(s), n^i(s))}{\partial n^i} \frac{dn^i(s)}{d\tau} \right) \\
&= \sum_s \pi(s) \frac{\partial u(c^i(s), n^i(s))}{\partial c^i(s)} \left(\frac{dc^i(s)}{d\tau} + \frac{\frac{\partial u\left(c^i(s), n^i(s)\right)}{\partial n^i}}{\frac{\partial u(c^i(s), n^i(s))}{\partial c^i(s)}} \frac{dn^i(s)}{d\tau} \right) \\
&= \sum_s \pi(s) \frac{\partial u(c^i(s), n^i(s))}{\partial c^i(s)} \left(-w^i(s) n^i(s) + (1-\tau) w^i(s) \frac{dn^i(s)}{d\tau} + \frac{dg}{d\tau} + \frac{\frac{\partial u\left(c^i(s), n^i(s)\right)}{\partial n^i}}{\frac{\partial u(c^i(s), n^i(s))}{\partial c^i(s)}} \frac{dn^i(s)}{d\tau} \right) \\
&= \sum_s \pi(s) \frac{\partial u(c^i(s), n^i(s))}{\partial c^i(s)} \left(-w^i(s) n^i(s) + \frac{dg}{d\tau} \right),
\end{aligned}$$

which corresponds to equation (22) in the text, using permanent consumption as lifetime welfare numeraire: $\lambda^i = \sum_s \pi(s) \frac{\partial u\left(c^i(s), n^i(s)\right)}{\partial c^i(s)}$.

Given the symmetry assumptions, in this economy

$$\frac{dW^\lambda}{d\tau} = \sum_i \omega^i \frac{dV^{i|\lambda}}{d\tau} = \sum_i \frac{dV^{i|\lambda}}{d\theta} = \Xi^E,$$

since $\omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i} \sum_s \pi(s) \frac{\partial u\left(c^i(s), n^i(s)\right)}{\partial c^i(s)}}{\frac{1}{I} \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \sum_s \pi(s) \frac{\partial u\left(c^i(s), n^i(s)\right)}{\partial c^i(s)}}$ (with $\frac{\partial \mathcal{W}}{\partial V^i} = 1$) is identical across individuals, and where

$$\frac{dV^{i|\lambda}}{d\tau} = \sum_s \omega^i(s) \frac{dV_1^{i|\lambda}(s)}{d\theta}, \quad \text{where} \quad \frac{dV_1^{i|\lambda}(s)}{d\theta} = -w^i(s) n^i(s) + \frac{dg}{d\tau}.$$

Therefore

$$\frac{dW^\lambda}{d\tau} = \Xi^E = \underbrace{\sum_s \omega_1(s) \sum_i \frac{dV_1^{i|\lambda}(s)}{d\theta}}_{\Xi^{AE}} + \underbrace{\sum_s \omega_1(s) \mathbb{C}ov_i^{\Sigma} \left[\frac{\omega_1^i(s)}{\omega_1(s)}, \frac{dV_1^{i|\lambda}(s)}{d\theta} \right]}_{\Xi^{RS}},$$

where $\omega_1^i(s) = \frac{\pi(s)\frac{\partial u\left(c^i(s),n^i(s)\right)}{\partial c^i(s)}}{\sum_s \pi(s)\frac{\partial u\left(c^i(s),n^i(s)\right)}{\partial c^i(s)}}$, $\omega_1^i(s) = \frac{1}{I}\sum_i \omega_1^i(s)$, and where

$$\Xi^{AE} = \tau \sum_s \omega_1(s) \sum_i w^i(s) \frac{dn^i(s)}{d\tau}$$
$$\Xi^{RS} = \sum_s \omega_1(s) \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_1^i(s)}{\omega_1(s)}, -w^i(s) n^i(s)\right],$$

which corresponds to equation (23) in the text.

C.3 Application 3

In addition to market clearing, an equilibrium in this economy is characterized by i) the borrowing/saving optimality conditions for both individuals:

$$u'\left(c_0^i\right) q_0 - \beta \sum_s \pi(s) u'\left(c_1^i(s)\right) = \eta^i,$$

where $\eta^i \geq 0$ denotes the Lagrange multiplier in the borrowing constraint (with $\eta^2 = 0$), and ii) the investment optimality condition for investors:

$$u'\left(c_0^1\right) \Upsilon'\left(k_0^1\right) - \beta \sum_s \pi(s) u'\left(c_1^1(s)\right) z(s) = 0.$$

Provided that the returns to investment are sufficiently attractive (which we always assume), the investor's borrowing constraint binds whenever $\bar{b}$ is sufficiently low, but ceases to bind at a level of $\bar{b}$ we denote by b^u.

We can express individual lifetime welfare gains $\frac{dV^i}{d\bar{b}}$ as

$$\frac{dV^i}{d\bar{b}} = u'\left(c_0^i\right) dc_0^i - \beta \sum_s \pi(s) u'\left(c_1^i(s)\right) dc_1^i(s),$$

where consumption changes are given by

$$dc_0^i = \frac{dq_0}{d\bar{b}} b_0^i + q_0 \frac{db_0^i}{d\bar{b}} - \Upsilon'\left(k_0^i\right) \frac{dk_0^i}{d\bar{b}}$$
$$dc_1^i(s) = z(s) \frac{dk_0^i}{d\bar{b}} - \frac{db_0^i}{d\bar{b}}.$$

Hence, normalized individual lifetime welfare gains take the form

$$\frac{dV^{i|\lambda}}{d\bar{b}} = \frac{\frac{dV^i}{d\bar{b}}}{\lambda^i} = \omega_0^i dc_0^i + \omega_1 \sum_s \omega_1^i(s) dc_1^i(s),$$

where $\omega_0^i = \frac{u'\left(c_0^i\right)}{\lambda^i}$ and $\omega_1^i = \frac{\beta \sum_s \pi(s) u'\left(c_1^i(s)\right)}{\lambda^i}$ with $\lambda^i = u'\left(c_0^i\right) + \beta \sum_s \pi\left(s\right) u'\left(c_1^i\left(s\right)\right)$ and where $\omega_1^i\left(s\right) = \frac{\pi(s) u'\left(c_1^i(s)\right)}{\sum_s \pi(s) u'\left(c_1^i(s)\right)}$.

Hence, in this economy

$$\frac{dW^\lambda}{d\bar{b}} = \sum_i \omega^i \frac{dV^{i|\lambda}}{d\bar{b}} = \sum_i \frac{dV^{i|\lambda}}{d\bar{b}} + \mathbb{C}ov_i^\Sigma \left[\omega^i, \frac{dV^{i|\lambda}}{d\bar{b}}\right],$$

where $\omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i}\left(u'\left(c_0^i\right)+\beta \sum_s \pi(s) u'\left(c_1^i(s)\right)\right)}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\left(u'\left(c_0^i\right)+\beta \sum_s \pi(s) u'\left(c_1^i(s)\right)\right)}$ with $\frac{\partial \mathcal{W}}{\partial V^i} = 1$. Moreover

$$\Xi^E = \Xi^{AE} + \Xi^{RS} + \Xi^{IS},$$

where

$$\begin{aligned}
\Xi^{AE} &= \omega_0 \sum_i \frac{dc_0^i}{d\bar{b}} + \omega_1 \sum_s \omega_1\left(s\right) \sum_i \frac{dc_1^i\left(s\right)}{d\bar{b}} \\
\Xi^{RS} &= \omega_1 \sum_s \omega_1\left(s\right) \mathbb{C}ov_i^\Sigma \left[\frac{\omega_1^i\left(s\right)}{\omega_1\left(s\right)}, \frac{dc_1^i\left(s\right)}{d\bar{b}}\right] \\
\Xi^{IS} &= \omega_0 \mathbb{C}ov_i^\Sigma \left[\frac{\omega_0^i}{\omega_0}, \frac{dc_0^i}{d\bar{b}}\right] + \omega_1 \mathbb{C}ov_i^\Sigma \left[\frac{\omega_1^i}{\omega_1}, \sum_s \omega_1^i\left(s\right) \frac{dc_1^i\left(s\right)}{d\bar{b}}\right]
\end{aligned}$$

where $\omega_1^i = \frac{1}{I}\sum_i \omega_1^i$ and $\omega_1^i\left(s\right) = \frac{1}{I}\sum_i \omega_1^i\left(s\right)$. Note that the impact of the perturbation on aggregate consumption, which is the key input into the aggregate efficiency component, is given by

$$\begin{aligned}
\sum_i \frac{dc_0^i}{d\bar{b}} &= -\Upsilon'\left(k_0^1\right) \frac{dk_0^1}{d\bar{b}} \\
\sum_i \frac{dc_1^i\left(s\right)}{d\bar{b}} &= z\left(s\right) \frac{dk_0^1}{d\bar{b}}.
\end{aligned}$$

Note that we can use individual optimality (i.e., the envelope theorem) to express lifetime welfare gains as

$$\begin{aligned}
\frac{dV^i}{d\bar{b}} &= u'\left(c_0^i\right) dc_0^i - \beta \sum_s \pi\left(s\right) u'\left(c_1^i\left(s\right)\right) dc_1^i\left(s\right) \\
&= u'\left(c_0^i\right)\left(\frac{dq_0}{d\bar{b}} b_0^i + q_0 \frac{db_1^i}{d\bar{b}} - \Upsilon'\left(k_0^i\right) \frac{dk_0^i}{d\bar{b}}\right) - \beta \sum_s \pi\left(s\right) u'\left(c_1^i\left(s\right)\right)\left(z\left(s\right) \frac{dk_0^i}{d\bar{b}} - \frac{db_0^i}{d\bar{b}}\right) \\
&= \underbrace{\left(u'\left(c_0^i\right) q_0 - \beta \sum_s \pi\left(s\right) u'\left(c_1^i\left(s\right)\right)\right) \frac{db_0^i}{d\bar{b}}}_{\text{Direct Effect}} + \underbrace{u'\left(c_0^i\right) \frac{dq_0}{d\bar{b}} b_0^i}_{\text{Distributive Pecuniary Effect}},
\end{aligned}$$

where the direct effect is weakly positive for investors and zero for lenders, and the distributive pecuniary effect is negative for investors and positive for lenders, although zero sum in the aggregate at date 0 since $\sum_i \frac{dq_0}{db} b_0^i = 0$ (Dávila and Korinek, 2018). While this formulation is useful to understand how individual lifetime utility changes, it is not useful to decompose welfare assessments in the way introduced in this paper.

D Extensions: Generalized Environments

In this section, we describe how to extend our results to more general environments.

