MULTINATIONAL BANKS AND FINANCIAL STABILITY[*]

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Abstract

We study the scope for international cooperation in macroprudential policies. Multinational banks contribute to and are affected by fire sales in countries they operate in. National governments setting quantity regulations non-cooperatively fail to achieve the globally efficient outcome, under-regulating domestic banks and over-regulating foreign banks. Surprisingly, non-cooperative national governments using revenue-generating Pigouvian taxation can achieve the global optimum. Intuitively, this occurs because governments internalize the business value of foreign banks through the tax revenue collected. Our theory provides a unified framework to think about international bank regulations and yields concrete insights with the potential to improve on the current policy stance.

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1 Introduction

The banking industry is multinational in its scope: banks that are headquartered in one country lend to, borrow from, and are owned by agents across country borders.[1] In the aftermath of the 2008 financial crisis, financial stability concerns from cross-border banking have motivated regulators to extend post-crisis macroprudential regulatory regimes – such as equity capital and liquidity requirements – to foreign banks operating domestically, and to apply capital control measures – such as residency based transaction taxes – to manage capital flows.[2] It has also led to concerns that uncoordinated financial regulation may not be efficient, motivating international cooperative regulatory regimes involving common regulatory standards (Basel III) and common supervision and resolution (European Banking Union, Single Point of Entry (SPOE) resolution).[3] Despite the prominence of these agreements and their attention in the policy world, there is relatively limited formal economic analysis studying the need for macroprudential cooperation in the presence of financial stability concerns from cross-border banking, or to guide policymakers in forming cooperative agreements.

The main contribution of this paper is to show that in a setting with cross-border banking and country-level fire sales, non-cooperative national governments that use Pigouvian taxation to regulate banks are able to achieve the globally efficient outcome, provided there are no monopoly rents at the country level, eliminating the need for international cooperation.[4] The key mechanism that leads to efficiency is that taxes on foreign banks generate revenues for the domestic government. Even though the domestic government places no direct value on the welfare of foreigners, we show that revenue collection allows the domestic government to internalize both the benefits to

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[1]For example, more than 30% of global bank claims are on foreign counterparties as of 2019, with more than half of foreign claims being on the non-bank private sector. Bank for International Settlements (BIS) Consolidated Banking Statistics (CBS), among reporting countries.

[2]For example, the Intermediate Holding Company requirement in the US applies prudential standards of The Dodd-Frank Wall Street Reform and Consumer Protection Act (Dodd-Frank Act) to foreign banks with large operations in the US.

[3]See BIS (2010) for an overview of Basel III, ECB (2018) for an overview of the EU Single Supervisory Mechanism responsible for common supervision, Financial Stability Board (2013a) for a discussion of SPOE, and Tucker (2016) for a discussion of motivations for cooperation.

[4]We frame our paper in terms of banks, but it also applies to broader classes of financial intermediaries.

foreign banks from domestic operations and the spillovers to foreign banks from the domestic fire sale. By contrast if governments use revenue-neutral taxes, they fail to achieve the efficient outcome. We further show in our main model that the outcome under revenue-neutral taxes is also achieved in a model with explicit quantity restrictions, motivating existing cooperative agreements that are designed around use of quantity restrictions. We discuss the robustness of our Pigouvian tax efficiency result in more general banking environments as well as its limitations.

Our paper develops a simple three-period economic framework to study the regulation of cross-border banks in the presence of fire sales. Despite its simplicity, our model captures key features of the global banking industry and real economy, and we show that its insights extend to a more general environment. In our model, banks issue debt in order to finance domestic and cross-border investment at the initial date. Banks experience shocks to the value of their investment positions at the interim date, and must then roll over or repay their initial debt. When faced with binding collateral constraints, banks are forced to liquidate part of their total investment portfolio prematurely. Banks choose which investments to liquidate, and generate fire sales in the countries they sell assets in. Fire sales in a country affect all banks that invest and sell assets in that country, leading to cross border spillovers. These fire sale spillovers, which are not internalized by banks, motivate the consideration of macroprudential regulation.

We begin our discussion in Section 3 by studying the problem of a global planner who sets regulatory policy for all banks. The policies adopted by this global planner provide an efficiency benchmark to which we can compare the policies adopted by national governments. We show that globally efficient regulation involves placing state-contingent "wedges" (or taxes) on bank asset liquidations in each country. There are two key properties of the global optimum. First, the magnitude of the state-contingent liquidation taxes are set to the total fire sale spillovers to both domestic and foreign banks generated by asset liquidations. Second, there is equal regulatory treatment in that the same taxes are applied to all banks, regardless of their domicile.

We next introduce country level governments, or "country planners," who design regulation for their respective countries in a non-cooperative manner, in Section 4. In practice, countries have

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regulatory jurisdiction not only over all activities of domestic banks, but also over the domestic activities of foreign banks. This means that multiple countries have regulatory jurisdiction over the same bank. In the absence of cooperative agreements, national regulators will set macroprudential policies independently to maximize national welfare. Our framework captures this common agency problem and allows us to study whether country planners are able to achieve the globally efficient outcome without international cooperation.

Country planners are endowed with the same regulatory taxes that were available to the global planner. Our main results relate to the differentiation between outcomes under two different rules for remission of revenues collected from the taxes. First, we study *revenue-neutral* taxes, under which revenue collected from taxes on foreign banks is remitted lump-sum back to foreign banks. Second, we study *revenue-generating* taxes, under which revenue collected from foreign banks is instead remitted lump-sum to domestic banks. Both of these instruments in principle would allow planners in our model to implement the globally efficient allocation. We show in the appendix that a game with explicit quantity restrictions achieves the same outcome as revenue-neutral taxes in our baseline model. We therefore refer to revenue-neutral taxes in our model as *quantity restrictions*. By contrast, we term revenue-generating taxes to be *Pigouvian taxation*.

We first show that if country planners use revenue-neutral taxes ("quantity restrictions"), they fail to achieve the globally efficient outcome without international cooperation. There are two key departures from global efficiency. First, taxes placed on asset liquidations by domestic banks are too small, accounting for domestic fire sale spillovers but not for foreign fire sale spillovers. Second, taxes placed on domestic liquidations by foreign banks are too large, ensuring that foreign banks do not contribute to domestic fire sales. The equivalent implementation using explicit quantity restrictions applies ceiling restrictions on liquidations that are too flexible for domestic banks, and too strict for foreign banks. This non-cooperative optimum motivates a cooperative agreement that increases regulation of domestic banks and ensures equal treatment of foreign banks, and helps to rationalize the broad architecture and goals of existing international cooperative arrangements.

The main and most surprising result of our paper is that *non-cooperative* national governments

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using revenue-generating taxes ("Pigouvian taxation") can implement the globally efficient outcome, eliminating the need for cooperation. In particular, country planners set tax rates that coincide with globally optimal policy, up to a monopolistic revenue extraction distortion. When countries' monopoly power is zero due to sufficient substitutability, non-cooperative Pigouvian taxation is globally efficient. The mobility of global banking assets and the presence of large offshore financial centers suggests that low monopoly revenues at the country level are a plausible description of the world.[5]

In contrast to revenue-neutral taxes (quantity restrictions), use of revenue-generating taxes (Pigouvian taxation) results in efficiency precisely because of the motivation for the domestic planner to collect tax revenues. When combined with the standard motivation to correct domestic externalities, this motive to collect tax revenue leads to efficient outcomes.[6] The intuition is that a country planner is willing to allow foreign banks to engage in socially costly domestic activities because she can collect more tax revenue as a result. This aligns preferences between the domestic planner and foreign banks because in equilibrium, the marginal tax rates on foreign banks' domestic activities are equal to the marginal benefit to foreign banks of those activities. And since domestic fire sales reduce the marginal benefit to foreign banks of domestic activities, they also reduce tax revenue collection. With an incentive to generate tax revenue, domestic planners internalize not only the benefits derived by foreign banks from domestic activities, but also the costs imposed on them by domestic fire sales. By contrast, because quantity regulation does not generate revenues, the domestic planner does not consider the welfare impacts of regulation on the value of foreign banks, which accrues to foreigners.

The efficiency of non-cooperative Pigouvian taxation has implications for both macroprudential policies and capital controls. Although in practice macroprudential policies often take the form of quantity regulation rather than Pigouvian taxation, we speculate that this may have arisen in part due to a combination of perceived duality between these instruments and political obstacles to taxation.

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[5]For example, see the work by Coppola et al. (2021) on global capital flows and tax havens.

[6]That is, in our model efficiency results from the "double dividend" of Pigouvian taxes – they correct externalities and generate revenues (Tullock 1967).

Our results suggest that adopting a (non-cooperative) tax-based approach to bank regulation can potentially be an alternative to explicit cooperative agreements over quantity restrictions.

We next study the practical policy implications of our results. In our baseline model of Sections 2-4, planners achieve efficiency by using state contingent taxes on ex post bank asset liquidations. However, in practice regulators generally use *ex-ante* macroprudential tools, such as equity capital and liquidity requirements. For this reason, much of the prior literature and policy debate have focused on these instruments. In Section 5, we use a variant of our baseline model to study four types of policies that are central to Basel III and the European Banking Union: regulation of debt and illiquid asset positions; liquidity regulation; regulation of cross-border bank resolution; and provision of fiscal backstops (or "bailouts"). In this environment, non-cooperative national governments again fail to achieve efficiency under revenue-neutral taxes, and as in the baseline model we show how the same outcome is achieved using explicit quantity restrictions. Conversely, non-cooperative governments achieve the efficient outcome, absent monopoly rents, using revenue-generating (Pigouvian) taxes. Our results suggest concretely that non-cooperative regulators may be able to improve efficiency by: (i) using taxes on debt and illiquid assets, rather than equity capital or leverage requirements; (ii) using taxes on liquid asset holdings, rather than liquidity requirements such as the Liquidity Coverage Ratio (LCR); (iii) charging banks a fee based on organizational structure (e.g. MPOE versus SPOE), rather than imposing orderly resolution requirements; and, (iv) charging banks a fee based on expected fiscal support, for example a deposit insurance premium.

Finally, we study the robustness and limitations of our main result. In Section 6, we show that our results extend to a broader class of externality problems featuring multinational banks (or "agents"). We present a general model of these externality problems. We characterize two classes of externalities: *local* and *global*. Local externalities, such as spillovers to the domestic economy or to a domestic deposit guarantee scheme, only affect domestic agents. Global externalities, such as fire sales or climate change, affect both domestic and foreign agents. The efficiency of non-cooperative Pigouvian (revenue-generating) taxation extends to the class of local externalities, following the same logic as in the main model. By contrast, the efficiency of non-cooperative

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Pigouvian taxation under global externalities depends on the precise nature of international spillovers. When a domestic externality spreads endogenously through cross-border activities, as with fire sales, Pigouvian taxation results in efficient outcomes. However, when an externality generates international spillovers even in the absence cross-border activities, as with climate change, or spreads through agents who cannot be regulated,[7] Pigouvian taxation is not generally efficient.

Related Literature. First, we relate to a large empirical literature on the determinants and properties of capital flows and cross border banking, including home bias.[8] These empirical observations help motivate the assumptions underlying our baseline banking model.

Second, we relate to a large literature on macroprudential regulation and capital controls in domestic and small open economies,[9] and to a smaller literature on optimal regulatory cooperation in international banking and financial markets.[10] Gersbach et al. (2020) show in a two country model that country regulators using a combination of capital requirements and a bank tax achieve the efficient outcome when foreign households can deposit and own equity in domestic banks, and so generate a domestic spillover to the domestic deposit guarantee scheme. However, they do not consider fire sales or broader classes of externality problems, and do not have common agency. Caballero and Simsek (2018, 2020) show that fickle capital flows can be a valuable source of liquidity to distressed countries. National regulators neglect this benefit and ban capital inflows to mitigate domestic fire sales. Korinek (2017) provide a first welfare theorem in a model in which

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[7]This limitation is to agents who ("technologically") cannot be regulated, and not to agents who can be regulated but are unregulated *in equilibrium*. For example, a (price) spillover to a foreign firm from reduced credit availability would be internalized even though the *equilibrium* tax on that firm is zero if it does not itself contribute to externalities.

[8]For example, Avdjiev et al. (2018), Broner et al. (2013), Caballero et al. (2008) Davis and Van Wincoop (2018), Forbes and Warnock (2012), and Milesi-Ferretti and Tille (2011), Maggiori (2017), Maggiori et al. (2020), De Marco et al. (2019), Mendoza et al. (2009), Niepmann (2015), and Shen (2019). See Coeurdacier and Rey (2013) for a broader overview of the home bias literature.

[9]For example, Bianchi (2011), Bianchi (2016), Bianchi and Mendoza (2010), Bianchi et al. (2016), Bianchi and Mendoza (2018), Caballero and Krishnamurthy (2001), Chari and Kehoe (2016), Dávila and Korinek (2018), Farhi et al. (2009), Farhi and Tirole (2012), Farhi and Werning (2016), Korinek and Simsek (2016), Lorenzoni (2008), Schmitt-Grohé and Uribe (2016), and Stein (2012). See Erten et al. (2021) for a survey of the capital controls literature.