D.1 Heterogeneous Beliefs

Here, we describe how to use the framework introduced in this paper to make welfare assessments in environments with heterogeneous beliefs.[18] To model heterogeneous beliefs, we assume that individual preferences take the form

$$V^i = \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t^i \left(s^t \middle| s_0 \right) u_t^i \left(c_t^i \left(s^t \right), n_t^i \left(s^t \right), s^t \right),$$

where $\pi_t^i \left(s^t \middle| s_0 \right)$ denotes the beliefs held by individual i over histories, which are now individual-specific. In this case, welfarist welfare assessments (respecting individual beliefs) are as described in the body of the paper, simply using the following normalized weights:

$$\omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i} \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t^i \left(s^t \middle| s_0 \right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}}{\frac{1}{I} \sum_i \frac{\partial \mathcal{W}}{\partial V^i} \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t^i \left(s^t \middle| s_0 \right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}}$$

$$\omega_t^i = \frac{\left(\beta^i\right)^t \sum_{s^t} \pi_t^i \left(s^t | s_0 \right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}}{\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t^i \left(s^t \middle| s_0 \right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}}$$

$$\omega_t^i \left(s^t \right) = \frac{\pi_t^i \left(s^t | s_0 \right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}}{\sum_{s^t} \pi_t^i \left(s^t | s_0 \right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}}.$$

Alternatively, a paternalistic planner is a DS-planner — introduced in Section (4) — who computes welfare using different beliefs than those held by investors (potentially using a common belief), simply

[18] A recent literature has explored how to make normative assessments in environments with heterogeneous beliefs. See, among others, Brunnermeier, Simsek and Xiong (2014), Gilboa, Samuelson and Schmeidler (2014), Dávila (2023), Blume et al. (2018), Caballero and Simsek (2019), and Dávila and Walther (2023).

computes welfare assessment using the following normalized weights:

$$\omega^{i}=\frac{\frac{\partial\mathcal{W}}{\partial V^{i}}\sum_{t}\left(\beta^{i}\right)^{t}\sum_{s^{t}}\pi_{t}^{i,P}\left(s^{t}\middle|\,s_{0}\right)\frac{\partial u_{t}^{i}\left(s^{t}\right)}{\partial c_{t}^{i}}}{\frac{1}{I}\sum_{i}\frac{\partial\mathcal{W}}{\partial V^{i}}\sum_{t}\left(\beta^{i}\right)^{t}\sum_{s^{t}}\pi_{t}^{i,P}\left(s^{t}\middle|\,s_{0}\right)\frac{\partial u_{t}^{i}\left(s^{t}\right)}{\partial c_{t}^{i}}}$$

$$\omega_{t}^{i}=\frac{\left(\beta^{i}\right)^{t}\sum_{s^{t}}\pi_{t}^{i,P}\left(s^{t}|s_{0}\right)\frac{\partial u_{t}^{i}\left(s^{t}\right)}{\partial c_{t}^{i}}}{\sum_{t}\left(\beta^{i}\right)^{t}\sum_{s^{t}}\pi_{t}^{i,P}\left(s^{t}\middle|\,s_{0}\right)\frac{\partial u_{t}^{i}\left(s^{t}\right)}{\partial c_{t}^{i}}}$$

$$\omega_{t}^{i}\left(s^{t}\right)=\frac{\pi_{t}^{i,P}\left(s^{t}|s_{0}\right)\frac{\partial u_{t}^{i}\left(s^{t}\right)}{\partial c_{t}^{i}}}{\sum_{s^{t}}\pi_{t}^{i,P}\left(s^{t}|s_{0}\right)\frac{\partial u_{t}^{i}\left(s^{t}\right)}{\partial c_{t}^{i}}},$$

where $\pi_{t}^{i,P}\left(s^{t}|s_{0}\right)$ denotes the beliefs used by the planner to compute welfare for individual i at a particular history. In a single belief case, $\pi_{t}^{i,P}\left(s^{t}|s_{0}\right)=\pi_{t}^{P}\left(s^{t}|s_{0}\right)$, $\forall i$. See e.g. Dávila and Walther (2023) for an application of this approach to compute optimal leverage regulation.

D.2 General Preferences

D.2.1 Recursive utility: Epstein-Zin Preferences

Here, we describe how to allow for recursive preferences. In particular, we consider the widely used Epstein-Zin preferences, defined recursively as follows:

$$V^{i}\left(s\right)=\left(\left(1-\beta^{i}\right)\left(u^{i}\left(c^{i}\left(s\right),n^{i}\left(s\right)\right)\right)^{1-\frac{1}{\psi^{i}}}+\beta^{i}\left(\sum_{s'}\pi\left(s'|s\right)\left(V^{i}\left(s'\right)\right)^{1-\gamma^{i}}\right)^{\frac{1-\frac{1}{\psi^{i}}}{1-\gamma^{i}}}\right)^{\frac{1}{1-\frac{1}{\psi^{i}}}},$$

where γ^{i} modulates risk aversion and ψ^{i} modulates intertemporal substitution. We use s and s' to denote any two recursive states (Ljungqvist and Sargent, 2018).

In this case, we can recursively express the lifetime welfare gains of a perturbation in utils, as follows:

$$\frac{dV^{i}\left(s\right)}{d\theta}=\frac{\partial V^{i}\left(s\right)}{\partial c^{i}\left(s\right)}\frac{du^{i|\lambda}\left(s\right)}{d\theta}+\sum_{s'}\frac{\partial V^{i}\left(s\right)}{\partial V^{i}\left(s'\right)}\frac{dV^{i}\left(s'\right)}{d\theta},\tag{OA5}$$

where

$$\frac{\partial V^{i}\left(s\right)}{\partial c^{i}\left(s\right)}=\left(1-\beta^{i}\right)\left(V^{i}\left(s\right)\right)^{\frac{1}{\psi^{i}}}\left(u^{i}\left(s\right)\right)^{-\frac{1}{\psi^{i}}}\frac{\partial u^{i}\left(s\right)}{\partial c^{i}}$$

$$\frac{\partial V^{i}\left(s\right)}{\partial V^{i}\left(s'\right)}=\beta^{i}\left(V^{i}\left(s\right)\right)^{\frac{1}{\psi^{i}}}\left(\sum_{s'}\pi\left(s'|s\right)\left(V^{i}\left(s'\right)\right)^{1-\gamma^{i}}\right)^{\frac{\gamma^{i}-\frac{1}{\psi^{i}}}{1-\gamma^{i}}}\pi\left(s'|s\right)\left(V^{i}\left(s'\right)\right)^{-\gamma^{i}},$$

and where

$$\frac{du^{i|\lambda}(s)}{d\theta} = \frac{dc_t^i(s)}{d\theta} + \frac{\frac{\partial V^i(s)}{\partial n^i(s)}}{\frac{\partial V^i(s)}{\partial c^i(s)}} \frac{dn_t^i(s)}{d\theta}.$$

The structure of Equation (OA5) immediately implies that $\frac{dV^i(s)}{d\theta}$ can be expressed as a linear transformation of instantaneous welfare gains, which in turn guarantees that $\frac{dV^i(s)}{d\theta}$ can be written as in Lemma (1). It is easiest to leverage equation (13) to compute normalized weights via state-prices for any date and state, as follows:

$$q^i(s'|s) = \frac{\frac{\partial V_i(s)}{\partial c^i(s')}}{\frac{\partial V^i(s)}{\partial c^i(s)}} = \frac{\frac{\partial V^i(s)}{\partial V^i(s')} \frac{\partial V^i(s')}{\partial c^i(s')}}{\frac{\partial V^i(s)}{\partial c^i(s)}} = \beta^i \pi(s'|s) \left(\frac{V^i(s')}{H(s)}\right)^{\frac{1}{\psi^i} - \gamma^i} \left(\frac{c^i(s')}{c^i(s)}\right)^{-\frac{1}{\psi^i}} \frac{\frac{\partial u^i(s')}{\partial c^i}}{\frac{\partial u^i(s)}{\partial c^i}},$$

where $H(s) = \left(\sum_{s'} \pi(s'|s) \left(V^i(s')\right)^{1-\gamma^i}\right)^{\frac{1}{1-\gamma^i}}$. It is straightforward to define DS-weights for even more general preferences, including preferences that are not time-separable or recursive, as we do next.

D.2.2 General Non-separable Preferences

It is possible to consider general non-expected utility non-time separable preferences of the form (we abstract from factor supply only for simplicity, the results extend straightforwardly to that case):

$$V^i = U^i\left(\left\{c_t^i\left(s^t\right)\right\}_{t,s^t}\right).$$

Individual lifetime welfare gains take the form

$$\frac{dV^i}{d\theta} = \sum_t \sum_{s^t} \frac{\partial U^i}{\partial c_t^i(s^t)} \frac{dc_t^i(s^t)}{d\theta}.$$

From here it is evident that Lemma 1 applies, with normalized weights of the form

$$\omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i} \sum_t \sum_{s^t} \frac{\partial U^i}{\partial c_t^i(s^t)}}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i} \sum_t \sum_{s^t} \frac{\partial U^i}{\partial c_t^i(s^t)}}$$

$$\omega_t^i = \frac{\sum_{s^t} \frac{\partial U^i}{\partial c_t^i(s^t)}}{\sum_t \sum_{s^t} \frac{\partial U^i}{\partial c_t^i(s^t)}}$$

$$\omega_t^i\left(s^t\right) = \frac{\frac{\partial U^i}{\partial c_t^i(s^t)}}{\sum_{s^t} \frac{\partial U^i}{\partial c_t^i(s^t)}}.$$

D.3 Multiple Goods/Factors

Here, we extend the result to economies with multiple goods/factors. We extend the baseline environment by assuming individuals consume $J \geq 1$ goods, indexed by $j \in \mathcal{J} = \{1, \ldots, J\}$, and supply $F \geq 0$ factors, indexed by $f \in \mathcal{F} = \{1, \ldots, F\}$, At all dates and histories. In this case, preferences are given by

$$V^i = \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0\right) u_t^i \left(\left\{c_t^{ij}\left(s^t\right)\right\}_j, \left\{n_t^{if}\left(s^t\right)\right\}_f; s^t\right), \qquad \text{(Preferences)}$$

where $c_t^{ij}(s^t)$ and $n_t^{if}(s^t)$ respectively denote the consumption of good j and the amount of factor f supplied by individual i at history s^t.

In this case, individual lifetime welfare gains are given by

$$\frac{dV^i}{d\theta} = \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0\right) \lambda_t^i \left(s^t\right) dV_t^{i|\lambda}\left(s^t\right),$$

where

$$dV_t^{i|\lambda}\left(s^t\right) = \sum_j \frac{\frac{\partial u_t^i(s^t)}{\partial c_t^{ij}}}{\lambda_t^i(s^t)} \frac{dc_t^{ij}(s^t)}{d\theta} + \sum_f \frac{\frac{\partial u_t^i(s^t)}{\partial n_t^{if}}}{\lambda_t^i(s^t)} \frac{dn_t^{if}(s^t)}{d\theta},$$

which generalizes equation (6) in the text. Given this, Proposition 1 and all the other results follow straightforwardly.

D.4 Perturbations to Probabilities

In this section, we describe how to use DS-weights in environments in which policy changes affect probabilities. Starting from Equation (4), we can express $\frac{dV^i}{d\theta}$ as

$$\frac{dV^i}{d\theta} = \sum_t \left(\beta^i\right)^t \sum_{s^t} \left(\pi_t\left(s^t\middle| s_0\right)\left(\frac{\partial u_t^i(s^t)}{\partial c_t^i}\frac{dc_t^i(s^t)}{d\theta} + \frac{\partial u_t^i(s^t)}{\partial n_t^i}\frac{dn_t^i(s^t)}{d\theta}\right) + \frac{d\pi_t\left(s^t\middle| s_0\right)}{d\theta} u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right)\right).$$

Hence, the definition of Ξ^{RD} and Ξ^{IS} apply unchanged, with the addition of the new term that includes how the change in probabilities impacts lifetime and date t welfare gains, respectively. The split between aggregate efficiency and risk-sharing now includes a third term that takes the form

$$\sum_t \omega_t \sum_i \sum_{s^t} \zeta_t^i(s) \frac{d\pi_t\left(s^t\middle| s_0\right)}{d\theta}, \quad \text{where} \quad \zeta_t^i = \frac{u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right)}{\sum_{s^t} \pi_t\left(s^t\middle| s_0\right) \frac{\partial u_t^i(s^t)}{\partial c_t^i}}.$$

Since $\sum_{s^t} \frac{d\pi_t(s^t|s_0)}{d\theta} = 0$, a stochastic decomposition of this additional term can be written in terms of $\mathbb{C}ov_{s^t}\left[\zeta_t^i, \frac{d\pi_t(s^t|s_0)}{d\theta}\right]$.

D.5 Zeros

D.5.1 Zero Weights

In the body of the paper, we exclusively consider SWFs in which $\frac{\partial \mathcal{W}}{\partial V^i} > 0$, and implicitly, since we assume that individual marginal utilities of consumption are strictly positive at all times, normalized dynamic and stochastic weights such that $\omega_t^i > 0$ and $\omega_t^i \left(s^t\right) > 0$. Here, we present a generalized decomposition that accommodates normalized weights to be zero for particular individuals, dates, or histories. To allow for this possibility, we must appropriately define the set of individuals over which sums and covariances are computed.[19]

Formally, a normalized welfare assessment takes the form:

$$\frac{dW^\lambda}{d\theta} = \sum_i \omega^i \frac{dV^{i|\lambda}}{d\theta} = \underbrace{\sum_{i|\omega^i>0} \frac{dV^{i|\lambda}}{d\theta}}_{\Xi^E} + \underbrace{\mathbb{C}ov^{\Sigma}_{i|\omega^i>0}\left[\omega^i, \frac{dV^{i|\lambda}}{d\theta},\right]}_{\Xi^{RD}},$$

where $\mathbb{C}ov^{\Sigma}_{i|\omega^i>0}\left[\omega^i, \frac{dV^{i|\lambda}}{d\theta}\right] = \sum_i \mathbb{I}\left[\omega^i>0\right] \mathbb{C}ov_{i|\omega^i>0}\left[\omega^i, \frac{dV^{i|\lambda}}{d\theta}\right]$, where $\mathbb{I}\left[\cdot\right]$ denotes an indicator, and where $\omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}{\frac{1}{\sum_i \mathbb{I}\left[\omega^i>0\right]}\sum_{i|\omega^i>0}\frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}$. In this case, both efficiency and redistribution exclusively account for the lifetime welfare gains of those individuals for whom $\omega^i > 0$. Following the same steps as in the baseline case, the efficiency component can be expressed as

$$\Xi^E = \sum_{i|\omega^i>0} \frac{dV^{i|\lambda}}{d\theta} = \sum_t \sum_{i|\omega^i>0} \omega_t^i \frac{dV_t^{i|\lambda}}{d\theta} = \sum_t \omega_t \sum_{i|\omega^i,\omega_t^i>0} \frac{dV_t^{i|\lambda}}{d\theta} + \underbrace{\sum_t \omega_t \mathbb{C}ov^{\Sigma}_{i|\omega^i,\omega_t^i>0}\left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}}{d\theta}\right]}_{\Xi^{IS}},$$

where $\Xi^{AE} = \sum_t \omega_t \Xi_t^{AE}$ and $\Xi^{RS} = \sum_t \omega_t \Xi_t^{RS}$ with

$$\begin{aligned}\sum_{i|\omega^i,\omega_t^i>0} \frac{dV_t^{i|\lambda}}{d\theta} &= \sum_{s^t} \sum_{i|\omega^i,\omega_t^i>0} \omega_t^i\left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} \\ &= \underbrace{\sum_{s^t} \omega_t\left(s^t\right) \sum_{i|\omega^i,\omega_t^i,\omega_t^i(s^t)>0} \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}}_{\Xi_t^{AE}} + \underbrace{\sum_{s^t} \omega_t\left(s^t\right) \mathbb{C}ov^{\Sigma}_{i|\omega^i,\omega_t^i,\omega_t^i(s^t)>0}\left[\frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]}_{\Xi_t^{RS}},\end{aligned}$$

[19] We repeatedly use the fact that

$$\sum_i x^i y^i = \frac{1}{I^+} \sum_{i|x^i>0} x^i \sum_{i|x^i>0} y^i + \sum_{i|x^i>0} \left(x_i - \frac{1}{I^+}\sum_{i|x^i>0} x^i\right)\left(y_i - \frac{1}{I^+}\sum_{i|x^i>0} y^i\right),$$

where $I^+ = \sum_i \mathbb{I}\left[x^i>0\right]$.

where $\omega_t = \sum_{i|\omega^i,\omega_t^i>0} \omega_t^i$ and $\omega_t(s^t) = \sum_{i|\omega^i,\omega_t^i,\omega_t^i(s^t)>0} \omega_t^i(s^t)$, and where

$$\mathbb{C}ov^{\Sigma}_{i|\omega^i,\omega_t^i>0}\left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}}{d\theta}\right] = \sum_i \mathbb{I}\left[\omega^i, \omega_t^i > 0\right] \mathbb{C}ov_{i|\omega^i,\omega_t^i>0}\left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}}{d\theta}\right].$$

$$\mathbb{C}ov^{\Sigma}_{i|\omega^i,\omega_t^i,\omega_t^i(s^t)>0}\left[\frac{\omega_t^i(s^t)}{\omega_t(s^t)}, \frac{dV_t^{i|\lambda}(s^t)}{d\theta}\right] = \sum_i \mathbb{I}\left[\omega^i, \omega_t^i, \omega_t^i(s^t) > 0\right] \mathbb{C}ov_{i|\omega^i,\omega_t^i,\omega_t^i(s^t)>0}\left[\frac{\omega_t^i(s^t)}{\omega_t(s^t)}, \frac{dV_t^{i|\lambda}(s^t)}{d\theta}\right].$$

In this case, only individuals with positive dynamic or stochastic weights enter into Ξ^{AE}, Ξ^{RS}, and Ξ^{IS}.

It is worth highlighting that Proposition 2a) no longer holds when $\omega^i = 0$ for individuals. For instance, a dictator who exclusively cares about individual $i = 1$, attributes all welfare gains to the efficiency component (actually Ξ^{AE}) for individual 1, but the efficiency component for such a dictator is different from the efficiency component of a utilitarian planner or any other welfarist planner who puts strictly positive weight on all individuals. Propositions 2b) and c), as well as Propositions 4, 5, and Proposition 6a) through d) still hold, but Proposition 6e) also fails.

The central takeaway from these results is that the welfare decomposition must be interpreted only for the individuals for positive weights when i) planners completely disregard the welfare gains by specific individuals, or ii) individuals do not value at all welfare gains at particular dates or states. For instance, the redistribution component for a planner who exclusively cares about individuals $i = 1$ and $i = 2$ is exclusively based on the lifetime welfare gains of these two individuals, disregard the rest. The same logic applies to the remaining terms of the decomposition.

D.5.2 Zero Welfare Gains

One may be tempted to also condition the covariance decomposition on the welfare gains terms to be non-zero, e.g. $\frac{dV^i}{d\theta} \neq 0$, but this would lead to erroneous conclusions. For instance, it may be that $\frac{dV^{i|\lambda}}{d\theta} = 0$ when $\frac{dV_t^{i|\lambda}}{d\theta} \neq 0$ or $\frac{dV_t^{i|\lambda}(s^t)}{d\theta} \neq 0$, which would yield incorrect results. An implication of always considering all individuals with $\omega^i > 0$, even when $\frac{dV^{i|\lambda}}{d\theta} = 0$, is that Ξ^{RD}, as well as the split of the efficiency among its three constituents will depend on the normalized weights of all individuals in the economy, including those unaffected directly by the perturbation. However, the efficiency component as a whole will not.

D.6 Other considerations

Idiosyncratic/Aggregate States In recursive economies with idiosyncratic (and potentially aggregate) states (i.e., Aiyagari or Krusell-Smith style economies) individuals can be ex-ante heterogeneous at the time of making a welfare assessment for two different reasons. First, individuals can be heterogeneous ex-ante (e.g., individuals can have different time-invariant preferences or face shocks that come from different distributions). Second, individuals can be heterogeneous ex-post (e.g., individuals can have different endowments or asset holdings at the time of the welfare assessment, even though they face identical problems starting from a given idiosyncratic state). This distinction is important to interpret correctly some of the results in this paper. For instance, 6d) only applies when all individuals are identical because of predetermined reasons and when they all have the same initial state. Formally, ex-ante heterogeneity of either form is captured by the index i in this paper. It is possible to further refine the composition in environments that differentiate between idiosyncratic and aggregate states along the lines of section E.

Continuum of Individuals, Continuous Time, Continuum of States In order to highlight the differences between averages and signs, we have considered an environment with countable individuals, dates, and histories. It is straightforward to extend the results to environments with a continuum of individuals, continuous time, and a continuum of histories. In fact, earlier versions of this paper included examples of all three cases.