[10]Additional contributions include Acharya (2003), Beck et al. (2013), Beck and Wagner (2016), Calzolari et al. (2019), Dell'Ariccia and Marquez (2006), Farhi and Werning (2017), Morrison and White (2009), Niepmann and Schmidt-Esenlohr (2013), and Segura and Vicente (2019). See also the literature on principal-agent problems with common agency, e.g. Bernheim and Whinston (1986) and Dixit et al. (1997). In common with much of the literature, we apply a Nash equilibrium concept rather than contractible contracts (as in e.g. Szentes 2015).

country planners control domestic agents, who interact on global markets. The environment in which this welfare theorem holds does not allow for domestic prices to appear in foreign constraints, as in our model, and does not feature common agency as regulators have no direct controls over foreign agents. Farhi and Tirole (2018) show that national regulators loosen bank supervision to dilute existing international creditors, motivating supranational supervision. Bolton and Oehmke (2019) study the trade-off between single- and multi-point-of-entry in bank resolution. Bengui (2014) and Kara (2016) consider regulatory cooperation when banks' operations are domestic but the asset resale market is global. Our main contribution to this literature is to show that in an environment with international spillovers from fire sales, national governments using Pigouvian taxation can achieve the global optimum even though they fail to do so using quantity regulation.

Differences between quantity regulations and Pigouvian taxes have long been recognized by multiple literatures. The key distinction in our paper derives from the "double dividend" – taxes both correct for externalities and generate revenues for the planner (Tullock 1967). The international trade literature has long recognized that quotas (quantities) and tariffs (taxes) can be dual in the sense of generating the same allocation, but a tariff allocates marginal surplus to the government whereas a quota may instead allocate surplus to foreigners (Bhagwati 1965, Bhagwati 1968, Shibata 1968, Magee 1972).[11] This insight has implications for (political) lobbying for tariff revenues (Bhagwati and Srinivasan 1980, Cassing and Hillman 1985), uncertainty (Fishelson and Flatters 1975, Pelcovits 1976, Dasgupta and Stiglitz 1977), and regulatory evasion (Falvey 1978). In our model, the revenue from the Pigouvian tax allows country planners to internalize fire sale spillovers to foreign banks. Another perspective emphasizes that uncertainty over private benefits and social costs leads to a trade-off between quantity regulation and (revenue-neutral) linear Pigouvian taxation (Weitzman 1974, Perotti and Suarez 2011). While our model assumes full information, we discuss the potential impacts of Weitzman (1974) considerations after presenting our main result.

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[11]For example, consider a foreign company with fixed marginal cost of 1 selling to domestic consumers with CES demand $c = 1/p^\sigma$. With a per unit import tariff $\tau > 0$ on the consumer, we have producer price $p^* = \frac{\sigma}{\sigma-1} + \frac{1}{\sigma-1}\tau$, an after-tax consumer price $p = p^* + \tau$, consumption $c^* = 1/p^\sigma$, and tariff revenue τc^* for the government. By contrast with an import quota $c \leq c^*$ on consumers, we have a producer price $p = p^* + \tau$, a consumer price p, and no revenue for the government. Hence the end price faced by consumers is the same in both cases, but tariff allocates the revenues τc^* to the government whereas the quota allocates the "revenues" τc^* to the firm.

2 Model

In this section, we present our baseline model, which is our leading example of a cross-border banking environment. In Section 6, we consider a more general environment, and discuss the robustness and limitations of our main results.

There are three dates, $t = 0, 1, 2$. The world economy consists of a unit continuum of countries, indexed by $i \in [0, 1]$. All countries are small and of equal measure, but are not necessarily symmetric or otherwise identical.

Each country is populated by a representative bank and a representative arbitrageur. Banks raise funds from global investors to finance investment in both their home country and in foreign countries. Arbitrageurs are second-best users of bank investment projects, and purchase bank investments that are liquidated prior to maturity. Arbitrageurs and global investors exist in our model to solve for the general equilibrium prices that banks face, but are not the primary focus of the model. Accordingly, we make their decision problems as simple as possible.

A global state $s \in S$, with continuous density $f(s)$, is realized at date 1, at which point uncertainty resolves. It captures all shocks in the model, including global, regional, and country-level shocks.

2.1 Banks

Banks are risk-neutral and do not discount the future. Banks only consume at date 2, with final consumption denoted by $c_i(s)$.

2.1.1 Bank Activities

Banks are able to undertake an investment project (or "asset") in each country at date 0. Projects are illiquid, and suffer a loss when liquidated (sold) prior to maturity. We denote by I_{ij} the (date 0) investment scale undertaken by country i banks in the country j project, with $I_i = \{I_{ij}\}_j$

denoting a bank's investment portfolio.[12] We assume that bank investment is *home biased*: domestic investment, $I_{ii} \in \mathbb{R}_+$, is a mass point, whereas foreign investment, $I_{ij} : [0,1] \to \mathbb{R}_+$, is a density.[13] Home bias can arise in the model when domestic banks specialize in domestic activities. Because fire sales will be a core focus of the model, we use the assumption of home bias to ensure that domestic banks retain a substantial exposure to the domestic economy, creating a motivation for domestic regulation. Assuming that banks retain only a marginal exposure to foreign countries is a simplifying assumption to maintain tractability.[14]

Banks operate a technology at date 0 which uses $\Phi_{ij}(I_{ij})$ units of the numeraire to produce I_{ij}, where Φ_{ij} is increasing and weakly convex. Banks' total investment cost is therefore $\Phi_{ii}(I_{ii}) + \int_j \Phi_{ij}(I_{ij})dj$.

At date 1, all projects in country j experience a quality shock $R_j(s)$, transforming the scale of projects operated by country i banks in country j to $R_j(s)I_{ij}$. Projects do not yield dividends at date 1, but yield $1 + r_{ij} \geq 1$ units of the consumption good per unit of scale when held to maturity at date 2. Intuitively, $R_j(s)$ captures a common risk exposure while r_{ij} captures different specializations (comparative advantages) in bank lending.

Projects may be liquidated at date 1, prior to maturity. We denote project liquidations by L_i, defined analogously to I_i, with $0 \leq L_{ij}(s) \leq R_j(s)I_{ij}$. Liquidated projects are sold to arbitrageurs at price $\gamma_j(s) \leq 1$, with the final return r_{ij} being lost.[15]

2.1.2 Bank Financing

Banks finance investment using an initial endowment $A_i > 0$ and by issuing external debt D_i from risk-neutral global investors at price 1.[16] For expositional simplicity, we consolidate banks' balance

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[12]Our results on Pigouvian efficiency generalize to the case where there are multiple investment goods in each country and fire sale discounts are interconnected across assets.

[13]See Caballero and Simsek (2020) for a similar assumption. As highlighted in the introduction, home bias is an empirical regularity.

[14]In Appendix E.7, we study a game with a finite number of countries whose banks maintain large exposures in foreign countries.

[15]For simplicity, we do not allow banks to purchase assets liquidated by other banks, which would bolster the liquidation price in cases where banks were not in correlated distress. In Section 6, we show that allowing banks to both purchase and sell assets does not alter our main Pigouvian efficiency result.

[16]We assume that risk-neutral global investors are deep pocketed, and therefore always finance debt at a price of 1.

sheets across countries and operations. Given a fixed debt price of 1, the liquidation prices γ are the only endogenous prices in the model. The bank uses its total funds to finance its investment portfolio at date 0, so that the date 0 bank budget constraint is

$$\Phi_{ii}(I_{ii}) + \int_j \Phi_{ij}(I_{ij})dj \leq A_i + D_i. \tag{1}$$

At date 1, banks can roll over debt at a price of 1, meaning that D_i is also the amount of new debt issued at date 1. Consolidating the dates 1 and 2 budget constraints yields

$$c_i(s) \leq \mathcal{R}_{ii}(s) + \int_j \mathcal{R}_{ij}(s)dj - D_i, \tag{2}$$

where $\mathcal{R}_{ij}(s) = \gamma_j(s)L_{ij}(s) + (1 + r_{ij})(R_j(s)I_{ij} - L_{ij}(s))$ is the total return to investment in country j for country i banks from both date 1 liquidations and date 2 final payoffs.

Collateral Constraint. Banks with no restrictions on debt rollover would never choose to liquidate assets, since liquidations always reduce bank value. To introduce a role for liquidations and fire sales, we impose a date 1 collateral constraint, which is a standard method of capturing forced deleveraging (e.g. Kiyotaki and Moore (1997)). The date 1 collateral constraint requires banks to back debt issued at date 1 with collateral, and is given by

$$D_i \leq \underbrace{\gamma_i(s)L_{ii}(s) + \int_j \gamma_j(s)L_{ij}(s)dj}_{\text{Funds From Liquidations}} + \underbrace{(1 - h_i(s))\mathcal{C}_{ii}(s) + \int_j (1 - h_j(s))\mathcal{C}_{ij}(s)dj}_{\text{Funds From Collateral}}, \tag{3}$$

where $\mathcal{C}_{ij}(s) = \gamma_j(s)\left[R_j(s)I_{ij} - L_{ij}(s)\right]$ is the market value of collateral at date 1. The collateral haircut $h_j(s) \in [0,1]$ reflects the extent to which investors discount a project's collateral value, and can reflect economic (e.g. uncertainty) and political (e.g. expropriation) concerns about collateral quality. Banks that cannot roll over their entire liabilities using collateral must liquidate assets to

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repay investors.[17]

2.1.3 Bank Optimization

At date 0, banks choose a contract (c_i, D_i, I_i, L_i) with commitment in order to maximize expected utility $\int_s c_i(s) f(s) ds$ subject to the budget constraints (1) and (2), and the collateral constraint (3). Banks take equilibrium prices γ as given. We discuss the role of bank and planner commitment in Section 4.4.2.

2.2 Arbitrageurs and Liquidation Values

Country i arbitrageurs are second best users of country i projects.[18] At date 1, they purchase an amount $L_i^A(s)$ of bank projects and convert them into the consumption good using an increasing and (weakly) concave technology $\mathcal{F}_i\left(L_i^A(s), s\right)$. Arbitrageur technology is inefficient in the sense that $\frac{\partial \mathcal{F}_i(L_i^A(s), s)}{\partial L_i^A(s)} \leq 1$, so that selling projects to arbitrageurs never results in a resource gain.

Arbitrageurs obtain surplus $c_i^A(s) = \mathcal{F}_i\left(L_i^A(s), s\right) - \gamma_i(s) L_i^A(s)$ from purchasing projects. Arbitrageurs are price takers, so that the equilibrium liquidation value is

$$\gamma_i(s) = \frac{\partial \mathcal{F}_i\left(L_i^A(s), s\right)}{\partial L_i^A(s)}, \quad L_i^A(s) = L_{ii}(s) + \int_j L_{ji}(s) dj \tag{4}$$

where $L_i^A(s)$ is equal in equilibrium to total country i projects sold by *all* banks, including foreign ones. There is a *fire sale spillover* when additional liquidations reduce liquidation values, that is when the marginal product of bank projects in arbitrageur technology is strictly decreasing. The extent of the fire sale spillover reflects the ability of the economy to absorb liquidations by banks, with deeper fire sales arising when limited market depth allocates liquidated bank projects to

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[17]We can obtain similar qualitative results if the future price $1 + r_{ij}$ is used to value collateral (rather than the current price $\gamma_j(s)$), provided that in at least some countries haircuts are large enough that banks are forced to liquidate assets (that is, $(1 - h_j(s))(1 + r_{ij}) < \gamma_j(s)$). Quantitatively, the final term $(1 - h_j(s))[R_j(s)I_{ij} - L_{ij}(s)]$ in equation (9) in Proposition 1, which reflects revaluing of collateral, would drop out and hence tend to dampen the magnitude of regulation.

[18]The empirical regularity of retrenchment by foreigners coinciding with domestic distress is suggestive of a role for local arbitrageurs (e.g. Broner et al. 2013). We discuss the possibility that banks also purchase assets in Section 6.4.

increasingly less efficient users.

2.3 Motivations for Cross-Border Banking

In order to map our model into economically important applications, we characterize the motivations for cross-border banking in the competitive equilibrium of the model.[19]

The optimal liquidation rule $L_{ij}(s)$ is given by the first order condition

$$0 = \underbrace{\lambda_i^1(s)\left(\gamma_j(s) - (1 + r_{ij})\right)}_{\text{Resource Loss}} + \underbrace{\Lambda_i^1(s)h_j(s)\gamma_j(s)}_{\text{Collateral Haircut}} + \underline{\xi}_{ij}(s) - \overline{\xi}_{ij}(s) \tag{5}$$

where the (non-negative) Lagrange multipliers are $\lambda_i^1(s)$ on the date 1 budget constraint (2), $\Lambda_i^1(s)$ on the date 1 collateral constraint (3), and $\underline{\xi}_{ij}(s), \overline{\xi}_{ij}(s)$ (respectively) on the constraints $0 \le L_{ij}(s) \le R_j(s)I_{ij}$ in state s. Equation (5) shows that because liquidations result in resource losses, that is $(1 + r_{ij}) - \gamma_j(s) > 0$, banks only liquidate assets when the collateral constraint binds, that is $\Lambda_i^1(s) > 0$. Banks prefer to liquidate assets with lower liquidation discounts, $(1 + r_{ij}) - \gamma_j(s)$, and higher collateral haircuts, $h_j(s)$.