Non-differentiabilities It is possible to generalize the results to environments in which lifetime or instantaneous utilities are not differentiable. For lifetime utilities, it is necessary to consider global assessments, as described in Section F.3. For instantaneous utilities, it is typically possible to incorporate non-differentiabilities using Leibniz rule — see Dávila and Goldstein (2023) for an application.

E Extensions: Subdecompositions and Alternative Decompositions

In this section, we describe how to further decompose the components of the welfare decomposition introduced in this paper. At times, we refer to two properties of covariances:

$$\mathbb{C}ov_i^{\Sigma}\left[x^i, y^i z^i\right] = \mathbb{E}_i\left[y^i\right] \mathbb{C}ov_i^{\Sigma}\left[x^i, z^i\right] + \mathbb{E}_i\left[z^i\right] \mathbb{C}ov_i^{\Sigma}\left[x^i, y^i\right] + \sum_i \left[\left(x^i - \mathbb{E}_i\left[x^i\right]\right)\left(y^i - \mathbb{E}_i\left[y^i\right]\right)\left(z^i - \mathbb{E}_i\left[z^i\right]\right)\right] \tag{OA6}$$

$$\mathbb{C}ov_i^{\Sigma}\left[x^i, y^i\right] = \sum_i \left[\mathbb{C}ov_i\left[x^i, y^i \mid z^i\right]\right] + \mathbb{C}ov_i^{\Sigma}\left[\mathbb{E}_i\left[x^i \mid z^i\right], \mathbb{E}_i\left[y^i \mid z^i\right]\right], \tag{OA7}$$

where X, Y, and Z denote random variables. The first property is established in Bohrnstedt and Goldberger (1969). The second is the Law of Total Covariance, and is standard. Figure OA-3 illustrates the decompositions introduced in Subsections E.4 and E.5.

It is worth highlighting that Lemma 1 implies that any decomposition of welfare assessments

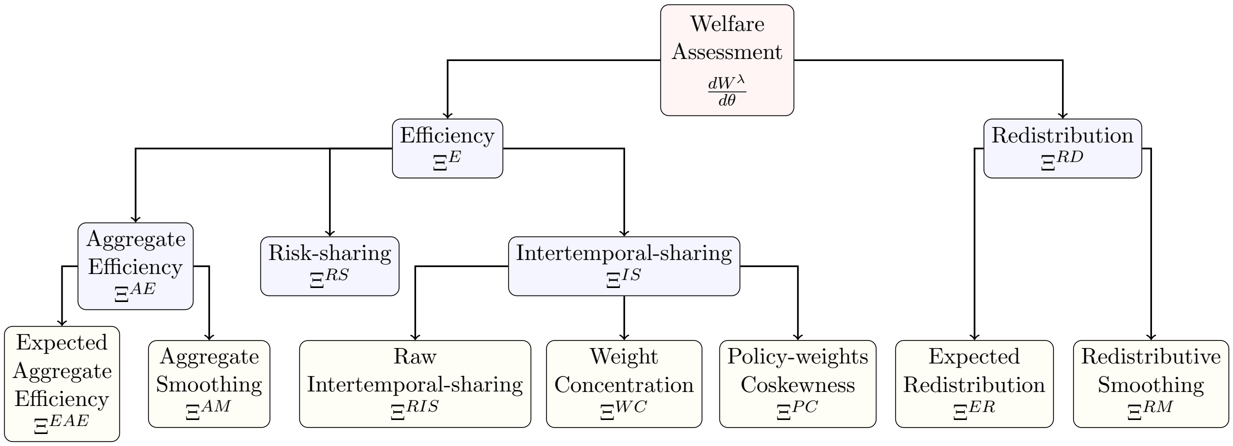

Figure OA-3: Subdecomposition

Note: This figure illustrates how the welfare decomposition can be subdecomposed. Section E describes multiple ways of subdecomposing the welfare decomposition and discusses alternative decompositions.

boils down to defining particular groupings of the triple sum:

$$\frac{dW^\lambda}{d\theta} = \sum_i \omega^i \sum_t \omega_t^i \sum_{s^t} \omega_t^i \left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}.$$

E.1 Term Structure and Related Results

Here, we show that the welfare decomposition and each of its components has a term structure. That is, it is possible to attribute welfare gains in the aggregate or for each of the components to particular dates in the future. Formally, note that

$$\frac{dW^\lambda}{d\theta} = \sum_t \omega_t \frac{dW_t^\lambda}{d\theta} \quad \text{where} \quad \frac{dW_t^\lambda}{d\theta} = \Xi_t^{AE} + \Xi_t^{RS} + \Xi_t^{IS} + \Xi_t^{RD}, \tag{OA8}$$

where

$$\Xi_t^{AE} = \sum_{s^t} \omega_t\left(s^t\right) \Xi_t^{AE}\left(s^t\right) \quad \text{where} \quad \Xi_t^{AE}\left(s^t\right) = \sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}$$

$$\Xi_t^{RS} = \sum_{s^t} \omega_t\left(s^t\right) \Xi_t^{RS}\left(s^t\right) \quad \text{where} \quad \Xi_t^{RS}\left(s^t\right) = \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]$$

$$\Xi_t^{IS} = \sum_{s^t} \omega_t\left(s^t\right) \Xi_t^{IS}\left(s^t\right) \quad \text{where} \quad \Xi_t^{IS}\left(s^t\right) = \mathbb{C}ov_i^{\Sigma}\left[\frac{\omega_t^i}{\omega_t}, \frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)} \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]$$

$$\Xi_t^{RD} = \sum_{s^t} \omega_t\left(s^t\right) \Xi_t^{RD}\left(s^t\right) \quad \text{where} \quad \Xi_t^{RD}\left(s^t\right) = \mathbb{C}ov_i^{\Sigma}\left[\omega^i, \frac{\omega_t^i}{\omega_t} \frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)} \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right].$$

This formulation shows that a welfare assessment can be interpreted as the discounted sum, using an aggregate discount factor — of date-specific welfare assessments, where each of these date-specific assessments can be further decomposed in aggregate efficiency, risk-sharing, intertemporal-sharing, and redistribution.

Transition vs. Steady State Welfare Gains. Equation (OA8) also allows us to decompose the transition and steady-state impact of perturbations for aggregate assessments and each of the components of the welfare decomposition. Formally, under the assumption that an economy reaches a new steady-state at date $T^{\star}$, it is possible to decompose welfare assessments into transition welfare effects and steady-state welfare effects:

$$\frac{dW^{\lambda}}{d\theta} = \underbrace{\sum_{t=0}^{T^{\star}} \omega_t \frac{dW_t^{\lambda}}{d\theta}}_{\text{transition welfare gains}} + \underbrace{\sum_{t=T^{\star}}^{T} \omega_t \frac{dW_t^{\lambda}}{d\theta}}_{\text{steady-state welfare gains}} .$$

It is worth highlighting that convergence to a new steady-state in terms of allocations does not guarantee convergence of normalized weights. To facilitate comparisons, it seems more natural to report the value of steady-state welfare effects expressed in permanent dollars starting at $T^{\star}$, rather than starting at date-0, that is: $\frac{\sum_{t=T^{\star}}^{T} \omega_t \frac{dW_t^{\lambda}}{d\theta}}{\sum_{t=T^{\star}}^{T} \omega_t}$.

Stochastic Structure. Finally note that it is possible to express a welfare assessment as

$$dW^{\lambda} = \sum_t \omega_t \sum_{s^t} \omega_t \left(s^t\right) \left(\Xi_t^{AE}\left(s^t\right) + \Xi_t^{RS}\left(s^t\right) + \Xi_t^{IS}\left(s^t\right) + \Xi_t^{RD}\left(s^t\right)\right). \tag{OA9}$$

This formulation shows that a welfare assessment can be interpreted as the discounted sum, using aggregate time and stochastic discount factors — of date-specific welfare assessments, where each of these date-specific assessments can be further decomposed in aggregate efficiency, risk-sharing, intertemporal-sharing, and redistribution. This formulation allows us to attribute welfare gains due to each of the components of the welfare decomposition to specific histories.

E.2 Individual Structure

Since each of the components of the welfare decomposition can be expressed as a triple-sum (over individuals, dates, and histories), it is also possible to compute the individual contribution of particular individuals to each of the components of the welfare decomposition. Formally, we can write

$$\Xi^{AE}=\sum_i \Xi^{i,AE} \quad \text{where} \quad \Xi^{i,AE}=\sum_t \omega_t \sum_{s^t} \omega_t\left(s^t\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}$$

$$\Xi^{RS}=\sum_i \Xi^{i,RS} \quad \text{where} \quad \Xi^{i,RS}=\sum_t \omega_t \sum_{s^t} \omega_t\left(s^t\right)\left(\frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)}-1\right) \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}$$

$$\Xi^{IS}=\sum_i \Xi^{i,IS} \quad \text{where} \quad \Xi^{i,IS}=\sum_t \omega_t\left(\frac{\omega_t^i}{\omega_t}-1\right) \frac{dV_t^{i|\lambda}}{d\theta}$$

$$\Xi^{RD}=\sum_i \Xi^{i,RD} \quad \text{where} \quad \Xi^{i,RD}=\left(\omega^i-1\right) \frac{dV^{i|\lambda}}{d\theta},$$

where, by construction, $\sum_i \xi^{i,AE}=1$ with $\xi^{i,AE}=\frac{\Xi^{i,AE}}{\Xi^{AE}}$; and analogously for the other three components.

E.3 Reallocation and Growth

In order to separate welfare gains due to reallocation from those due to changes in aggregates, it may be useful to decompose how the changes in the level of consumption (analogously, factor supply) are due to changes in the share of consumption across individuals or to changes in aggregate consumption. Formally, we can define consumption and factor supply shares at a given history by $\psi_{t,c}^i\left(s^t\right)=\frac{c_t^i(s)}{c_t(s^t)}$, and $\psi_{t,n}^i\left(s^t\right)=\frac{n_t^i(s)}{n_t(s^t)}$where $c_t\left(s^t\right)=\sum_i c_t^i\left(s^t\right)$ and $n_t\left(s^t\right)=\sum_i n_t^i\left(s^t\right)$ Hence, by applying the product rule, we can express $\frac{dc_t^i(s^t)}{d\theta}$ and $\frac{dn_t^i(s^t)}{d\theta}$ as

$$\frac{dc_t^i\left(s^t\right)}{d\theta}=\underbrace{\frac{d\psi_{t,c}^i\left(s^t\right)}{d\theta} c_t\left(s^t\right)}_{=\text{Reallocation}}+\underbrace{\psi_{t,c}^i\left(s^t\right) \frac{dc_t\left(s^t\right)}{d\theta}}_{=\text{Growth}}$$

$$\frac{dn_t^i\left(s^t\right)}{d\theta}=\underbrace{\frac{d\psi_{t,n}^i\left(s^t\right)}{d\theta} n_t\left(s^t\right)}_{=\text{Reallocation}}+\underbrace{\psi_{t,n}^i\left(s^t\right) \frac{dn_t\left(s^t\right)}{d\theta}}_{=\text{Growth}}.$$

Hence, combining these definitions with the definition of instantaneous welfare gains in (6) or (OA10), it is possible to subdecompose each of the components of the welfare decomposition into a term that capture reallocation of consumption (factor supply) and a term that captures aggregate growth.

E.4 Stochastic Decompositions

As implied, for instance, by equation (OA9), each of the components of the welfare decomposition includes aggregate valuation considerations. Here, we formalize this insight by further decomposing i) the aggregate efficiency component into an expected aggregate efficiency component and an aggregate smoothing component, and ii) the redistribution component into an expected redistribution and a redistributive smoothing component. Similar decompositions can be constructed for intertemporal-sharing and risk-sharing.

Aggregate Efficiency. The aggregate efficiency component, Ξ^{AE}, can be decomposed into i) an expected aggregate efficiency component, Ξ^{EAE}, and ii) an aggregate smoothing component, Ξ^{AM}. Formally, at date t:

$$\Xi_t^{AE} = \underbrace{\mathbb{E}_{\pi_t(s^t)}\left[\omega_t\left(s^t\right)\right]\mathbb{E}_{\pi_t(s^t)}\left[\sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]}_{=\Xi_t^{EAE}\text{ (Expected Aggregate Efficiency)}} + \underbrace{\mathbb{C}ov_{\pi_t(s^t)}\left[\frac{\omega_t\left(s^t\right)}{\pi_t\left(s^t\right)}, \sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]}_{=\Xi_t^{AM}\text{ (Aggregate Smoothing)}},$$

where $\Xi^{AE} = \sum \omega_t \Xi_t^{AE}$. This decomposition is the standard asset pricing decomposition into the expected payoff and a risk compensation. The expected aggregate efficiency component, Ξ_t^{EAE}, captures the expected welfare gain across histories at a particular date. The aggregate smoothing component, Ξ_t^{AM}, captures whether aggregate efficiency gains take place in histories that a planner values more in aggregate terms. It should be evident that aggregate smoothing, based on the covariance of aggregate welfare gains across histories, is logically different from the risk-sharing and intertemporal-sharing components, Ξ^{RS} and Ξ^{IS}, based on cross-sectional covariances.

The welfare gains associated with eliminating aggregate business cycles in a representative-agent economy, (Lucas, 1987), arise from aggregate smoothing considerations. Finally, note it is possible to generate a similar decomposition that captures the part of aggregate efficiency welfare gains that are due to front-loading welfare gains, by using a covariance decomposition across dates.

Redistribution. Similarly to the aggregate efficiency component, the redistribution component Ξ^{RD} is shaped by valuation considerations, in this case, at the individual level. Here, we decompose Ξ^{RD} into i) an expected redistribution component, Ξ^{ER}, and a redistributive smoothing component, Ξ^{RM}. Formally, at date t:

$$\Xi_t^{RD} = \underbrace{\mathbb{C}ov_i\left[\omega^i, \sum_t \omega_t^i \mathbb{E}_{\pi_t(s^t)}\left[\omega_t\left(s^t\right)\right]\mathbb{E}_{\pi_t(s^t)}\left[\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]\right]}_{=\Xi^{ER}\text{ (Expected Redistribution)}}$$
$$+ \underbrace{\mathbb{C}ov_i\left[\omega^i, \sum_t \omega_t^i \mathbb{C}ov_{\pi_t(s^t)}\left[\frac{\omega_t\left(s^t\right)}{\pi_t\left(s^t\right)}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]\right]}_{=\Xi^{RM}\text{ (Redistributive Smoothing)}}.$$

This is, again, a standard asset pricing decomposition. The expected redistribution component, Ξ^{ER}, captures the welfare gains due to the expected welfare gains across histories at a particular date. When individuals with a high individual weight have higher expected welfare gains, a planner attributes this to the expected redistribution component. The redistributive smoothing component, Ξ^{RM}, captures whether individual welfare gains take place in histories that are more desirable for individuals with a higher individual weight. That is, the redistributive smoothing component will be non-zero for perturbations that smooth individual consumption for individuals with high individual weights.

E.5 Alternative Cross-Sectional Decompositions

Here, we provide two alternative cross-sectional decompositions of the risk-sharing and intertemporal-sharing components.

First, using equation (OA6), it is possible to decompose Ξ^{IS} into i) a raw intertemporal-sharing component, ii) a weight concentration component, and iii) a policy-weights coskewness component, as follows

$$\begin{aligned}\Xi^{IS} = & \underbrace{\sum_t \omega_t \sum_{s^t} \omega_t \left(s^t\right) \mathbb{C}ov_i^{\Sigma} \left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]}_{=\Xi^{RIS} \text{ (Raw Intertemporal-sharing)}} \\ & + \underbrace{\sum_t \omega_t \sum_{s^t} \omega_t \left(s^t\right) \mathbb{C}ov_i \left[\frac{\omega_t^i}{\omega_t}, \frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)}\right] \sum_i \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}}_{=\Xi^{WC} \text{ (Weight Concentration)}} \\ & + \underbrace{\sum_t \omega_t \sum_{s^t} \omega_t \left(s^t\right) \sum_i \left(\frac{\omega_t^i}{\omega_t} - 1\right) \left(\frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)} - 1\right) \left(\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} - \mathbb{E}_i \left[\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right]\right)}_{=\Xi^{PC} \text{ (Policy-weights Coskewness)}}.\end{aligned}$$

The first component of Ξ^{IS}, Ξ^{RIS}, can be interpreted as an intertemporal-sharing component in which welfare gains at date t are not risk-discounted (i.e., raw). Note that the history s^t determinant of Ξ^{IS} relative to Ξ^{RIS} compare as follows

$$\mathbb{C}ov_i^{\Sigma} \left[\frac{\omega_t^i}{\omega_t}, \frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)} \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right] \quad \text{vs.} \quad \mathbb{C}ov_i^{\Sigma} \left[\frac{\omega_t^i}{\omega_t}, \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta}\right],$$

where it is clear that intertemporal-sharing corrects welfare gains by risk through $\frac{\omega_t^i\left(s^t\right)}{\omega_t\left(s^t\right)}$, while raw intertemporal-sharing does not. Hence, the remaining two components, Ξ^{WC} and Ξ^{PC}, precisely capture the difference due to such risk correction.

The Ξ^{WC} component corrects for the fact that dynamic and stochastic weights are cross-sectionally correlated. Even though one may consider including Ξ^{WC} in the aggregate efficiency component, there are two good reasons not to do so. First, it would require knowledge of the cross-section of the dynamic and stochastic weights, which goes against expressing the aggregate efficiency component exclusively as a function of aggregate statistics. Second, $\Xi^{WC} = 0$ when markets are complete, which highlights that Ξ^{WC} relies on valuation differences across individuals.

The Ξ^{PC} component is based on the coskewness between dynamic and stochastic weights and the instantaneous welfare gain. Coskewness is a measure of how much three random variables jointly change. For instance, Ξ^{PC} could be non-zero even when $\mathbb{C}ov_i\left[\frac{\omega_t^i}{\omega_t}, \frac{\omega_t^i(s^t)}{\omega_t(s^t)}\right] = 0$. Also, coskewness is zero when the random variables are multivariate normal (Bohrnstedt and Goldberger, 1969), so it relies on higher-order moments.[20] Ξ^{WC} is also zero if one of ω_t^i, $\omega_t^i\left(s^t\right)$, or $\frac{dV_t^{i|\lambda}(s^t)}{d\theta}$ is constant across individuals.

F Extensions: Additional Results

In this section, we discuss additional results.

F.1 General Welfare Numeraires

In the body of the paper, we immediately adopt a triple of consumption-based welfare numeraires. Here, we proceed to derive the counterpart of Lemma 1 for general welfare numeraires. The main difference with the body of the paper is that we introduce a triple of normalizing factors (lifetime, date, and instantaneous) to allow for general welfare numeraires: λ^i, λ_t^i, and $\lambda_t^i\left(s^t\right)$.

The first step is to express a welfare assessment as

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i \frac{dV^{i|\lambda}}{d\theta}, \quad \text{where} \quad \frac{dV^{i|\lambda}}{d\theta} = \frac{\frac{dV^i}{d\theta}}{\lambda^i}$$

denotes individual lifetime welfare gains in units of the lifetime welfare numeraire, and where λ^i is the normalizing factor — with units $\frac{\text{individual } i \text{ utils}}{\text{lifetime welfare numeraire}}$ — that allows us to express individual lifetime welfare gains in a common unit. The only restriction when choosing the lifetime welfare numeraire is that λ^i must be strictly positive for all individuals affected by the perturbation.

Next, to meaningfully compare welfare gains at particular dates or histories across individuals in a common unit, we select date and instantaneous welfare numeraire for each date and history. Formally, individual lifetime welfare gains in units of the lifetime welfare numeraire, $\frac{dV^{i|\lambda}}{d\theta}$, can be expressed as

$$\frac{dV^{i|\lambda}}{d\theta} = \sum_t \frac{\lambda_t^i}{\lambda^i} \sum_{s^t} \frac{\left(\beta^i\right)^t \pi_t\left(s^t \middle| s_0\right) \lambda_t^i\left(s^t\right)}{\lambda_t^i} \frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta},$$

[20] These terms are likely to be important in models that emphasize higher moments of the distribution of risks (e.g., Guvenen, Ozkan and Song (2014)).

where

$$\frac{dV_t^{i|\lambda}\left(s^t\right)}{d\theta} = \frac{\frac{\partial u_t^i\left(s^t\right)}{\partial c_t^i}}{\lambda_t^i\left(s^t\right)} \frac{dc_t^i\left(s^t\right)}{d\theta} + \frac{\frac{\partial u_t^i\left(s^t\right)}{\partial n_t^i}}{\lambda_t^i\left(s^t\right)} \frac{dn_t^i\left(s^t\right)}{d\theta} \tag{OA10}$$

denotes normalized instantaneous welfare gains at history s^t, expressed in units of the instantaneous welfare numeraire, and where $\lambda_t^i\left(s^t\right)$ is the instantaneous normalizing factor — with units $\frac{\text{instantaneous individual } i \text{ utils}}{\text{instantaneous welfare numeraire at } s^t}$ — that allows us to express instantaneous welfare gains at history s^t in a common unit at that history and λ_t^i is the date normalizing factor — with units $\frac{\text{individual } i \text{ utils}}{\text{date welfare numeraire at } t}$ — that allows us to express welfare gains at all date-t histories in a common unit at that date. The only restriction when choosing the date and instantaneous welfare numeraires is that λ_t^i and $\lambda_t^i\left(s^t\right)$ must be strictly positive for all individuals affected by the perturbation at a particular date and history.