The investment decision I_{ij} of banks is given by the first order condition

$$0 \ge \overbrace{-\lambda_i^0 \frac{\partial \Phi_{ij}}{\partial I_{ij}} + E[\lambda_i^1]E\left[(1 + r_{ij})R_j\right]}^{\text{Specialization}} + \overbrace{\text{cov}\left(\lambda_i^1, (1 + r_{ij})R_j\right)}^{\text{Diversification}} + \overbrace{E\left[\overline{\xi}_{ij}R_j\right]}^{\text{Liquidity}} \tag{6}$$

$$+ \underbrace{E\left[\Lambda_i^1(1 - h_j)\gamma_j R_j\right]}_{\text{Collateral}}$$

where λ_i^0 is the non-negative multiplier on the date 0 budget constraint (1). "Specialization" indicates that banks invest in assets with low marginal costs, $\frac{\partial \Phi_{ij}}{\partial I_{ij}}$, and high returns, r_{ij}. For example, banks may specialize in certain lending markets or may lend in under-serviced markets. "Diversification"

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[19]Formally, a competitive equilibrium of the global economy is a vector of allocations (c, D, I, L, L^A) and prices γ such that: (1) the contract (c_i, D_i, I_i, L_i) is optimal for country i banks, given prices; (2) purchases L_i^A are optimal for country i arbitrageurs, given prices; and the markets for liquidations clear. See Appendix A.1 for derivations of first order conditions in this section.

indicates that banks value assets that pay off in states where the value of bank wealth, $\lambda_i^1(s)$, is high. Although banks are risk neutral in their preferences, a binding collateral constraint induces a higher marginal value of wealth as banks seek to avoid forced asset sales. "Liquidity" measures the value to banks of having more of an asset available to be liquidated when faced with binding collateral constraints, and implies banks value liquid assets with a high Lagrange multiplier $\overline{\xi}_{ij}(s)$ (with a positive Lagrange multiplier indicating a corner solution, where the bank would prefer to liquidate more investment if it had more to liquidate). "Collateral" measures the value of an asset as collateral for rolling over debt at date 1, and is decreasing in both the haircut, $h_{ij}(s)$, and the liquidation discount, $1 - \gamma_j(s)$.

3 Globally Optimal Policy

In this section, we study the optimal policy that would be adopted by a global planner looking to correct the pecuniary externalities that arise from the presence of prices in banks' constraints. This provides a natural benchmark of a globally Pareto efficient allocation, which would be achieved with international cooperation. We later contrast our main results in Section 4 with this cooperative benchmark to study conditions under which independent country regulators can achieve the cooperative outcome even without international cooperation.[20]

The global planning problem is a constrained-efficient problem of choosing activities of all banks (c_i, D_i, I_i, L_i for all i) in order to maximize a weighted sum of bank welfare, $\int_i \omega_i \int_s c_i(s) f(s) ds di$, subject to the same constraints (equations (1), (2), (3)) as faced by banks, but internalizing the equilibrium pricing equation (4).[21] For expositional purposes, we place welfare weights of zero on arbitrageurs, and show in Section 6.3 that our main result on Pigouvian efficiency still holds under positive welfare weights. As with banks, the global planner in our model solves the constrained efficient planning problem with commitment.

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[20]For expositional purposes, results in the main text are presented for interior solutions.

[21]As usual, the constrained efficient allocation may also involve lump sum transfers between countries at date 0 to guarantee Pareto efficiency. See e.g. Dávila and Korinek (2018) and Korinek (2017).

We characterize the solution of the global planning problem by its decentralization: the complete set of date 0 wedges $\tau = \{\tau_i^c, \tau_i^D, \tau_i^I, \tau_i^L\}$ and date-0 lump sum transfers $\mathcal{T}_i$ that implement the globally optimal allocation. The complete set of wedges placed on country i banks consists of: a wedge $\tau_i^c(s)$ on consumption in state s; a wedge τ_i^D on date 0 debt; a wedge τ_{ij}^I on investment in country j; and a wedge $\tau_{ij}^L(s)$ on liquidations of country j assets in state s. A complete wedge approach is a conventional approach to studying constrained efficient planning problems. However, in Appendix D.3, we show that the global optimum of Proposition 1 can also be implemented with explicit quantity restrictions, rather than wedges. Moreover, we show in Section 5 that our results also apply to more conventional macroprudential instruments: Our main results and the economic forces behind them extend to the case where ex-ante controls over assets and liabilities are allowed, but ex-post controls over liquidations are not.

Formally, wedges are taxes on banks at date 0 (recall banks also solve with commitment), whose proceeds are remitted lump sum to the banks they are collected from. The total *date 0 wedge burden* borne by country i banks (excluding remissions) is $T_i = T_{ii} + \int_j T_{ij}dj$, where $T_{ii} = \tau_{ii}^c c_i + \tau_{ii}^D D_i + \tau_{ii}^I I_i + \tau_{ii} L_i$ is the burden from their domestic activities and where $T_{ij} = \tau_{ij}^I I_j + \tau_{ij}^L L_{ij}$ is the burden from their foreign activities in country j. To ease exposition, we have adopted inner product notation, where for example $\tau_{ii}^c c_i = \int_s \tau_{ii}^c(s) c_i(s) f(s) ds$. The equilibrium value T_i^* of revenue from wedges is remitted lump sum to country i banks at date 0, so that their date 0 budget constraint accounting for wedges and remissions is

$$\Phi_{ii}(I_{ii}) + \int_j \Phi_{ij}(I_{ij})dj \leq A_i + D_i - T_i + T_i^*. \tag{7}$$

Throughout the paper, we maintain the asterisk notation to denote revenue remissions.

Because the global planner has a complete set of (revenue-neutral) wedges for every aspect of banks' decision problems, the planner can incentivize banks to adopt the socially optimal allocation rules by setting the wedge equal to the gap between the marginal social value of a change in that activity, the social planner's FOC, and the marginal private value of a change in that activity. See the

proof for a formal representation. This gives us a standard representation of decentralizations of constrained efficient planning problems. The following proposition characterizes the decentralization of the globally constrained efficient allocation in terms of these wedges.

Proposition 1. *The globally efficient allocation can be decentralized using liquidation wedges*

$$\tau_{ji}^L(s) = -\underbrace{\Omega_{ii}(s)}_{\text{Domestic Spillovers}} - \underbrace{\int_{i'}\Omega_{i'i}(s)di'}_{\text{Foreign Spillovers}} \qquad \underbrace{\forall j}_{\text{Equal Treatment}} \tag{8}$$

where $\Omega_{ij}(s) \leq 0$ is the spillover to bank i from changes in total liquidations in foreign country j, given by

$$\Omega_{ij}(s) = \underbrace{\frac{\partial \gamma_i(s)}{\partial L_i^A(s)}}_{\text{Price Impact}} \left[\underbrace{\frac{\lambda_i^1(s)}{\lambda_i^0} L_{ij}(s)}_{\text{Distributive Externality}} + \underbrace{\frac{\Lambda_i^1(s)}{\lambda_i^0}\left(L_{ij}(s) + (1-h_j(s))\left[R_j(s)I_{ij} - L_{ij}(s)\right]\right)}_{\text{Collateral Externality}} \right] \tag{9}$$

All other wedges are 0.

The proof of Proposition 1, along with all other proofs, is in the Appendix. The globally efficient allocation corrects a fire sale spillover problem: higher liquidations reduce liquidation prices and collateral values, tightening banks' collateral constraints further and forcing further liquidations. Because both domestic and foreign banks hold domestic investment, the fire sale impacts both domestic banks ("Domestic Spillovers") and all foreign banks ("Foreign Spillovers"). The impact on any individual bank is the product of the marginal change in the liquidation price ("Price Impact") and the total impact of that price change on that bank. That total impact consists of two standard pecuniary externalities: distributive externalities and collateral externalities (Dávila and Korinek 2018). Distributive externalities reflect that an increase in the liquidation price increases the recovery value to banks from liquidating the asset, which is weighted by the marginal value of wealth, $\lambda_i^1(s)$, in that state. Distributive externalities are larger when banks liquidate more of that asset, that is $L_{ij}(s)$ is high, or when the marginal value of date 1 wealth is high due to a more severely binding collateral constraint. Collateral externalities reflect the impact of the change in price on the binding

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collateral constraint. An increase in the liquidation price relaxes the collateral constraint both because liquidations generate a greater recovery value to repay debt holders, and because the collateral value for debt rollover increases. As a result, foreign spillovers are particularly large when foreign banks are forced to liquidate domestic assets or face binding collateral constraints at the same time that the domestic liquidation price is particularly sensitive to additional liquidations.

Because both domestic and foreign banks can contribute to the domestic fire sale via liquidations, globally efficient policy applies wedges to both domestic and foreign banks. Moreover, globally efficient policy applies *equal treatment*: the wedge placed on liquidations of the country i asset does not depend on the domicile of the bank liquidating it. This is because both domestic and foreign banks generate the same total spillover by liquidating a domestic project. Although foreign banks can contribute to domestic instability by retrenching, they are not treated differently from domestic banks under the globally efficient policy.

4   Non-Cooperative Policies

The globally efficient policy of Section 3 is predicated on a global planner setting policy. However, in practice individual countries have regulatory jurisdiction over banks within their borders. In this section, we present the main result of our paper: whereas independent governments using *quantity regulation* are unable to achieve efficient policy, independent governments using *Pigouvian taxation* are able to achieve the efficient outcome provided that monopoly rents are zero.

4.1   Country Planners

Each country has a designated government, or "social planner," who represents and acts in the interests of domestic agents. The social welfare function of country planner i is equal to domestic bank welfare, $\int_s c_i(s)f(s)ds$.[22] The social planner of each country has a complete set of wedges on both domestic banks and domestic allocations of foreign banks. The wedges of the country i

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[22]We have assumed banks are wholly domestically owned. Appendix D.1 allows for partial foreign ownership of domestic banks, and shows that Pigouvian taxation remains efficient.

planner on country i banks are $\tau_{i,i} = (\tau_{i,i}^c, \tau_{i,i}^D, \tau_{i,i}^I, \tau_{i,i}^L)$, and are fully contingent as in Section 3. The fully contingent wedges of the country i planner on country j banks are $\tau_{i,j} = (\tau_{i,ji}^I, \tau_{i,ji}^L)$, reflecting that the country i planner can only directly influence the domestic activities of foreign banks.[23] To clarify notation, the index prior to the comma indicates the identity of the country planner placing the wedge, while the indexing after the comma (combined with the superscript) indicates the bank and activity the wedge is being placed on. Furthermore, observe that there is common agency: both the planner of country i and the planner of country j have wedges over the investment ($\tau_{i,ij}^I$ and $\tau_{j,ij}^I$, respectively) and liquidations ($\tau_{i,ij}^L$ and $\tau_{j,ij}^L$) of country i banks in country j.

As in Section 3, these wedges are taxes from the perspective of banks, meaning that revenue is collected from their use. The total *date 0* wedge burden borne by country i banks (excluding remissions) is $T_{i,i} + \int_j T_{j,ij} dj$, where $T_{i,i} = \tau_{i,i}^c c_i + \tau_{i,i}^D D_i + \tau_{i,i}^I I_i + \tau_{i,i} L_i$ is the wedge burden owed by domestic banks to the domestic planner and $T_{j,ij} = \tau_{j,ij}^I I_{ij} + \tau_{j,ij}^L L_{ij}$ is the wedge burden owed by domestic banks to foreign planner j. Note that the index prior to the comma again refers to the planner to whom wedge revenues are owed.

Quantity Regulation versus Pigouvian Taxation. In Section 3, proceeds from these wedges were remitted lump-sum to the banks they were collected from. In this section, we differentiate between two instruments – quantity regulation and Pigouvian taxation – based on the revenue remission rule for revenues collected from wedges on foreign banks. The equilibrium tax revenue $T_{i,i}^*$ collected from domestic banks is always remitted lump-sum to domestic banks.

We refer to *revenue-neutral* wedges on foreign banks as quantity restrictions, appealing to duality results between revenue-neutral taxes and quantity restrictions in problems with a single regulator.[24] Moreover, in Appendix D.3, we verify that the optimum characterized in Proposition 3 is also attained when country planners utilize explicit quantity restrictions, rather than revenue-neutral wedges. Under quantity regulation (i.e. revenue-neutral wedges), tax revenue collected from

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[23]Because wedges are the means of controlling allocations, we rule out explicit side payments.

[24]For example, Erten et al. (2021) argues that "the principle of dualism...implies that every quantity-based control corresponds to an equivalent price-based control.

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foreign banks is remitted globally to foreign banks according to a remission rule $T_{i,-i}^{*,\text{Quantity}} = T_i^G$, which is taken as given by country i.[25] In particular, planner i does not internalize the impact of how domestic taxes on foreign banks change the remitted tax revenue $T_{i,-i}^{*,\text{Quantity}}$. Wedges under the quantity regulation remission rule are used to control allocations, but do not generate revenues for the domestic planner.

By contrast, we refer to *revenue-generating* wedges as Pigouvian taxation. Under Pigouvian taxation, the equilibrium tax revenue collected from foreign banks is remitted to domestic banks. This generates total remissions $T_{i,-i}^{*,\text{Pigou}} = \int_j T_{i,ji} dj$ to domestic banks. In contrast to quantity regulation, the country i planner now accounts for how changes in policy affect revenue collected from foreign banks because it translates directly into changes in revenues remitted to domestic banks.

In both cases, taxes appear in the banks' date 0 budget constraint, now given by

$$\Phi_{ii}(I_{ii}) + \int_j \Phi_{ij}(I_{ij}) dj \leq A_i + D_i \underbrace{- T_{i,i} - \int_j T_{j,ij} dj}_{\text{Total Tax Burden}} + \underbrace{T_{i,i}^* + T_{i,-i}^*}_{\text{Total Remissions}} \tag{10}$$

where $T_{i,-i}^* \in \{T_{i,-i}^{*,\text{Quantity}}, T_{i,-i}^{*,\text{Pigou}}\}$, depending on the policy regime. Banks optimally choose contracts as in Section 2, now taking into account the additional tax burden. Notice that equation (10) is identical to the budget constraint in Section 3, up to the different remission rules. This means that collectively, country planners have the same total set of instruments that the global planner does. In contrast to the global planner, however, different country planners set different instruments independently of one another, and there are some instruments that multiple country planners possess (common agency).