In the body of the paper, we assume that λ^i, λ_t^i and $\lambda_t^i\left(s^t\right)$ are given by

$$\lambda^i = \sum_t \lambda_t^i \qquad \text{and} \qquad \lambda_t^i = \sum_{s^t} \lambda_t^i\left(s^t\right) \qquad \text{and} \qquad \lambda_t^i\left(s^t\right) = \frac{\partial u_t^i\left(s^t\right)}{\partial c_t^i}, \tag{OA11}$$

which ensures that ω_t^i and $\omega_t^i\left(s^t\right)$ define normalized discount factors and risk-neutral probabilities.

In general, the counterparts of the normalized individual, dynamic, stochastic weights in equations (7), (8), and (9) for general welfare numeraires are

$$\omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i}\lambda^i}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\lambda^i} \qquad \text{and} \qquad \omega_t^i = \frac{\lambda_t^i}{\lambda^i} \qquad \text{and} \qquad \omega_t^i\left(s^t\right) = \frac{\beta^t \pi_t\left(s^t|s_0\right)\lambda_t^i\left(s^t\right)}{\lambda_t^i},$$

where the interpretation of the weights as marginal rates of substitution is as in the body of the paper, but now using different units.

More generally, the nominal unit (e.g., dollars) or particular commodities or bundles of commodities may also be reasonable choices for welfare numeraires. An alternative choice of lifetime welfare numeraire in models with a single consumption good is date-0 consumption, so $\lambda^i = \frac{\partial u_0^i\left(s^0\right)}{\partial c_0^i}$. In this case, the normalized stochastic weights remain unchanged, while the normalized individual and dynamic weights take the form

$$\omega^i = \frac{\frac{\partial \mathcal{W}}{\partial V^i}\frac{\partial u_0^i\left(s^0\right)}{\partial c_0^i}}{\frac{1}{I}\sum_i \frac{\partial \mathcal{W}}{\partial V^i}\frac{\partial u_0^i\left(s^0\right)}{\partial c_0^i}} \qquad \text{and} \qquad \omega_t^i = \frac{\sum_{s^t}\beta^t \pi_t\left(s^t|s_0\right)\lambda_t^i\left(s^t\right)}{\frac{\partial u_0^i\left(s^0\right)}{\partial c_0^i}}.$$

The main difference with respect to using permanent consumption as the lifetime welfare numeraire is that now the efficiency component is expressed in terms of willingness-to-pay at date 0. This may be desirable in particular circumstances.

It is worth making three final remarks. First, note that welfare numeraires always exist: at worst, one could choose a bundle of all goods/factors with non-negative marginal utility at a given history, date, or on a lifetime basis. Second, one could potentially pick different numeraires in different dates or histories, but it seems natural to choose a consistent numeraire to yield easily interpretable results. Finally, while the choice of welfare numeraires does not change the directional welfare assessment of a welfarist planner, the interpretation of the elements of the welfare decomposition is contingent on such choice.

F.2 Role of Transfers

Here, we explain how the ability to costlessly transfer resources across individuals impacts the welfare decomposition. Formally, if a DS-planner has access to a set of transfers $T_i^i\left(s^t\right)$ in units of the instantaneous welfare numeraire (here assumed to be consumption), so that individual budget constraints have the form

$$c_t^i\left(s^t\right) = T_i^i\left(s^t\right) + \ldots.$$

it follows immediately that the social value of such transfer equals the DS-weight of individual at that particular history:

$$\frac{dW^{DS}}{dT_i^i\left(s^t\right)} = \omega^i \omega_t^i \omega_t^i\left(s^t\right) = \tilde{\omega}_t^i\left(s^t\right).$$

When a planner can transfer resources costlessly across individuals, subject to $\sum_i T_i^i\left(s^t\right) = 0$, the availability of transfers endogenously restricts the variation of DS-weights across different individuals. For instance, a welfarist planner who can transfer resources freely across all individuals, at all dates and histories will equalize the DS-weights across all individuals, at all dates and histories. Given Proposition 5, this implies that this planner will only value aggregate efficiency. Similar conclusions can be reached when a DS-planner only has access to a subset of transfers.

F.3 Global Assessments

The body of the paper focuses on marginal welfare assessments because marginal welfare gains can be computed unambiguously — see e.g. Schlee (2013), which shows that consumer surplus, equivalent variation, and compensating variation are identical for marginal changes in a classical demand setup. However, it is important to understand how to make non-marginal welfare assessments.

Even for a single individual, there is no unambiguous approach to measure welfare gains or losses for non-marginal changes in meaningful units (money-metric) — see e.g., Silberberg (1972) or Mas-Colell, Whinston and Green (1995). This phenomenon is typically illustrated by the discrepancy between consumer surplus, equivalent variation, and compensating variation in classic demand theory. The same logic extends to aggregate welfare assessments and to the welfare decomposition. Despite this hurdle, it is possible to make judicious global welfare assessments.

In practice, it is possible to study global changes by parameterizing perturbations using a line integral, as illustrated in Section 6. Assuming that policy changes can be scaled by $\theta \in [0,1]$, where $\theta = 0$ corresponds to the status-quo and $\theta = 1$ corresponds to a global non-marginal change, it is possible to define a non-marginal welfare change as follows:

$$W^{DS}(\theta = 1) - W^{DS}(\theta = 0) = \int_0^1 \frac{dW^{DS}(\theta)}{d\theta} d\theta,$$

where θ is an explicit argument of $\frac{dW^{DS}(\theta)}{d\theta}$, defined as in (3) or (14). That is, by recomputing $\frac{dW^{DS}(\theta)}{d\theta}$ or $\frac{dW^{\lambda}(\theta)}{d\theta}$ along a particular path, it is possible to come up with a social welfare measure that is akin to consumer surplus, with the same logic applying to each of the components of the welfare decomposition. While using different paths will typically yield different global answers to the question of what are the gains from a global multidimensional perturbation, in practice it is often possible to find monotonic paths of integration, as defined by Zajac (1979) and Stahl (1984). In that case, there is no ambiguity on whether $\theta = 0$ is socially preferred to $\theta = 1$, or vice versa.

Two additional remarks are worth making. First, while the approach outlined here is the easiest to implement, it is possible to follow Alvarez and Jermann (2004) to consider global equivalent/compensating variation-like assessments for welfarist planners within the DS-weights framework. This will only be valid for aggregate assessments, not necessarily each of the components of the welfare decomposition.

Second, the potential for ambiguity of global assessments is not relevant if one is interested in using DS-planners to solve optimal policy problems, since $\frac{dW^{DS}}{d\theta}$ is unambiguously defined for any policy perturbation. Hence, if there is a point at which $\frac{dW^{DS}}{d\theta} = 0$ given the set of policy instruments, this will be a critical point and, under suitable second-order conditions, a local optimum. If there is a single local optimum and it is possible to establish that the optimum is interior, this optimum will be global. If there are multiple local optima, one could use the value of the SWF to rank them in the welfarist case. So welfarist planners can unambiguously rank any two policies globally. Outside of the welfarist case, one can look for monotonic paths of integration (Zajac, 1979; Stahl, 1984) to rank different local optima, so it is only when this is not possible to find such paths that there may be some global ambiguity when ranking two particular policies for DS-planners.[21] In general, one can choose a set of reasonable policy paths (e.g., linear paths or bounded paths) and compare the predictions for the associated welfare assessments both in aggregate and for each of the components of the welfare decomposition.

[21] Stahl (1984) proves that there always exist monotonic paths of integration in a classical demand context. While a formal proof of the existence of such paths for the general framework considered here is outside of the scope of this paper, there is no reason to believe this result cannot be extended to more general environments.

F.4 Inequality and Bounds

Concerns related to inequality often take a prominent role when assessing the welfare impact of policies. The welfare decomposition introduced in this paper highlights which particular forms of inequality matter for the determination of welfare assessments and their components.

Formally, by using the Cauchy-Schwarz inequality — which states that $|\mathbb{C}ov[x, y]| \leq \sqrt{\mathbb{V}ar[x]}\sqrt{\mathbb{V}ar[y]}$ — it is possible to provide bounds for Ξ^{RS}, Ξ^{IS}, and Ξ^{RD} based on the cross-sectional dispersion of normalized weights and the welfare gains, as follows:

$$\begin{aligned}
\left|\Xi^{RS}\right| &\leq \sum_{t} \omega^{t} \sum_{s^{t}} \mathbb{SD}_{i}^{\Sigma}\left[\omega_{t}^{i}\left(s^{t}\right)\right] \cdot \mathbb{SD}_{i}^{\Sigma}\left[\frac{dV_{t}^{i}\left(s^{t}\right)}{d\theta}\right] \\
\left|\Xi^{IS}\right| &\leq \sum_{t} \mathbb{SD}_{i}^{\Sigma}\left[\omega_{t}^{i}\right] \cdot \mathbb{SD}_{i}^{\Sigma}\left[\frac{dV_{t}^{i}}{d\theta}\right] \\
\left|\Xi^{RD}\right| &\leq \mathbb{SD}_{i}^{\Sigma}\left[\omega^{i}\right] \cdot \mathbb{SD}_{i}^{\Sigma}\left[\frac{dV^{i}}{d\theta}\right],
\end{aligned}$$

where $\mathbb{SD}_{i}^{\Sigma}[\cdot]$ denotes a cross-sectional standard deviation, where the variance is computed in sum form. This result shows that inequality considerations matter for the aggregate assessments of policies via the cross-sectional dispersion of normalized or the impact of a perturbation by itself. While cross-sectional standard deviations can bound the welfare effect of perturbations, the welfare decomposition is a function of covariances.

These bounds are helpful in practice because they can be computed using univariate statistics, i.e., cross-sectional standard deviations, and do not require the joint distribution of DS-weights and normalized consumption-equivalent effects, which are necessary to compute cross-sectional covariances (a multivariate statistic).