Equilibrium Concept. A non-cooperative equilibrium of the model is a Nash equilibrium between country planners, in which every country planner optimally chooses wedges $\tau_i = (\tau_{i,i}, \{\tau_{i,j}\})$

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[25]In particular, there is the globally remitted revenue $T^G = \int_i \int_j T_{i,ji}^* dj di$ arising from the wedges, which corresponds to remitting revenue to foreigners. We assume this is remitted according to some allocation rule $\int_i T_i^G di = T^G$.

to maximize domestic social welfare, taking as given the wedges τ_{-i} set by all other country planners.

Implementability. Banks are subject to wedges by planners in all countries they operate in. Moreover, although country planner i has a complete set of controls over domestic banks, she has only a partial set of controls over foreign banks. Nevertheless, we provide an implementability result that allows us to solve the problem using a standard approach of directly choosing allocations, and then backing out the wedges that implement that allocation.

Lemma 2 (Implementability). *Under both quantity regulation and Pigouvian taxation, the optimization problem of country planner i can be written as maximizing social welfare by directly choosing allocations $(c_i, D_i, I_i, L_i, \{I_{ji}, L_{ji}\}_j)$, subject to equations ([10](#)), ([2](#)), ([3](#)), and ([4](#)), taking as given τ_{-i} and γ_{-i}. The implementing wedges for the domestic allocations of foreign banks are*

$$\tau_{i,ji}^I = -\tau_{j,ji}^I - \frac{\partial \Phi_{ji}}{\partial I_{ji}} + E\left[\frac{\lambda_j^1}{\lambda_j^0}(1+r_{ji})R_i\right] + \frac{1}{\lambda_j^0}E\left[\Lambda_j^1(1-h_i)\gamma_i R_i\right] \tag{11}$$

$$\tau_{i,ji}^L(s) = -\tau_{j,ji}^L(s) + \frac{\lambda_j^1(s)}{\lambda_j^0}\left(\gamma_i(s) - (1+r_{ji})\right) + \frac{1}{\lambda_j^0}\Lambda_j^1(s)h_i(s)\gamma_i(s) \tag{12}$$

where country planner i takes the Lagrange multipliers $\lambda_j^0, \lambda_j^1(s), \Lambda_j^1(s)$ as given (for $j \neq i$).

4.2 Non-Cooperative Quantity Regulation

We now characterize the non-cooperative equilibrium under quantity regulation, where revenue from wedges on foreign banks is remitted to foreign banks.

Proposition 3. *The non-cooperative equilibrium under quantity regulation has the following features.*

    1. The domestic liquidation wedges on domestic banks are

$$\tau_{i,ii}^L(s) = - \underbrace{\Omega_{ii}(s)}_{\text{Domestic Spillovers}} \qquad (13)$$

where Ω_{ii} is defined as in Proposition 1.

2. *The domestic liquidation wedges on foreign banks generate an allocation rule*

$$L_{ji}(s) \underbrace{\Omega_{ii}(s)}_{\text{Domestic Spillovers}} = 0.$$

In other words, if $|\Omega_{ii}(s)| > 0$ then $\tau_{i,ji}^L(s)$ is set high enough that foreign banks do not liquidate domestic assets in state s.

3. *All other wedges on domestic and foreign banks are* 0.

Proposition 3 reflects how country planners use quantity regulation to manage fire sale spillovers. First, country planners place wedges on domestic liquidations by domestic banks that account for the fire sale spillover cost to *domestic* banks. Because planners do not care about the welfare of foreign banks, the domestic wedges do not account for spillovers to foreign banks.

Second, country planners place wedges on liquidations by foreign banks. Because planners again do not care about foreign bank welfare, they find it optimal to prohibit foreign banks from contributing to the domestic fire sale whenever there is an adverse domestic spillover, even while allowing domestic banks to liquidate domestic assets. This effective ban on domestic liquidations by foreign banks (e.g. a ban on outflows) is too strong in practice, and arises because there is no domestic benefit to foreign investment in the baseline model. Less strong versions of this result arise under the same logic if foreign banks generate some benefits to the domestic economy: because country planners are not concerned with the welfare of foreign banks, they continue to under-regulate domestic banks (neglecting spillovers to foreign banks) and impose unequal treatment (neglecting benefits to foreign banks).[26]

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[26]See Examples 1 and 2 of Section 6 as well as Appendix D.5 for details. Unlike in Proposition 3, unequal treatment can take the form of foreign banks also being under-regulated relative to the global optimum, but still being regulated differently from domestic banks.

Finally, the domestic planner does not tax *foreign* liquidations by domestic banks. This happens because the investment presence of domestic banks in any single foreign country is marginal, so that country planners do not internalize their fire sale impact in foreign countries.

Optimal Cooperation. Non-cooperative quantity regulation differs from globally efficient policy in two important ways. First, non-cooperative quantity regulation does not account for foreign spillovers, so that the globally efficient wedge $\tau_{ii}^L(s)$ is generally *higher* than the non-cooperative wedge $\tau_{i,ii}^L(s)$. Non-cooperative regulation features too little regulation of domestic banks due to the foreign spillovers from domestic fire sales. This is a *multilateral* problem, as the domestic fire sale potentially affects all foreign countries investing domestically. Second, non-cooperative quantity regulation results in unequal treatment of foreign banks for domestic activities – foreign banks are regulated more stringently than domestic banks. This regulatory gap $\tau_{i,ji}^L(s) - \tau_{i,ii}^L(s)$ reflects a *bilateral* problem: the marginal benefit to foreign banks of liquidating the domestic asset outweighs the marginal cost to the domestic economy. Nevertheless, foreign liquidations are banned because that positive surplus accrues to foreign banks, and not to the domestic economy.

4.3 Non-Cooperative Pigouvian Taxation

We now characterize the non-cooperative equilibrium under Pigouvian taxation, where wedge revenues from foreign banks are remitted domestically. Recall that the change in the remission rule is the only difference relative to quantity regulation. Our next two propositions provide our main result: that country planners using non-cooperative Pigouvian taxation achieve the globally efficient outcome, absent monopoly rents.

Proposition 4. *The non-cooperative equilibrium under Pigouvian taxation has the following features.*

 1. *The domestic liquidation wedges on domestic* and *foreign banks are*

$$\tau^L_{i,ii}(s) = \tau^L_{i,ji}(s) = -\underbrace{\Omega_{ii}(s)}_{\text{Domestic Spillovers}} - \underbrace{\int_{i'}\Omega_{i'i}(s)di'}_{\text{Foreign Spillovers}} \underbrace{\forall j}_{\text{Equal Treatment}} \qquad (14)$$

where $\Omega_{ij}(s)$ are as defined in Proposition 1.

2. *The wedges on domestic investment by foreign banks are*

$$\tau^I_{i,ji} = \underbrace{\frac{\partial^2 \Phi_{ji}}{\partial I^2_{ji}}I_{ji}}_{\text{Monopolist Motive}} \geq 0$$

3. *All other wedges are* 0.

In contrast to quantity regulation, *non-cooperative* planners using Pigouvian taxation implement the efficient wedges on asset liquidations. This difference arises from the motive to collect revenue from foreign banks. To build intuition, we begin with an informal and economic discussion of how the revenue motive leads to the efficient outcome. We will then show the formal steps for why it leads to fully efficient policy. For expositional purposes, in the arguments that follow we set $h_i(s) = 1$, that is banks are unable to use assets as collateral. The general case is contained in the proof.

Economically, there are three consequences for tax revenue collection of allowing foreign bank j to increase liquidations $L_{ji}(s)$ of the domestic asset. The first two, the "Direct Effect" (DE) and the "Monopoly Effect" (ME), capture the impact of the change $dL_{ji}(s)$ on revenue collected from bank j. The third, the "Price Effect" (PE), is the impact of the change $dL_{ji}(s)$ on revenue collected from all other foreign banks i' due to changes in the equilibrium liquidation price $\gamma_i(s)$.

Consider first the direct effect (DE). In the non-cooperative equilibrium, country planner i collects total revenue $\tau^L_{i,ji}(s)L_{ji}(s)$ from bank j for asset liquidations in state s. This means that by allowing an increase $dL_{ji}(s)$ in asset liquidations, there is a direct increase $\tau^L_{i,ji}(s) \cdot dL_{ji}(s)$ in revenue collected from bank j. In equilibrium, the tax rate $\tau^L_{i,ji}(s)$ charged to bank j for liquidations (i.e. the private marginal cost) must be equal to the private marginal benefit to bank j of liquidating the domestic asset. In other words, DE captures the foregone private marginal surplus to bank j as a

result of regulation that rules out an increase $dL_{ji}(s)$.[27]

The second effect, the monopoly effect (ME), is the second aspect of the change in revenue collected from bank j: in response to an increase $dL_{ji}(s)$, there is an impact $\frac{\partial \tau_{i,ji}^L(s)}{\partial L_{ji}(s)}$ on the tax rate that can be charged.[28] From Lemma 2, we see that $\frac{\partial \tau_{i,ji}^L(s)}{\partial L_{ji}(s)} = 0$, meaning that the partial equilibrium elasticity of liquidations with respect to the liquidation tax rate is infinite, that is a small increase in the tax rate leads bank j to stop liquidating the country i asset. In other words, this is a conventional case of perfect competition between countries in their respective markets for liquidations, as an attempt by one country to further increase their tax rate leads foreign banks to instead liquidate in other countries. The monopoly effect is therefore zero.

The third effect, the "Price Effect" (PE), reflects that a change in the equilibrium price $\gamma_i(s)$ as a result of a change in liquidations affects the tax rate that can be charged to a bank. This has a corresponding impact $\frac{\partial \gamma_i(s)}{\partial L_i^A(s)} \cdot \frac{\partial \tau_{i,i'i}^L(s)}{\partial \gamma_i(s)} \cdot L_{i'i}(s)$ on revenue collected from bank i' for liquidations. In contrast to ME, this term is not zero: an increase in the price $\gamma_i(s)$ increases the marginal value to bank j of liquidating that asset, and so increases the tax rate that can be charged. In other words, mitigating the domestic fire sale serves to increase the revenue that can be collected from foreign banks. Drawing from the tax formulas of Lemma 2, we can see that this effect is given by

$$
\frac{\partial \gamma_i(s)}{\partial L_i^A(s)} \cdot \frac{\partial \tau_{i,i'i}^L(s)}{\partial \gamma_i(s)} \cdot L_{i'i}(s) = \frac{\partial \gamma_i(s)}{\partial L_i^A(s)} \left[\underbrace{\frac{\lambda_{i'}^1(s)}{\lambda_{i'}^0} L_{i'i}(s)}_{\text{Distributive Externality}} + \underbrace{\frac{1}{\lambda_{i'}^0} \Lambda_{i'}^1(s) L_{i'i}(s)}_{\text{Collateral Externality}} \right] = \Omega_{i'i}(s).
$$

The first term captures the effect of the price change on revenue collected from asset liquidations and corresponds to the distributive externality of Proposition 1. The second term captures the ability of liquidations to relax the collateral constraint, and so reflects the entire collateral externality of Proposition 1 when $h_i(s) = 1$ and debt cannot be rolled over. Thus, the combination of these revenue impacts generates the spillover $\Omega_{i'i}(s)$.

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[27]The direct effect is thus analogous to the distinction identified in the trade literature in the introduction. Viewed in that context, we might think of $\tau_{i,ji}^L(s)$ as a tariff on the foreign bank, with the marginal surplus of an additional marginal unit of liquidations being collected by the country i planner as tariff revenue, rather than being retained by the foreign bank. This shifting of the marginal surplus from the foreign bank to the domestic government is behind the direct effect.

[28]There is also an analogous effect on $\tau_{i,ji}^I$, which we omit here for exposition but is detailed below.

We will now put these terms together into the formal argument. Given revenue collected $\Pi_i = \int_{i'} \left[\tau^I_{i,i'i} I_{i'i} + \tau^L_{i,i'i} L_{i'i} \right] di'$, the first order condition of planner i for bank j liquidations $L_{ji}(s)$ is given by

$$
0 = \underbrace{\Omega_{ii}(s)}_{\text{Domestic Spillovers}} + \underbrace{\tau^L_{i,ji}(s) + \overbrace{\frac{\partial \tau^I_{i,ji}}{\partial L_{ji}(s)} I_{ji} + \frac{\partial \tau^L_{i,ji}}{\partial L_{ji}(s)} L_{ji}}^{\text{ME}} + \overbrace{\frac{\partial \gamma_i(s)}{\partial L^A_i(s)} \int_{i'} \left[\frac{\partial \tau^I_{i,i'i}}{\partial \gamma_i(s)} I_{i'i} + \frac{\partial \tau^L_{i,i'i}}{\partial \gamma_i(s)} L_{i'i} \right] di'}^{\text{PE}}}_{\text{Impact on Revenue}}
$$

$$
= \underbrace{\Omega_{ii}(s)}_{\text{Domestic Spillovers}} + \underbrace{\overbrace{\tau^L_{i,ji}(s)}^{\text{DE}} + \overbrace{0}^{\text{ME}} + \overbrace{\int_{i'} \Omega_{i'i}(s) di'}^{\text{PE}}}_{\text{Impact on Revenue}}
$$

resulting in efficiency. Notice that in this case with $h_i(s) = 1$, we had $\frac{\partial \tau^I_{i,ji}}{\partial L_{ji}(s)} = 0$. This is because investment has no value as collateral when $h_i(s) = 1$ and so debt cannot be rolled over.

The monopoly effect (ME) in the market for liquidations was zero, reflecting perfect competition between country planners in this market. When considering the analogous effect in the market for initial investment, we have the term $\frac{\partial \tau^I_{i,ji}}{\partial I_{ji}} = -\frac{\partial^2 \Phi_{ji}}{\partial I^2_{ji}}$.[29] Thus to ensure absence of monopoly rents in the market for investment, we need $\frac{\partial^2 \Phi_{ji}}{\partial I^2_{ji}} = 0$, that is there is also perfect competition in this market. Notably, non-cooperative regulators set wedges on liquidations according to the correct equation even with monopoly rents for initial investment. As such, even if there is an investment scale distortion, regulators using Pigouvian taxes achieve the correct Pigouvian tax on liquidations, after accounting for the distorted investment scale.

Efficiency of Non-Cooperative Pigouvian Taxation. Under Pigouvian taxation, the non-cooperative equilibrium differs from efficient policy only due to the monopolist motive that leads to taxes on foreign investment. If countries are substitutable with other countries from an investment perspective, monopoly power will be small. In the limit where monopoly power is zero, non-cooperative

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[29]Note that we have expressed this motive as a derivative of price (tax rate) in the quantity. This is equivalent to expressing it in a more familiar way of a derivative of quantity (demand) in price. In our model, it is simpler to solve for quantities, and then back out the implementing prices (taxes).

taxation implements the globally efficient outcome, eliminating the need for cooperation.

Proposition 5. *Suppose that for all i and $j \neq i$, $\frac{\partial^2 \Phi_{ij}}{\partial I_{ij}^2} = 0$. Then, the non-cooperative equilibrium under taxation is globally efficient. There is no scope for cooperation.*

Proposition 5 suggests an alternative to cooperative regulatory agreements exists in the model. If countries switch to Pigouvian taxation to manage fire sale spillovers, country planners can achieve the cooperative outcome in a non-cooperative manner. They do so even though each country maximizes domestic welfare only, even though domestic liquidation prices appear in foreign bank constraints, and even though domestic planners have market power over domestic liquidation prices.

The sufficient condition of Proposition 5 requires a notion of substitutability between countries. The condition $\frac{\partial^2 \Phi_{ij}}{\partial I_{ij}^2} = 0$ implies that the (partial equilibrium) elasticity of investment with respect to the tax rate is infinite. The infinite elasticity is a limiting case in which countries have no monopoly power over foreign banks, and so implement an efficient outcome.

Proposition 4 provides an exact efficiency result in a limiting case of an infinite elasticity. Even if countries have some monopoly power, Pigouvian taxation provides three potential advantages. First, it restricts the need for cooperation to cooperation over regulation of *foreign* activities of banks. Second, it transforms the source of inefficiency from a *multilateral* spillover problem into a *bilateral* monopolist problem, which may be able to be solved for example by tax treaties. Third, it changes the information required to determine the need for and terms of a cooperative agreement to a set of *partial equilibrium* elasticities of investment with respect to the tax rate (the cost of investment). Cooperation is required when the elasticity of investment in the tax rate (cost of investment) is low, and not required when it is high. By contrast, cooperation under quantity regulation requires evaluating a set of multilateral general equilibrium financial stability spillovers, and there may be substantial disagreements between countries as to their magnitudes and cross-country correlations.

4.4 Discussion

In practice, macroprudential regulatory requirements – such as minimum equity capital and liquidity requirements – commonly take the form of quantity restrictions. However, use of Pigouvian taxes, such as a tax on debt, has been discussed as an alternative (e.g. Cochrane 2014, De Nicoló et al. 2014, Kocherlakota 2010, and Tucker 2016). On the other hand, emerging markets in practice use both quantity- and priced-based capital control measures to manage capital flows (IMF 2012). Our model implies that, provided that monopoly power is low, *price-based* regulation and capital control measures are efficient, whereas *quantity-based* measures are not. We thus provide an efficiency based argument in favor of a Pigouvian tax approach to macroprudential policies, which we further develop in Section 5.

In addition to this normative implication for the design of bank regulation, our model also helps to understand the broad architecture of existing cooperative agreements. The model suggests that non-cooperative quantity regulation of domestic banks is overly lax while there is also unequal treatment of foreign banks. Both the Basel III accords and the European Banking Union aim to enhance bank regulatory standards to address cross-border stability risks, for example by strengthening bank capital and liquidity requirements. Moreover, equal treatment is recognized as an important aspect of cooperation.[30]

Importantly, cooperation is often perceived to be difficult when countries are sufficiently asymmetric.[31] Asymmetric agreements may require explicit international transfers, which may be politically difficult to implement. Pigouvian taxation implements the required transfers in a decentralized manner through the revenues collected, and may help facilitate efficient outcomes when countries are relatively asymmetric (e.g. developed economies and emerging markets).

One interesting property of our model is that non-cooperative regulators using quantity restrictions ban capital outflows (see also Caballero and Simsek 2020), but allow outflows (subject to a

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[30]For example, Basel III "rais[es] the resilience of the banking sector by strengthening the regulatory capital framework" (BIS 2010), and the ECB lists one of the goals of the SSM as "ensuring a level playing field and equal treatment of all supervised institutions" (ECB 2018).

[31]See for example Bolton and Oehmke (2019) and Dell'Ariccia and Marquez (2006).

tax) when using Pigouvian taxation. This suggests that after moving to a Pigouvian regime, we might expect an increase in observed capital retrenchment. However, this needs to be caveated. If there are domestic benefits from foreign banking, optimal quantity restrictions may result in unequal treatment but not a complete ban on outflows (see Appendix D.5). Non-cooperative regulators may in fact under-regulate foreign banks relative to the global optimum, accounting for the net positive benefit to the domestic economy but not for foreign spillovers. We might thus conjecture that following a switch to a Pigouvian regime, countries with small (large) benefits from foreign banking would see more (less) capital retrenchment.

4.4.1 Practical Concerns with Taxation

Financial regulation in practice commonly takes the form of quantity restrictions, rather than Pigouvian taxation. This leads to the natural concern that our model fails to account for the reasons why the current regulatory framework favors quantity restrictions over taxes. One possibility is that Pigouvian taxation may simply be perceived as roughly equivalent to quantity regulation. Even in academic debates, duality is a common assumption, particularly since quantity regulation can include tax-like features such as risk weights and capital surcharges.[32] Moreover, even though governments and regulators likely recognize the ability of Pigouvian taxes to raise revenues, it may not be appreciated that the revenue collection incentive can actually promote efficient regulation of cross-border banks, which is our main contribution.

Nevertheless, in practice quantity regulations and revenue-neutral *linear* Pigouvian taxes may not be dual when regulators face uncertainty (Weitzman 1974). For example, this violation of duality might arise in our model if regulators face uncertainty about bank productivity.[33] What follows is an illustrative verbal example. Suppose that bank productivity is known, and suppose that the global optimum allows greater fire sales when bank productivity is higher to allow banks to capitalize on higher productivity. This means the optimal tax on liquidations should increase in bank productivity since externalities are greater. Now, suppose that bank productivity is not known

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[32]See e.g. Greenwood et al. (2017) for a discussion of tax-like features of quantity regulation.

[33]See Perotti and Suarez (2011) for formal analysis along these lines, and Tucker (2016) for further discussion.

to the regulator. If the global planner imposes a quantity ceiling on liquidations, low-productivity banks fall below the ceiling and face an unconstrained choice, whereas high-productivity banks are pooled together at the binding ceiling. As a result, low-productivity banks that fall below the ceiling are underregulated in equilibrium, while particularly high-productivity banks that face a binding cap are overregulated. By contrast, a linear tax applies the same tax rate regardless of productivity, meaning that low-productivity banks are overregulated while high-productivity banks are underregulated. Uncertainty gives rise to a violation of duality in this setting which is different from the revenue motives we study, and in particular this violation of duality applies also to the global planner. Notably, the key issue here is that the optimal tax with certainty is nonlinear in productivity. The optimal regime in this setting may thus feature nonlinear taxation, rather than linear taxes or quantity restrictions (Roberts and Spence 1976, Spence 1977).

There are several additional economic and political concerns that may arise from use of taxation. One prominent practical concern is that a race to the bottom may undermine a regulatory use of taxes in practice. Race to the bottom is a common concern both in bank regulation (Dell'Ariccia and Marquez 2006) and in debates on corporate taxation.[34] Interestingly, competition among country planners in our model results in efficiency, rather than a race to the bottom. A second practical concern may be that setting the correct taxes could be difficult in practice. In our model, however, the same information – the social cost $\tau_i^L(s)$ – is required to set either the optimal tax or the optimal quantity restriction. This observation is further developed in Section 5, where optimal macroprudential quantity restrictions (e.g. leverage and liquidity requirements) are in fact characterized in terms of the optimal Pigouvian taxes. A third practical concern is that there may be important political impediments to our proposal. Pigouvian taxation may be politically more difficult to implement than quantity restrictions due to perceived unpopularity of taxes, particularly if the burden of the tax is perceived to be borne by consumers.[35] Moreover, application of taxes may lead to political lobbying for tax revenues (e.g. Bhagwati and Srinivasan 1980, Cassing and

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[34]For example, US Treasury Secretary Janet Yellen cited concerns about race-to-the-bottom in advocating for a global minimum corporate tax. "Yellen: 'Global race to the bottom' in corporate tax," BBC News, 23 March 2021.

[35]While outside the banking context, see e.g. Mankiw (2009) and Masur and Posner (2015). Baker III et al. (2017) argues that revenue neutrality is important to ensure political support for a carbon tax.

[Hillman 1985](), which might also undermine efficiency.

This subsection has taken a first step towards establishing and discussing several potential concerns with our proposal. These concerns, potentially among others, may represent important practical limitations that will have to be evaluated against the merits of the Pigouvian tax approach to bank regulation identified in our paper.

4.4.2 The Role of Commitment

In our model, banks commit to liquidation rules L_i and planners commit to taxes on liquidations at the same time as investment I_i is being undertaken. In Appendix D.4, we show that a time consistency problem can arise absent commitment because planners at date 1 neglect the impact of date 1 policies on the date 0 value of investment, which is partly driven by its value as collateral.[36] Because part of the revenue impact of liquidation taxes derives from the collateral value of investment, this can undermine Pigouvian efficiency.[37] The time consistency problem disappears when we consider macroprudential capital and liquidity regulation in Section 5, where policies are set at date 0. Section 5 also considers resolution and bailout policies, and we provide further discussion of commitment in those settings. In fact, in Section 5.4, we provide an alternative interpretation of the baseline model in which banks effectively commit to ex post liquidations policies through ex ante (date 0) organizational choices – for example, by using explicit debt guarantees, employing single- versus multi-point-of-entry resolution, or expansion using branches versus subsidiaries. In this interpretation, efficiency is achieved through an entry fee charged to banks based on date 0 organizational form, rather than through ex post liquidation taxes.

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[36]Notably, this time consistency arises because the collateral value is based on the date 1 price $\gamma_j(s)$, and so would not arise if it were based on the date 2 price $1 + r_{ij}$.

[37]The time consistency problem that arises is similar to the problem studied in [Farhi and Tirole (2018)](). In their paper, regulation is set after debt has been sold to foreign investors, leading the regulator to fail to internalize the effect of regulation on the price of debt. Here, if planners set taxes on liquidations after investment has been determined, then planners neglect the impact of liquidation taxes on revenue collected from taxes on investment.

5   Applications to Macroprudential Regulation

In this section, we apply our theory to more conventional regulatory tools – bank equity capital and liquidity requirements – and bank resolution policies – cross-border support and bailouts. These policies are important centerpieces of post-crisis regulation and cooperative arrangements.[38] Our applications shed light on the practical implications of our results for the design of regulation. Under non-cooperative quantity regulation, country planners engage in various forms of ring-fencing of foreign banks: excessive leverage restrictions, excessive liquidity requirements, hoarding of loss-absorbing capital, and under-provision of fiscal backstops. Moreover, country planners under-regulate domestic banks along each of these dimensions. This motivates a cooperative regime that provides for equal treatment of foreign banks and increases regulation of domestic banks. By contrast, non-cooperative planners using Pigouvian taxation implement the efficient outcome, absent monopoly rents. Efficiency is achieved through a combination of: (i) a tax on subsidiary debt issuance and a *subsidy* on illiquid assets, in place of a leverage requirement; (ii) a subsidy on liquid assets, in place of a liquidity requirement; (iii) a tax on loss-absorbing capital by which the subsidiary recapitalizes the parent, replacing an orderly resolution requirement; and, (iv) a fee charged to banks for the bailouts they expect to receive.

5.1   A Model with Macroprudential Regulation

We now formulate a variant of the baseline model under which ex ante restrictions on banks, rather than ex post liquidation taxes, serve as the method of controlling bank behavior. Doing so requires establishing how decisions at the country level impact liquidations in that country. To this end, we interpret bank i as a bank holding company, located in country i, which owns and operates subsidiaries in different countries, with ij denoting its subsidiary in country j. In this section we assume subsidiaries do not support each other in distress, an assumption we relax in Section 5.4.

At date 0, bank i allocates its total initial funds A_i across its subsidiaries, with E_{ij} denoting the

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[38]Common standards for equity capital and liquidity requirements are core elements of both Basel III and the European Banking Union, and common resolution and fiscal backstops are pillars of the European Banking Union.