G Relation to Existing Work

G.1 Relation to Saez and Stantcheva (2016)

The notion of DS-planners introduced in Section 4 nests the generalized weight approach in Saez and Stantcheva (2016) and extends it to dynamic stochastic environments. Formally, while that paper considers welfare objectives that directly define the individual weight ω^{i}, DS-planners also define (potentially non-welfarist) dynamic and stochastic generalized weights, ω_{t}^{i} and $\omega_{t}^{i}\left(s^{t}\right)$, for each individual.[22]

[22] In general, unless they are based on a SWF, welfare assessments based on generalized individual weights (those considered in Saez and Stantcheva (2016)) are non-welfarist, yet they are Paretian and non-paternalistic. Welfare assessments based on generalized dynamic and stochastic weights (those considered in Section 4 of this paper) are non-welfarist, and typically non-Paretian and paternalistic.

Hence, DS-planners in i) static environments or ii) environments that exclusively feature generalized individual weights (but welfarist dynamic and stochastic weights) can be interpreted as special cases of the generalized weight approach in Saez and Stantcheva (2016). Although in that second case, our results use a different choice of lifetime welfare numeraire: Saez and Stantcheva (2016) naturally choose instantaneous consumption as their numeraire, but this choice is more subtle in dynamic environments, as explained in Section F.1. DS-planners that feature generalized dynamic and/or stochastic weights are not considered in that paper. It is also worth highlighting that their paper does not feature any form of welfare decomposition, even for static environments, so every result in Section 3 of this paper is unrelated to the results in Saez and Stantcheva (2016).

A central insight in Saez and Stantcheva (2016) is that by using (individual) generalized weights it is possible to accommodate alternatives to welfarism, such as equality of opportunity, libertarianism, or Rawlsianism, among others. Since our approach nests theirs, it can also accommodate these possibilities. There is scope to integrate these alternatives into dynamic stochastic environments.

G.2 Relation to Lucas (1987) and Alvarez and Jermann (2004)

It is common in papers that make welfare assessments in dynamic stochastic environments to compute welfare gains using consumption-equivalents, as in Lucas (1987), who measures the welfare gains associated with a policy change — specifically, the welfare gains associated with eliminating business cycles. Our approach, built using marginal arguments, connects directly to the results in Alvarez and Jermann (2004), who provide a marginal formulation of the approach in Lucas (1987). While the Lucas (1987) and Alvarez and Jermann (2004) approach is easily interpretable in representative agent economies, it has the pitfall that consumption-equivalents cannot be meaningfully aggregated when there are heterogeneous individuals. See, for instance, how Atkeson and Phelan (1994), Krusell and Smith (1999), or Krusell et al. (2009) carefully avoid aggregating consumption-equivalent welfare gains across different individuals.

To illustrate these arguments, here we consider a perturbation for a given individual i, who could be a representative agent or not. We abstract from factor supply, for simplicity, and consider preferences of the form

$$V^i = \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0\right) u^i \left(c_t^i \left(s^t\right)\right).$$

We suppose that the consumption of individual i at date t and history s^t can be written as

$$c_t^i \left(s^t\right) = (1-\theta) \underline{c_t^i} \left(s^t\right) + \theta \overline{c_t^i} \left(s^t\right),$$

where both $\underline{c_t^i}\left(s^t\right)$ and $\overline{c_t^i}\left(s^t\right)$ are sequences measurable with respect to history s^t. The sequence $\underline{c_t^i}\left(s^t\right)$ can be interpreted as a given initial consumption path — when $\theta = 0$ — and the sequence

$\overline{c_t^i}\left(s^t\right)$ can be interpreted as a final consumption path — when $\theta = 1$. In the case of Lucas (1987), $\theta = 1$ corresponds to fully eliminating business cycles.

First, we compute the marginal gains from marginally reducing business cycles using a multiplicative consumption-equivalent, as in Lucas (1987) and Alvarez and Jermann (2004). Next, we compute the marginal gains using an additive consumption-equivalent.

Multiplicative Compensation. Lucas (1987) proposes using a time-invariant equivalent variation, expressed multiplicatively as a constant fraction of consumption at each date and history as follows

$$\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u^i\left(\underline{c_t^i}\left(s^t\right)\left(1+\Lambda^i(\theta)\right)\right) = \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u^i\left(\left(1-\theta\right)\underline{c_t^i}\left(s^t\right) + \theta\overline{c_t^i}\left(s^t\right)\right), \quad \text{(OA12)}$$

where $\Lambda^i(\theta)$ implicitly defines the welfare gains associated with a policy indexed by θ. The exact definition in Lucas (1987) corresponds to solving for $\Lambda^i(\theta = 1)$.[23]

Following Alvarez and Jermann (2004), the derivative of the RHS of Equation (OA12) is given by

$$\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u^{i\prime}\left(\left(1-\theta\right)\underline{c_t^i}\left(s^t\right) + \theta\overline{c_t^i}\left(s^t\right)\right)\frac{dc_t^i\left(s^t\right)}{d\theta} \quad \text{(OA13)}$$

where $\frac{dc_t^i\left(s^t\right)}{d\theta} = \overline{c_t^i}\left(s^t\right) - \underline{c_t^i}\left(s^t\right)$.

Analogously, the derivative of the LHS of Equation (OA12) is given by

$$\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u^{i\prime}\left(\underline{c_t^i}\left(s^t\right)\left(1+\Lambda^i(\theta)\right)\right)\underline{c_t^i}\left(s^t\right)\frac{d\Lambda^i}{d\theta}. \quad \text{(OA14)}$$

Hence, combining (OA13) and (OA14) and solving for $\frac{d\Lambda^i}{d\theta}$, yields the marginal cost of business cycles, as defined in Alvarez and Jermann (2004). Formally, we can express $\frac{d\Lambda^i}{d\theta}$ as

$$\frac{d\Lambda^i}{d\theta} = \frac{\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u^{i\prime}\left(\left(1-\theta\right)\underline{c_t^i}\left(s^t\right) + \theta\overline{c_t^i}\left(s^t\right)\right)\frac{dc_t^i\left(s^t\right)}{d\theta}}{\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u^{i\prime}\left(\underline{c_t^i}\left(s^t\right)\left(1+\Lambda^i(\theta)\right)\right)\underline{c_t^i}\left(s^t\right)} = \sum_t \sum_{s^t} \tilde{\omega}_t^i\left(s^t\right)\frac{dc_t^i\left(s^t\right)}{d\theta}, \quad \text{(OA15)}$$

where DS-weights are given by

$$\tilde{\omega}_t^i\left(s^t\right) = \frac{\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u^{i\prime}\left(\left(1-\theta\right)\underline{c_t^i}\left(s^t\right) + \theta\overline{c_t^i}\left(s^t\right)\right)}{\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u^{i\prime}\left(c_t^i\left(s^t\right)\left(1+\Lambda^i(\theta)\right)\right)\underline{c_t^i}\left(s^t\right)}. \quad \text{(OA16)}$$

[23] Alternatively, one could define a compensating variation as

$$\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u^i\left(\underline{c_t^i}\left(s^t\right)\right) = \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u^i\left(\left(\left(1-\theta\right)\underline{c_t^i}\left(s^t\right) + \theta\overline{c_t^i}\left(s^t\right)\right)\left(1+\Lambda^i(\theta)\right)\right).$$

Additive Compensation. Here, we would like to contrast the approach in Lucas (1987) to one that relies on a time-invariant equivalent variation, expressed additively in terms of consumption at each date and history as follows:

$$\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0\right) u_i \left(\underline{c}_t^i \left(s^t\right) + \mathcal{A}^i \left(\theta\right)\right) = \sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0\right) u_i \left(\left(1-\theta\right) \underline{c}_t^i \left(s^t\right) + \theta \overline{c}_t^i \left(s^t\right)\right).$$

Following the same steps as above to find the counterpart of Equation (OA15), we find that

$$\frac{d\mathcal{A}^i}{d\theta} = \frac{\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0\right) u^{i\prime} \left(\left(1-\theta\right) \underline{c}_t^i \left(s^t\right) + \theta \overline{c}_t^i \left(s^t\right)\right) \frac{dc_t^i\left(s^t\right)}{d\theta}}{\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0\right) u^{i\prime} \left(\underline{c}_t^i \left(s^t\right) + \mathcal{A}^i \left(\theta\right)\right) \underline{c}_t^i \left(s^t\right)} = \sum_t \sum_{s^t} \tilde{\omega}_t^i \left(s^t\right) \frac{dc_t^i \left(s^t\right)}{d\theta}, \tag{OA17}$$

where DS-weights are given by

$$\tilde{\omega}_t^i \left(s^t\right) = \frac{\left(\beta^i\right)^t \pi_t \left(s^t \middle| s_0\right) u^{i\prime} \left(\left(1-\theta\right) \underline{c}_t^i \left(s^t\right) + \theta \overline{c}_t^i \left(s^t\right)\right)}{\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0\right) u^{i\prime} \left(\underline{c}_t^i \left(s^t\right) + \mathcal{A}^i \left(\theta\right)\right)}. \tag{OA18}$$

Comparison and Implications. We focus on comparing Equations (OA15) and (OA17) when $\theta = 0$ — similar insights emerge when $\theta \neq 0$. When $\theta = 0$, Equations (OA16) and (OA18) become

$$\tilde{\omega}_t^i \left(s^t\right) = \frac{\left(\beta^i\right)^t \pi_t \left(s^t \middle| s_0\right) u^{i\prime} \left(\underline{c}_t^i \left(s^t\right)\right)}{\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0\right) u^{i\prime} \left(\underline{c}_t^i \left(s^t\right)\right) \underline{c}_t^i \left(s^t\right)} \quad \text{(multiplicative)} \tag{OA19}$$

$$\tilde{\omega}_t^i \left(s^t\right) = \frac{\left(\beta^i\right)^t \pi_t \left(s^t \middle| s_0\right) u^{i\prime} \left(\underline{c}_t^i \left(s^t\right)\right)}{\sum_t \left(\beta^i\right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0\right) u^{i\prime} \left(\underline{c}_t^i \left(s^t\right)\right)}. \quad \text{(additive)} \tag{OA20}$$

Two major insights emerge from Equations (OA19) and (OA20). First, the DS-weights defined for the additive case in Equation (OA20) exactly correspond to the product of the normalized dynamic and stochastic weights for a welfarist planner, as defined in (8) and (9). Second, the denominator of the DS-weights in the multiplicative case is multiplied by $\underline{c}_t^i \left(s^t\right)$ at all dates and histories. This captures the fact that the welfare assessment is computed as a fraction of consumption at each date and history, not in units of the consumption good. The presence of $\underline{c}_t^i \left(s^t\right)$ in the denominator is what complicates the aggregation of welfare assessments using the Lucas (1987) approach.