"equity" allocation to the subsidiary in country j and E_i denoting the portfolio of equity allocations. In exchange, bank i receives the entire equity claim of the subsidiary. Equity allocation is subject to the budget constraint $E_{ii} + \int_j E_{ij} dj = A_i$. In addition, subsidiary ij can issue debt D_{ij} to finance investment, so that the budget constraint of subsidiary ij is $\Phi_{ij}(I_{ij}) \leq E_{ij} + D_{ij}$.[39] The shock and return structure are the same as in the baseline model, but now subsidiary ij is responsible for its debt rollover and faces a collateral constraint $D_{ij} \leq \gamma_j(s)L_{ij}(s) + (1 - h_j(s))\gamma_j(s)[R_j(s)I_{ij} - L_{ij}(s)]$. Rearranging the collateral constraint, liquidations of subsidiary ij are given by

$$L_{ij}(s) = \frac{1}{h_j(s)\gamma_j(s)} \max\left\{D_{ij} - d_j^*(s)I_{ij}, 0\right\}, \tag{15}$$

where $d_j^*(s) \equiv (1 - h_j(s))\gamma_j(s)R_j(s)$ reflects the collateralizability of investment. Liquidations are increasing in leverage $d_{ij} \equiv \frac{D_{ij}}{I_{ij}}$, increasing in absolute debt level D_{ij}, and decreasing in scale I_{ij}. Define the region of distress of subsidiary ij as the set of states s in which it is forced to liquidate assets, that is

$$S_{ij}^D = \{s \in S | D_{ij} > d_j^*(s)I_{ij}\}. \tag{16}$$

We further define $d_j^* = \inf_{s \in S} d_j^*(s)$ as the highest leverage subsidiary ij can undertake without ever being forced to liquidate assets, that is so that $S_{ij}^D = \emptyset$.

The final equity payoff of subsidiary ij at date 2 is given by $c_{ij}(s) = \gamma_j(s)L_{ij}(s) + (1 + r_{ij})[R_j(s)I_{ij} - L_{ij}(s)] - D_{ij}$, so that the total equity payoff of bank i is $c_i(s) = c_{ii}(s) + \int_j c_{ij}(s)dj$. The problem of bank i is therefore to choose (E_i, D_i, I_i) in order to maximize expected equity payoff, $E[c_i(s)]$, subject to the budget constraint of the holding company, the budget constraints of the subsidiaries, and the liquidation rule of equation (15).

Commitment. In the baseline model, we solved the problem with commitment because liquidations occurred at date 1 and were regulated. Up through and including the liquidity model of Section 5.3, regulatory decisions considered in this section affect bank choices at date 0, meaning the

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[39]Notice that combining these equations by substituting out equity E_{ij} gives the consolidated budget constraint (1) in the baseline model.

assumption of commitment is now immaterial. We will provide further discussion of commitment in Section 5.4, when we introduce cross-border support.

Arbitrageurs and Aggregate Liquidations.   Arbitrageurs are defined analogously to the baseline model, but aggregate liquidations in state s by all subsidiaries in country i are now endogenous to the price through the collateral constraint. In particular, defining $D_i^A(s) = D_{ii}\mathbf{1}_{s \in S_{ii}^D} + \int_{i'|s \in S_{i'i}^D} D_{i'i}di'$ to be the aggregate debt of distressed subsidiaries in country i in state s (and similarly for $I_i^A(s)$), then aggregate liquidations in country i in state s are given by $L_i^A(s) = \frac{1}{h_i(s)\gamma_i(s)}\left[D_i^A(s) - d_j^*(s)I_i^A(s)\right]$.

5.2   Optimal Regulation

We now turn to characterizing optimal policy. Planners still have a complete set of instruments for the date 0 choices of banks, which now correspond to wedges on subsidiary debt, τ_{ij}^D, and on subsidiary investment, τ_{ij}^I. As before, we denote revenue-neutral wedges to be quantity regulation, and revenue-generating wedges to be Pigouvian taxation. We also provide mappings of optimal policy under revenue-neutral wedges into explicit quantity restrictions.

We begin by characterizing globally optimal regulation.

Proposition 6. *The globally efficient allocation can be decentralized using wedges*

$$\tau_{ji}^D = \underbrace{\overbrace{\Pr(s \in S_{ji}^D)}^{\text{Probability of Distress}} \times \overbrace{\mathbb{E}\left[\tau_i^L(s)\frac{1}{h_i(s)\gamma_i(s)}\middle| s \in S_{ji}^D\right]}^{\text{Social Cost of Debt in Distress}}}_{\geq 0}$$

$$\tau_{ji}^I = \Pr(s \in S_{ji}^D)\cdot\underbrace{\mathbb{E}\left[\tau_i^L(s)\frac{-d_i^*(s)}{h_i(s)\gamma_i(s)}\middle| s \in S_{ji}^D\right]}_{\leq 0}$$

where the total social cost $\tau_i^L(s) \geq 0$ of liquidations in country i in state s is

$$\tau_i^L(s) = \underbrace{\left| \frac{d\gamma_i(s)}{dL_i^A(s)} \right|}_{\text{Total Price Impact}} \cdot \underbrace{\frac{1}{h_i(s)\gamma_i(s)} \left[\frac{1}{\gamma_i(s)} D_i^A(s) - d_i^*(s)I_i^A(s) \right]}_{\text{Total Fire Sale Spillover}},$$

where the total price impact $\frac{d\gamma_i(s)}{dL_i^A(s)}$ is defined in the proof.

The intuition of Proposition 6 is similar to that of the baseline model. All else equal, an increase in the debt level D_{ji} of subsidiary ji increases liquidations in state s when it is in distress, which leads to a fire sale spillover to domestic and foreign banks. The tax on debt is thus given by the probability of distress times the expected social cost of debt (via greater liquidations) in distress. In contrast to debt, increases in project scale (holding fixed debt) reduce liquidations because they increase total collateral, resulting in a subsidy for scale. This subsidy arises because an increase in project scale, holding debt fixed, must be achieved by increasing equity E_{ji}.

The global optimum accounts for international spillovers: the social cost $\tau_i^L(s)$ includes spillovers to both domestic and foreign banks. It also features a form of equal treatment: the wedges placed on subsidiary ji depend on the identity j only through the region of distress S_{ji}^D, meaning two subsidiaries with the same region of distress face the same wedges.

Finally, there is a straightforward implementation of the global optimum using an explicit quantity restriction: the requirement $\tau_{ji}^D D_{ji} + \tau_{ji}^I I_{ji} \leq \tau_{ji}^D D_{ji}^* + \tau_{ji}^I I_{ji}^*$, where D_{ji}^*, I_{ji}^* are set to their globally optimal values.[40] Rearranging this requirement yields $d_{ji} \leq \left| \frac{\tau_{ji}^I}{\tau_{ji}^D} \right| + \left[d_{ji}^* - \left| \frac{\tau_{ji}^I}{\tau_{ji}^D} \right| \right] \cdot \frac{I_{ji}^*}{I_{ji}}$, which is a leverage requirement with a surcharge based on size (that is, the right-hand side decreases in I_{ji}). A size-based surcharge is required because liquidations increase not only in the leverage d_{ij}, but also in size I_{ij} (holding fixed leverage).[41]

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[40]The argument follows in the same manner as in Appendix D.3 for the baseline model, with the non-negative Lagrange multipliers for the bank regulatory constraints being $\kappa_{ij} = \lambda_i^0$.

[41]This is in keeping with surcharges for systemically important institutions in equity capital and TLAC requirements (e.g. 12 CFR Part 252 RIN 7100–AE37).

5.2.1 Country-level Regulation

We now consider the policies adopted by non-cooperative country planners. Because the formal characterizations largely mirror those of the baseline model, we provide a detailed characterization in Appendix B.1 and focus here on the key differences and policy implications.

Quantity Regulation. Under non-cooperative quantity regulation, optimal regulation of domestic banks follows the same formulas as in Proposition 6, but with the domestic distress costs Ω_{ii} in place of the total social cost τ_i^L. In other words, the domestic planner neglects spillovers to foreign banks, leading to under-regulation of domestic banks in the forms of a tax on debt and subsidy on investment that are both too low in magnitude relative to the global optimum. Regulation of domestic banks can equivalently be expressed by the quantity restriction $d_{ji} \leq \left| \frac{\Omega_{ji}^I}{\Omega_{ii}^D} \right| + \left[d_{ji}^* - \left| \frac{\Omega_{ji}^I}{\Omega_{ii}^D} \right| \right] \frac{I_{ji}^*}{I_{ji}}$, where D_{ji}^*, I_{ji}^* are evaluated at their non-cooperative optimal values. By contrast, regulation of foreign banks is overly restrictive and ensures that foreign subsidiaries always have sufficient collateral to never be in distress, that is $d_{ji} \leq d_i^*$. This reflects unequal treatment: $d_i^* < \left| \frac{\Omega_{ji}^I}{\Omega_{ii}^D} \right|$, that is the leverage requirement for foreign banks is tighter than for domestic banks, regardless of size. This combination motivates a cooperative regulatory agreement prescribing an increase in domestic bank regulation and equal regulatory treatment of foreign banks.[42]

Pigouvian Tax Efficiency. In absence of monopoly rents, non-cooperative Pigouvian taxation achieves the efficient outcome in this setting, a result formalized in the following proposition.

Proposition 7. *Suppose that for all i and $j \neq i$, $\frac{\partial^2 \Phi_{ij}}{\partial I_{ij}^2} = 0$. Then, the non-cooperative equilibrium under Pigouvian taxation implements the taxes and allocations of Proposition 6, and so is globally efficient. There is no scope for cooperation.*

Proposition 7 states that non-cooperative regulators can achieve efficiency through use of

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[42]Importantly, our model suggests that equal treatment means that the same social costs $\tau_i^L(s)$ are used to compute the leverage requirement for each bank based on its region of distress, but not that all banks are subject to the same leverage requirement.

Pigouvian taxes on debt and investment rather than quantity restrictions such as leverage require-ments.[43] Absent monopoly rents, these optimal taxes are given by their formulas in Proposition 6. This gives a foundation for thinking about the magnitude of the taxes that would need to be employed by country planners in practice, since the tax on debt (investment) is given by the product of the probability of distress and the expected marginal social cost of debt (investment) in distress. Moreover, it tells us that the social cost of debt in distress is related to the total value of the banking sector that will end up in distress, in terms of both aggregate debt of distressed banks, $D_i^A(s)$, and investment of distressed banks, $I_i^A(s)$. Thus, taxes and subsidies applied are particularly large in countries where the banking sector is large and distress is correlated across subsidiaries.

5.3 Liquidity Regulation

We now augment the model to study liquidity regulation. Suppose that in each country, subsidiary ij can also invest in a liquid project, denoted by T_{ij} ("treasury"). This project yields 1 unit of payoff per unit of scale with certainty and is fully liquid so that the unit of payoff can be obtained at either date 1 or date 2. At date 0, the cost of undertaking both projects is $\Phi_{ij}(I_{ij}, T_{ij})$. The "net debt" $ND_{ij} = D_{ij} - T_{ij}$ is the amount that needs to be repaid using collateral-backed debt rollover or asset liquidations at date 1, leading to a liquidation rule $L_{ij}(s) = \frac{1}{h_j(s)\gamma_j(s)} \max\left\{ND_{ij} - d_j^*(s)I_{ij}, 0\right\}$ and a region of distress $S_{ij}^D = \{s \in S | ND_{ij} > d_j^*(s)I_{ij}\}$. From here, the analysis proceeds as in the baseline model, with detailed formal characterizations left to Appendix B.2.

Globally Optimal Regulation. Globally optimal regulation of debt and illiquid project scale is given as in Proposition 6, except that net debt ND_{ij} now determines subsidiary distress and spillovers. The key new addition is liquidity regulation, which is given by $\tau_{ji}^T = -\tau_{ji}^D$ since liquid assets and debt have equal and opposite effects on net debt. Globally optimal regulation can be expressed as a liquidity requirement $D_{ji} \leq T_{ji} + \left|\frac{\tau_{ji}^I}{\tau_{ji}^D}\right| I_{ji} + \left[D_{ji}^* - T_{ji}^* - \left|\frac{\tau_{ji}^I}{\tau_{ji}^D}\right| I_{ji}^*\right]$. Concretely, consider this liquidity requirement in the context of the Liquidity Coverage Ratio (LCR), which requires

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[43]Intuitively, the subsidy on scale is optimal because greater stability increases the value of debt, and so increases revenue by increasing the tax that can be applied to subsidiary debt.

that banks have sufficient high quality liquid assets to cover creditor outflows over a 30 day stress period.[44] Calculation of the LCR requires a specification of the run-off rate, that is what fraction of bank creditors' claims will be withdrawn, as well as liquidity weights assigned to different assets used to cover withdrawals. Our results imply that the globally optimal LCR assigns a run-off rate of 100%, a liquidity weight of 1 to liquid assets, and a liquidity weight $\left| \frac{\tau^I_{ji}}{\tau^D_{ji}} \right|$ to illiquid assets. Moreover, the constant on the RHS acts as a size-based surcharge to the liquidity requirement.

Non-Cooperative Quantity Regulation. Non-cooperative regulators under quantity regulation again set liquid asset requirements on domestic banks to be too low, calibrating regulation accounting for domestic spillovers but neglecting foreign spillovers. They also set the requirement for foreign bank subsidiaries to be too high, imposing $D_{ji} \leq d^*_i I_{ji} + T_{ji}$, so that foreign subsidiaries must hold sufficient liquid assets to offset one for one debt D_{ji} that exceeds illiquid collateral $d^*_i I_{ji}$. In the context of LCR, our results imply that non-cooperative regulators impose an LCR on foreign bank subsidiaries that assigns a run-off rate of 100%, a liquidity weight of 1 to liquid assets, and a liquidity weight d^*_i to illiquid assets. This helps to understand cooperative agreements, such as Basel III, which provide common standards for LCR and NSFR. Moreover, it helps to understand concerns that uncoordinated liquidity requirements could lead to excessive liquidity ring fencing.[45]

Non-Cooperative Pigouvian Taxation. Non-cooperative regulators under Pigouvian taxation achieve the efficient outcome when monopoly rents are zero, employing taxes on debt, illiquid assets, and liquid assets. Concretely, our results suggest regulators can achieve efficiency by using subsidies of liquid asset holdings, rather than by imposing liquidity requirements such as LCR.

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[44]Similarly, the Net Stable Funding Ratio (NSFR) provides for liquidity coverage over a longer horizon.

[45]For example, a recent proposal by the US Federal Reserve Board to "impose standardized liquidity requirements on the U.S. branch and agency network of a foreign banking organization" (84 FR 59032) raised concerns that the proposal would lead other countries "to implement similar requirements" and could "lead to market fragmentation," meaning that "concerns regarding liquidity risk at branches and agencies should be further discussed and evaluated at the global level by international regulatory groups before any actions are taken at the national level" (84 FR 59230).

5.4 Cross-Border Support and Resolution

We now augment the model to study the possibility that subsidiaries in different countries may support one another during times of distress. In practice, this can happen in several ways: (i) bank i may use explicit guarantees of debts of its subsidiaries; (ii) bank i may structure itself for single-point-of-entry resolution (SPOE), under which the holding company, rather than operating subsidiaries, is resolved, and transfers between jurisdictions may be required to repay debts of subsidiaries;[46] and, (iii) bank i could expand using branches rather than subsidiaries, in which case the foreign bank would be liable for the debts of its branches.[47]

We model cross-border support as a "transfer" or "guarantee" of $G_{ij}(s)$ from bank i to its subsidiary ij in state s, where $G_{ij}(s) < 0$ indicates the subsidiary supporting the parent.[48] It is natural to consider state-contingent transfers in the context of cross-border support. For example, in a SPOE regime, a subsidiary with modest losses would support a subsidiary with large losses via resolution of the parent holding company. We assume commitment over transfers $G_{ij}(s)$. In practice, we think of committed transfers as arising from the organizational structure of the bank at date 0, for example: (i) guarantees of subsidiary debt; (ii) establishment of SPOE resolution; or, (iii) expansion via branches rather than subsidiaries.[49]

Transfers must be balanced budget, that is $G_{ii}(s) + \int_j G_{ij}(s)dj = 0$. The subsidiary liquidation rule is $L_{ij}(s) = \frac{1}{h_j(s)\gamma_j(s)} \max\left\{D_{ij} - G_{ij}(s) - d_i^*(s)I_{ij}, 0\right\}$ and the region of distress is $S_{ij}^D = \{s \in S | D_{ij} > d_j^*(s)I_{ij} + G_{ij}(s)\}$. Appendix B.3 contains detailed formal analysis for this section.

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[46]See Bolton and Oehmke (2019) for formal analysis of SPOE versus MPOE, and Tucker (2014) for further discussion. In this section, we focus on the transfers that occur between jurisdictions as part of the resolution process (i.e. the internal resolution process), rather than on the part of the resolution process involving write-downs of the external debt of the bank holding company.

[47]Although in practice branches are typically regulated by the home country, host country regulators in theory could also impose regulations, such as branch liquidity requirements (84 FR 59230 and 84 FR 59032). Nolle (2012) provides some background on organizational form for multinational banks.

[48]Results in this section generalize to cases where there are incomplete markets restrictions on feasible transfers.

[49]Notice that this model can be viewed as a reinterpretation of the baseline model. In the baseline model, bank i has debt at the holding company that could be rolled over using collateral or liquidations in any country. Here, liabilities are at the country level, but bank i can freely reallocate resources from collateral or liquidations between countries. In this sense, an alternative interpretation of commitment over taxes on ex post liquidations is that taxes are instead levied on the ex ante choices (SPOE vs MPOE, explicit guarantees, etc.) that lead to those ex post liquidations. Naturally, this exact analogy requires that ex ante decisions can be used to accomplish the same outcomes of ex post decisions, that is the presence of sufficiently rich instruments.

Global Optimum. The key new component of globally optimal regulation is regulation of transfers between subsidiaries, given by $\tau_{ji}^G(s) = -\tau_i^L(s) \frac{1}{h_i(s)\gamma_i(s)} \mathbf{1}_{s \in S_{ji}^D}$. Relative to the private optimum, the global optimum encourages banks to transfer money out of (possibly distressed) subsidiaries in countries with low spillovers, that is low $\tau_i^L(s)$, into distressed subsidiaries in countries with high spillovers. It can equivalently be viewed as a leverage requirement that adjusts for loss-absorbing capital, $d_{ji} - \mathbb{E}\left[\left|\frac{\tau_{ji}^G(s)}{\tau_{ji}^D}\right| \cdot g_{ji}(s)\right] \leq \left|\frac{\tau_{ji}^{iI}}{\tau_{ji}^D}\right| + \frac{T_{ji}^*}{I_{ji}}$.[50] Guarantees from the parent to the subsidiary in the region of distress provide support and so relax the leverage requirement, whereas funds from the subsidiary to the parent tighten the requirement. Broadly speaking, the global optimum is consistent with a single-point-of-entry resolution regime, under which losses and loss-absorbing capital are shared across subsidiaries at the international level.[51]

Non-Cooperative Quantity Regulation. Non-cooperative regulators using quantity regulation require that foreign bank subsidiaries satisfy $-G_{ji}(s) \leq d_i^*(s)I_{ji} - D_{ji}$ for all s, that is country planner i ensures foreign subsidiaries always have sufficient loss absorbing capacity to never have to liquidate the domestic asset. Economically, this requirement more closely resembles a multi-point-of-entry (MPOE) resolution regime, under which loss absorbing capital and resolution are conducted at the subsidiary, rather than at the holding company.[52] This motivates cooperative resolution, which allows for allocating losses to subsidiaries in countries with low spillovers.

Non-Cooperative Pigouvian Taxation. Absent monopoly rents, non-cooperative Pigouvian taxation implements the global optimum, with the addition of taxes $\tau_{ji}^G(s)$ on transfers. In practice, this Pigouvian tax could be implemented by charging an entry fee to a bank based on the organizational structure and the implied path of transfers. In particular, if a foreign bank establishes a structure that results in a future set of transfers $G_{ji}(s)$, our results say that the total fee that should be charged

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[50]Where $T_{ji}^* = \tau_{ji}^D D_{ji}^* + \tau_{ji}^I I_{ji}^* + \mathbb{E}\left[\tau_{ji}^G(s)G_{ji}^*(s)\right]$.

[51]For example, SPOE "may be more suitable to a firm that operates in a highly integrated manner (through, for example, centralized liquidity, trading, hedging and risk management)" (Financial Stability Board 2013b).

[52]This is also consistent with excessive liquidity ring fencing at the branch level. For example, commentators on the standardized liquidity requirement for foreign branches noted that it "could limit the ability of foreign banking organizations to deploy funds as needed, including during times of stress" (84 FR 59230).

is $\Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\tau_i^L(s)\frac{-G_{ji}(s)}{h_i(s)\gamma_i(s)}\,\middle|\,s \in S_{ji}^D\right]$. An isolated subsidiary, as in the baseline model, would not

generate transfers to or from its banking group, that is $G_{ji}(s) = 0$, and so would not be charged an

entry fee. By contrast, a bank organizing under SPOE or expanding via a branch would expect to

support or be supported by its parent in future resolution, with an entry fee charged as above.

5.5 Bailouts and Fiscal Backstops

Fiscal backstop measures (or "bailouts") – such as deposit insurance, lender of last resort (LOLR),

asset purchases, and debt guarantees – may be complementary to an effective regulatory regime in

safeguarding financial stability, and are an additional focus of cooperative regimes.[53] We model

bailouts as commitments to ex-post lump sum transfers $T_{ij}^1(s)$ by the government to subsidiary ij,

which are paid for by raising funds from taxpayers. For example, committed transfers can arise

from deposit insurance. Bailouts reduce the debt burden of subsidiary ij to $D_{ij} - T_{ij}^1(s)$, relaxing

the collateral constraint in a state-contingent manner. Both country planner i and country planner j

can provide backstops to subsidiary ij. Appendix B.4 provides detailed formal analysis.

In this environment, non-cooperative planners using quantity regulation also under-provide

fiscal backstops to both domestic and foreign banks, not internalizing the positive spillover effects

from greater financial stability to foreign banks. Moreover, a Pigouvian tax approach to regulation

is not on its own enough to achieve efficiency. The intuition is that bailouts are not priced, and

so the Pigouvian tax does not appropriately capture the "willingness to pay" of foreign banks for

bailouts. However, if planners also charge a Pigouvian tax (or fee) to banks for the bailouts they

expect to receive, then efficiency is restored. For example, this might be achieved through a deposit

insurance premium or a fee for ability to access the domestic LOLR.

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[53]For example, see Bianchi (2016), Clayton and Schaab (2021), Jeanne and Korinek (2020), and Keister (2016) for formal work and Geithner (2016) for a policy perspective on complementarities between bailouts and regulation. See Acharya et al. (2021) for discussion and analysis of the ECB as a common (EU wide) LOLR. See European Commision (2015) for a proposal for Common Deposit Insurance for the EU.

6 A General Framework

In this section, we both study the extent to which the insights of the baseline model generalize to broader banking environments and other banking externalities, and discuss limitations in its applicability. We show that, in addition to whether or not there is a monopoly problem, the applicability of the result depends on the form of the externality, which we further explore and discuss in the context of five examples.[54] "Local" externalities that only affect domestic agents, such a spillovers to the real economy (Example 1) or spillovers to surplus of local arbitrageurs (Example 2), can be addressed non-cooperatively under Pigouvian taxation, but not under quantity regulation when foreign banks contribute to them. By contrast, "global" externalities, which also affect foreign agents, may not be well-handled by Pigouvian taxation, unless they spread endogenously through the cross-border activities of banks (as with fire sales in the baseline model). In this context, we show that efficiency extends when foreign banks can also purchase domestic fire sold assets (Example 4). However, we illustrate how efficiency breaks down with global environmental externalities (Example 3) and, drawing on this example, show how efficiency also breaks down if there are unregulated international "shadow banks" (Example 5).

6.1 Model

Each country $i \in [0,1]$ has a representative multinational agent (or "bank"). The representative multinational agent has a vector $a_{ij} = \{a_{ij}(m)\}_{m \in M}$ of continuous and real-valued actions available in country j, where M is an indexing set and where $a_{ij}(m) \geq 0$.[55] The action $a_{ij}(m) = 0$ indicates not conducting activity m in country j. Multinational agents are home biased, so that domestic actions are a mass while foreign actions are a density.

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[54] Appendix E contains additional extensions to this section under which qualitatively similar results hold, including allowing for global traded goods (Appendix E.1), local constraints on foreign bank activities (Appendix E.2), heterogeneous within-country agents (Appendix E.3), non-linear country aggregates (Appendix E.4), general non-regulatory government actions (Appendix E.5), and preference misalignment between country planners and multinational agents (Appendix E.6).

[55] An example of an indexing set is $M = \{0\} \cup \{\{1\} \times S\}$, which denotes an action $a_{ij}(0)$ at date 0 and an action $a_{ij}(1,s)$ at date 1 in state s. We can impose that there are only actions $M' \subset M$ in country j by making actions $m \notin M'$ valueless.

We use country-level aggregates to capture spillover effects in the model. In particular, define $a_i^A(m) = a_{ii}(m) + \int_j a_{ji}(m)dj$ to be the aggregate action m in country i, with $a_i^A = \{a_i^A(m)\}$ denoting the vector of aggregates in country i. In the baseline model, the relevant aggregate for spillovers was aggregate liquidations $L_i^A(s)$ in each state s, which affected multinational banks by determining the liquidation price.

Country i multinational agents have a utility function $U_i\left(u_i(a_i), u_i^A(a_i, a^A)\right)$, where $u_i(a_i) = u_{ii}(a_{ii}) + \int_j u_{ij}(a_{ij})dj$ and $u_i^A(a_i, a^A) = u_{ii}^A(a_{ii}, a_i^A) + \int_j u_{ij}^A(a_{ij}, a_j^A)dj$.[56] This preference structure provides a flexible way to add up the utility impact of activities in different countries – for example, a consumption good in each country – while ensuring sufficient continuity so that a change in foreign activities generates a utility impact proportional to the measure of those activities. Multinational agents face constraint sets $\Gamma_i(W_i, \phi_i(a_i), \phi_i^A(a_i, a^A)) \geq 0$, where W_i is the wealth of the multinational agents (accounting for taxes), and where $\phi_i(a_i) = \phi_{ii}(a_{ii}) + \int_j \phi_{ij}(a_{ij})dj$ and $\phi_i^A(a_i, a^A) = \phi_{ii}^A(a_{ii}, a_i^A) + \int_j \phi_{ij}^A(a_{ij}, a_j^A)dj$. Taken together, the optimization problem of country i multinational agents is

$$\max_{a_i} U_i\left(u_i(a_i), u_i^A(a_i, a^A))\right) \quad \text{s.t.} \quad \Gamma_i\left(W_i, \phi_i(a_i), \phi_i^A(a_i, a^A)\right) \geq 0 \tag{17}$$

where all multinational agents take the vector a^A of aggregates as given.

6.2 Globally Efficient and Non-Cooperative Policies

We first characterize the globally efficient allocation. The global planner uses a complete set of wedges $\tau_{ji}(m)$ on each action m of each multinational agent j in each country i, which are taken out of the wealth level W_i of that multinational agent and are remitted lump sum to the agent.[57]

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[56]Note that u_i, u_i^A can be functions in a generalized sense – for example, a vector of real numbers or a vector of functions defined over an underlying state space.

[57]Any required lump sum transfers between countries are also done out of this wealth.