While both Lucas (1987) and Alvarez and Jermann (2004) study representative-agent environments, others have used a similar approach in environments with heterogeneity; see e.g., Atkeson and Phelan (1994), Krusell and Smith (1999), or Krusell et al. (2009), among many others. However, as highlighted by these papers, a well-known downside of the Lucas (1987) approach is that it does not aggregate meaningfully because individual welfare assessments are reported as constant shares of individual consumption. Our approach, implicitly based on an additive compensation, allows for meaningful aggregation among heterogeneous individuals.

Relation to EV, CV, and CS. Finally, note that the analysis in this section illustrates how the marginal approach relates to the conventional approaches in classic demand theory: equivalent variation (EV), compensating variation (CV), and consumer surplus (CS). The approach of Lucas (1987) and Alvarez and Jermann (2004), and the alternative version described in Footnote 23 are the dynamic counterpart of compensating and equivalent variations, expressed in proportional terms, in a dynamic stochastic environment. Hence, the analysis of this section shows that a DS-planner can be used to operationalize the counterpart of all three notions — either proportionally or additively — in dynamic stochastic environments. As expected, these considerations only matter away from the $\theta = 0$ case. However, the consumer surplus approach yields the most straightforward approach to making global assessments, as explained in Section F.3.

G.3 Relation to Existing Welfare Decompositions

Our paper is not the first to introduce a decomposition of welfare assessments in different components. In fact, most of the existing literature that applies welfare decompositions to specific environments follows versions of the decompositions introduced by Benabou (2002) and Floden (2001). There is also the more recent decomposition introduced by Bhandari et al. (2021). We discuss how our approach is related to both of these next.

Benabou (2002)/Floden (2001). The Benabou (2002)/Floden (2001) approach is based on first computing certainty-equivalent consumption levels for individuals and then building measures of inequality from the distribution of such certainty-equivalents. The starting point for the Benabou (2002)/Floden (2001) approach is the (incorrect) presumption that the welfarist approach cannot distinguish the effects of policy that operate via efficiency, missing markets, and redistribution. Benabou (2002) explicitly writes:

> "*I will also compute more standard social welfare functions, which are aggregates of (intertemporal) utilities rather than risk-adjusted consumptions. These have the clearly desirable property that maximizing such a criterion ensures Pareto efficiency. On the other hand, it will be seen that they cannot distinguish between the effects of policy that operate through its role as a substitute for missing markets, and those that reflect an implicit equity concern.*"

In this paper, we have shown that it is possible to distinguish — using standard Social Welfare

Functions — the effects of policy that operate through efficiency, including in economies with missing markets, and redistribution/equity. As Benabou (2002) points out, his approach may conclude that Pareto-improving policies are undesirable: this can never occur for welfarist planners, as explained in Section 3. It is only when considering non-welfarist planners — such as some DS-planners introduced in Section 4 — that perturbations that individuals find Pareto-improving are undesirable for a particular DS-planner. In those cases, our welfare decomposition is precise in the way in which such departures take place.

In terms of properties, it is evident that the Benabou (2002)/Floden (2001) approach does not satisfy Proposition 4a), in which we show that welfarist planners conclude that the risk-sharing and intertemporal-sharing components are zero when markets are complete; Proposition 4b), in which we show that welfarist planners conclude that intertemporal-sharing component is zero when individuals can freely trade a riskless bond; and Proposition 2a), in which we show that different welfarist planners exclusively disagree on the redistribution component, among others. The Benabou (2002)/Floden (2001) approach is only invariant to preference-preserving transformations because it is exclusively defined for environments in which all individuals have identical preferences.

Bhandari et al. (2021). The decomposition introduced by Bhandari et al. (2021) considers a utilitarian planner with arbitrary Pareto weights α_i, although it seems obvious to apply to general Social Welfare Functions. In contrast to Benabou (2002)/Floden (2001), the approach of Bhandari et al. (2021) is defined for general dynamic stochastic economies in which individuals may have different preferences.

For simplicity, we consider a scenario in which there is a single consumption good. In this environment, Bhandari et al. (2021) propose to first decompose the consumption of a given individual at a given date and history as

$$c_t^i\left(s^t\right) = C \times w_i \times \left(1 + \varepsilon_t^i\left(s^t\right)\right), \tag{OA21}$$

where C captures aggregate lifetime consumption, w_i captures the share of individual i's consumption relative to the aggregate and $1 + \varepsilon_t^i\left(s^t\right)$ captures any residual variation. While equation (OA21) may resemble the triple of individual, dynamics, and stochastic weights introduced in Lemma 1, it is conceptually different. In particular, the decomposition in equation (OA21) decomposes consumption, $c_t^i\left(s^t\right)$, while our weights decompose social marginal valuations. Only heuristically, the term w_i in (OA21) can be mapped to our normalized individual weight, while $1 + \varepsilon_t^i\left(s^t\right)$ can be mapped to both dynamic and stochastic weights.

Bhandari et al. (2021) then introduce a second-order Taylor expansion around a midpoint to write welfare differences (partially adopting the notation in that paper) as follows:

$$\mathcal{W}^B - \mathcal{W}^A \simeq \underbrace{\int \phi_i \Gamma di}_{\text{agg. efficiency}} + \underbrace{\int \phi_i \Delta_i di}_{\text{redistribution}} + \underbrace{\int \phi_i \gamma_i \Lambda_i di}_{\text{insurance}}, \tag{OA22}$$

where $\phi_i = \alpha_i \sum_t \sum_{s^t} \frac{\partial u_i(s^t)}{\partial c_t^i} c_t^i \left(s^t\right)$ denotes quasi-weights — using the terminology in Bhandari et al. (2021) — and γ_i is a measure of risk-aversion, $-c_t^i \left(s^t\right) \frac{\partial^2 u_i(s^t)}{\partial (c_t^i)^2} / \frac{\partial u_i(s^t)}{\partial c_t^i}$, and where $\Gamma = \ln C^B - \ln C^A$, $\Delta_i = \ln w_i^B - \ln w_i^A$, and $\Lambda_i = -\frac{1}{2}\left[\mathbb{V}ar_i \left[\ln c_i^B\right] - \mathbb{V}ar_i \left[\ln c_i^A\right]\right]$. That paper decomposes $\mathcal{W}^B - \mathcal{W}^A$ into three terms as follows:

$$1 = \underbrace{\frac{\int \phi_i \Gamma di}{\mathcal{W}^B - \mathcal{W}^A}}_{\text{agg. efficiency}} + \underbrace{\frac{\int \phi_i \Delta_i di}{\mathcal{W}^B - \mathcal{W}^A}}_{\text{redistribution}} + \underbrace{\frac{\int \phi_i \gamma_i \Lambda_i di}{\mathcal{W}^B - \mathcal{W}^A}}_{\text{insurance}}. \tag{OA23}$$

Bhandari et al. (2021) establish three properties of the decomposition in Equation (OA23): a) a welfare change that affects aggregate consumption C but not $\{w_i, \varepsilon_i\}_i$ is exclusively attributed to aggregate efficiency; b) a welfare change that affects expected shares $\{w_i\}_i$ but not C and $\{\varepsilon_i\}_i$ is exclusively attributed to redistribution; c) a welfare change that affects $\{\varepsilon_i\}_i$ but not C and $\{w_i\}_i$ is exclusively attributed to insurance. The insurance component in Bhandari et al. (2021) is heuristically related to the risk-sharing and intertemporal-sharing components in our paper. Bhandari et al. (2021) also establish a fourth property, reflexivity, which our approach also satisfies. These properties are conceptually the counterpart of Proposition 5a), since they consider properties of a decomposition for particular perturbations. However, it should be evident that properties a), b), and c) in Bhandari et al. (2021) neither imply nor are implied by the properties that we establish in Proposition 5a). This occurs because properties a), b), and c) consider proportional changes while Proposition 5a) considers changes in levels, with both the proportional and level approaches being different but reasonable.[24]

The decomposition of Bhandari et al. (2021) does not have a counterpart to Propositions 4 and 5. That is, it is possible to consider complete market economies in which the decomposition of

[24]Note that by writing $c_t^i \left(s^t\right) = C \times w_i \times \left(1 + \varepsilon_t^i \left(s^t\right)\right)$, we can express $\frac{dc_t^i(s^t)}{d\theta}$ as follows:

$$\frac{dc_t^i \left(s^t\right)}{d\theta} = \frac{dC}{d\theta} \times w_i \times \left(1 + \varepsilon_t^i \left(s^t\right)\right) + C \times \frac{dw_i}{d\theta} \times \left(1 + \varepsilon_t^i \left(s^t\right)\right) + C \times w_i \times \frac{d\left(1 + \varepsilon_t^i \left(s^t\right)\right)}{d\theta}.$$

In this case, even when $\frac{dw_i}{d\theta} = \frac{d\left(1+\varepsilon_t^i(s^t)\right)}{d\theta} = 0$, a change in $\frac{dC}{d\theta}$, by virtue of being *proportional* to existing consumption, does not imply a uniform change in $\frac{dc_t^i(s^t)}{d\theta}$ across individuals, dates, and histories. A similar logic applies to changes in $\frac{dw_i}{d\theta}$ and $\frac{d\varepsilon_t^i(s^t)}{d\theta}$. More generally, the decompositions yield different conclusions. For instance, the decomposition in Bhandari et al. (2021) attributes welfare gains associated to smoothing business cycles in a representative agent economy — as in Lucas (1987) — to insurance, while our decomposition attributes such gains to the aggregate insurance subcomponent of aggregate efficiency.

Bhandari et al. (2021) attributes welfare changes to their insurance component. More importantly, it follows from (OA23) that changing the Pareto weights α_i that a utilitarian planner assigns to an individual or simply multiplying the lifetime utility of a single individual by a constant factor — a preference-preserving transformation that has no impact on allocations — will change all three elements (aggregate efficiency, redistribution, insurance) of the decomposition introduced by Bhandari et al. (2021). Formally, it follows from the definition of ϕ_i above that a change in α_i or a linear transformation of utilities will change ϕ_i and consequently each of the three elements on the right-hand side of Equation (OA22). Critically, $\mathcal{W}^B - \mathcal{W}^A$ in Equation (OA22) (as well as ϕ_i) is expressed in utils, not consumption units or any other common numeraire.[25] Hence, changes in Pareto weights or utility transformations directly affect all the components of the decomposition, including aggregate efficiency and insurance in Equation (OA23).

[25] Bhandari et al. (2021) explain how $\mathcal{W}^B - \mathcal{W}^A$ is measured in utils as follows:

> *"Quasi-weights $\{\phi_i\}_i$ convert percent changes $\{\Gamma, \Delta_i, \Lambda_i\}_i$ that into a welfare change $\mathcal{W}^B - \mathcal{W}^A$, measured in utils."*