Proposition 8. *The globally efficient allocation can be decentralized by wedges*

$$\tau_{ji}(m) = - \underbrace{\Omega_{ii}(m)}_{\text{Domestic Spillovers}} - \underbrace{\int_{i'} \Omega_{i'i}(m) di'}_{\text{Foreign Spillovers}} \qquad \underbrace{\forall j}_{\text{Equal Treatment}}$$

where $\Omega_{i'i}(m) = \underbrace{\frac{\omega_{i'}}{\lambda_{i'}^0} \frac{\partial U_{i'}}{\partial u_{i'}^A} \frac{\partial u_{i'i}^A}{\partial a_i^A(m)}}_{\text{Utility Spillovers}} + \underbrace{\frac{1}{\lambda_{i'}^0} \Lambda_{i'} \frac{\partial \Gamma_{i'}}{\partial \phi_{i'}^A} \frac{\partial \phi_{i'i}^A}{\partial a_i^A(m)}}_{\text{Constraint Set Spillovers}}$ *is the spillover effect on country i' multina-*

tional agents from an increase in $a_i^A(m)$, where Λ_i' is the Lagrange multiplier on the constraint set of

country i' agents, and where $\lambda_{i'}^0 \equiv \Lambda_{i'} \frac{\partial \Gamma_{i'}}{\partial W_{i'}}$ is the marginal value of wealth to country i' multinational

agents.

Globally optimal policy in the general model features the same two core features as the baseline model. First, globally optimal policy enforces equal treatment of foreign agents, so that they are able to enjoy equally the benefits of cross-border activities. Second, globally optimal policy accounts for both domestic and foreign spillovers. There are two forms of spillovers in the general model that are reflected in $\Omega_{i'i}(m)$. The first set of spillovers is direct utility spillovers, a leading example of which is spillovers from banking activities into the real economy. The second of spillovers is constraint set spillovers, a leading example of which is fire sale externalities.

Non-Cooperative Quantity Regulation. The inefficiencies of non-cooperative quantity regulation are qualitatively similar to those identified in Propositions 3 and 11, and we leave formal characterization to Appendix C. Non-cooperative quantity regulation neglects international spillovers and results in unequal treatment, with foreign agent activities allowed only to the extent they benefit the domestic economy. Moreover, we show that non-cooperative quantity regulation is generically inefficient in settings with externalities arising from cross-border activities.

Non-Cooperative Pigouvian Taxation. The efficiency of non-cooperative Pigouvian taxation applies in the general model under two conditions. The first is a similar notion of no monopoly

rents, which carries the same intuition and is formalized in the Appendix.[58] Second, it requires the following assumption on how foreign aggregates can affect a domestic agent.

Assumption 9. For all i and $j \neq i$, u_{ij}^A and ϕ_{ij}^A are homogeneous of degree 1 in a_{ij}, holding a_j^A fixed. That is, $u_{ij}^A(\beta a_{ij}, a_j^A) = \beta u_{ij}^A(a_{ij}, a_j^A)$ and $\phi_{ij}^A(\beta a_{ij}, a_j^A) = \beta \phi_{ij}^A(a_{ij}, a_j^A)$.

Assumption 9 states that domestic agents' exposure to aggregates in a foreign country scales with their activities in that foreign country.[59] For example, in the case where action m has a local price $\gamma_j(a_j^A)$ attached to it, we obtain a linear form $\gamma_j(a_j^A) a_{ij}(m)$, which satisfies Assumption 9, as with fire sales. Notice, therefore, that homogeneity of degree 1 does *not* restrict the form of the pricing function γ_j, which may be nonlinear (as in the baseline model). Moreover, although Assumption 9 restricts the form of cross-border externalities, it places no restrictions on the form of *domestic* externalities affecting domestic agents. It also allows for multiple externalities, for example fire sales of multiple assets combined with spillovers to the real economy.

To understand the role of Assumption 9, we decompose the gap between the wedge that is set by the global planner and the Pigouvian tax set by country planner i. For expositional simplicity, we illustrate the decomposition for utility spillovers alone,[60] which is given by

$$\tau_{ji}(m) - \tau_{i,ji}(m) = \int_{i'} \frac{1}{\lambda_{i'}^0} \omega_{i'} \frac{\partial U_{i'}}{\partial u_{i'}^A} \left(\underbrace{\frac{\partial}{\partial a_i^A(m)} \left[\frac{\partial u_{i'i}^A}{\partial a_{i'i}} a_{i'i} \right]}_{\text{Tax Revenue Derivative}} - \underbrace{\frac{\partial u_{i'i}^A}{\partial a_i^A(m)}}_{\text{Externality}} \right) di' \tag{18}$$

when monopoly rents are zero. This gap is determined by the gap between the true externality effect on foreign agents, $\frac{\partial u_{i'i}^A}{\partial a_i^A(m)}$, and the change in tax revenue collected from foreign agents, $\frac{\partial}{\partial a_i^A(m)} \left(\frac{\partial u_{i'i}^A}{\partial a_{i'i}} a_{i'i} \right)$, that arises because changes in domestic aggregates affect the willingness-to-pay

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[58]Note that the requirement of no monopoly rents implies that our theory does not provide a solution to terms of trade manipulation, given that the monopolist distortion is similar to terms of trade manipulation. More subtly, it implies that Pigouvian taxation may have trouble addressing problems of domestic market power of foreign multinational agents. This is because when a multinational agent earns monopoly rents in a country, the country planner may in turn gain some monopoly power over it.

[59]Assumption 9 requires linear scaling due to the fact that the Pigouvian tax is linear.

[60]Including constraint set spillovers simply adds a second and analogous term to the decomposition.

of foreign agents for domestic activities. Homogeneity of degree 1 (Assumption 9) implies that the tax revenue derivative and externality effect are precisely equal, $\frac{\partial u_{i'i}^A}{\partial a_{i'i}} a_{i'i} = u_{i'i}^A$, leading to efficiency.

Assumption 9 gives rise to a natural classification of types of externalities into local and global externalities, which we now discuss.

6.3 Pigouvian Efficiency under Local Externalities

We first examine the efficiency of Pigouvian taxation under a class of *local* externalities. Local externalities arise when the domestic aggregates a_i^A only affect domestic multinational agents.[61] For example, local externalities might include spillovers from the financial sector to the real economy of that country or costs to the domestic deposit insurance entity. Under quantity regulation, local externalities result in unequal treatment of foreign banks. By contrast, under Pigouvian taxation, because a_i^A does not affect foreign agents, under the decomposition of equation (18) we have $\frac{\partial u_{i'i}^A}{\partial a_i^A(m)} = \frac{\partial}{\partial a_i^A(m)} \left(\frac{\partial u_{i'i}^A}{\partial a_{i'i}} a_{i'i} \right) = 0$. A Pigouvian tax approach results in the efficient outcome, provided monopoly rents are zero, for the entire class of local externalities problems, even though foreign agents can contribute to these externalities through their activities. Cooperation is not required. We can state this result formally as follows.

Corollary 10. *Suppose that there are only local externalities, that is $u_{ij}^A = \phi_{ij}^A = 0$ for all i and $j \neq i$. Then absent monopoly rents, the non-cooperative equilibrium under Pigouvian taxation is globally efficient.*

Corollary 10 is a corollary of Proposition 27 in Appendix C, which shows Pigouvian efficiency when Assumption 9 holds and there are no monopoly rents. Corollary 10 follows because $u_{ij}^A = \phi_{ij}^A = 0$ for $j \neq i$ trivially satisfies Assumption 9. We now provide two examples that incorporate local externalities into the baseline model and show that Pigouvian tax efficiency continues to hold.

Example 1: Real Economy Spillovers. Suppose that banking activities have spillovers into the domestic economy, for example changes in credit available to SMEs. We model these spillovers

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[61]Of course, actions a_{ji} can still appear in utility functions and constraints of country j multinational agents.

in the baseline model as an additive term $u_i^A(I_i^A, L_i^A)$ (where $I_i^A = I_{ii} + \int_j I_{ji}$) in the utility of the representative agent, so that social welfare now includes both bank consumption and the real economy spillovers. This additional term only affects domestic agents, not foreign ones, meaning that it is fully internalized by the domestic planner in the domestic spillover. There is no change in foreign spillovers, meaning Pigouvian efficiency holds by the same logic as the baseline model. In the general framework of this section, this term satisfies Assumption 9, which places no restrictions on the form of domestic spillovers.

In Appendix D.5, we discuss the implications of real economy spillovers (and also the subsequent Example 2, Arbitrageur Welfare) for non-cooperative quantity regulation. In contrast to the baseline model, non-cooperative quantity regulation can lead to under-regulation of foreign banks if the benefit of foreign banks to the domestic economy exceeds their cost in terms of domestic fire sale spillovers. However, the domestic planner still neglects the value of foreign banks to foreigners, and so continues to under-regulate domestic banks and impose unequal treatment.

Example 2: Arbitrageur Welfare.   Suppose that in the baseline model, arbitrageurs in country i have utility $\theta_i w_i^A + E[c_i^A]$, where $\theta_i > 0$ and where w_i^A is date 0 wealth of arbitrageurs. Arbitrageurs cannot borrow or save. Country i social welfare is $E[c_i] + \omega_i^A \left[\theta_i w_i^A + E[c_i^A] \right]$ for Pareto weight ω_i, with $c_i^A(s) = \mathcal{F}_i(L_i^A(s), s) - \gamma_i(s) L_i^A(s)$. This is simply another form of local spillover, satisfying Assumption 9, so that Pigouvian tax efficiency holds. Given a Pareto efficient allocation has $\omega_i^A \theta_i = \lambda_i^0$ (equalizing marginal value of wealth across agents), the additional spillover is

$$\Omega_{ii}^A(s) = \frac{1}{\lambda_i^0} \omega_i^A \frac{\partial c_i^A(s)}{\partial L_i^A(s)} = -\frac{1}{\theta_i} \frac{\partial \gamma_i(s)}{\partial L_i^A(s)} L_i^A(s),$$

that is fire sales have a *positive* spillover to arbitrageurs. This positive spillover reduces the magnitude of the liquidation wedge, but as in Example 1 this is only a change in spillovers to domestic agents. Pigouvian taxation is still efficient by the same logic as the baseline model, which

is the limiting case $\theta_i \to +\infty$. Assumption 9 again holds, as in Example 1.[62]

6.4 Pigouvian Efficiency under Global Externalities

We next discuss the efficiency of Pigouvian taxation under a class of *global* externalities. Global externalities are externalities that also affect foreign agents – the domestic aggregates a_i^A appear in the utility functions or constraint sets of foreign agents.

The fire sale externality of the baseline model was a form of global externality: the domestic aggregate $L_i^A(s)$ appeared in the constraint sets of foreign banks through the liquidation price $\gamma_i(s)$. However, this global pecuniary externality satisfied Assumption 9: foreign banks' exposure to the domestic externality scaled linearly with their domestic activities. As a result, even though the externality took on a global dimension, its endogenous spread through banks' cross-border activities allowed the domestic planner to internalize its global impact through revenue collections. This suggests that Pigouvian efficiency extends to a broader class of pecuniary externality problems resulting from domestic prices appearing in the constraints of foreign banks. For example, Appendix D.2 studies a case where local investment has to be undertaken using a local capital good, with a local price. Whereas country planners under quantity regulation engage in protectionism to shield their own banks from competition, under Pigouvian taxation they achieve the efficient outcome.

Example 3: Climate Change. To help understand the limitations implied by Assumption 9, we start with a simple example unrelated to banking: climate change. Consider an economy with a single activity in each country, "production" a_{ij}. Production benefits agents but produces carbon emissions, so that country i welfare is $u_{ii}(a_{ii}) + \int_j u_{ij}(a_{ij})dj + W_i - \int_j a_j^A dj$. It is intuitive why the revenue collection motive fails to account for climate change spillovers: the spillover $\int_j a_j^A dj$ is separable in utility. Aggregate carbon emissions a_i^A from country i therefore do not impact the marginal value to bank j of domestic production, a_{ji}, and hence do not change revenue collected.

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[62]As is usual in constrained efficient planning problems with fire sales, an increase in regulation that reduced liquidations would make arbitrageurs worse off. This implies optimal policy would combine a Pigouvian tax with lump sum transfers from banks to arbitrageurs at date 0. This is true under both the globally efficient and Pigouvian regimes.

Formally, observe that the spillover does not satisfy Assumption 9. To see formally the failure of non-cooperative Pigouvian taxation, in the decomposition in equation (18) we have a spillover effect $\frac{\partial u_{i'i}^A}{\partial a_i^A(m)} = -1$. However, because climate change is separable in utility, we have $\frac{\partial}{\partial a_i^A(m)}\left(\frac{\partial u_{i'i}^A}{\partial a_{i'i}}a_{i'i}\right) = 0$, that is it does not affect tax revenues. As a result, non-cooperative Pigouvian taxation does not account at all for climate change.

Example 4: Global Banks and Arbitrage. In the baseline model, foreign banks were not able to purchase the domestic asset. However in practice, international banks can potentially buy domestic fire sold assets and support the domestic price, rather than depress it. We extend our baseline model to allow banks to both buy and sell assets, and show our main result on Pigouvian efficiency still holds, with the same intuition. The spillover effects generated by asset purchases and the spillover effects onto purchasing banks are the same as those of asset sales, but with opposite signs.

Formally, banks can purchase assets at date 1, denoted $P_{ij}(s) \geq 0$. Purchased assets yield a final payoff of $F_{ij}(P_{ij}(s),s)$ at date 2, with $F'_{ij}(0,s) \leq 1+r_{ij}$ so that the marginal return to purchased assets is no larger than the marginal return on retained assets.[63] Because the cost to purchases is $\gamma_j(s)P_{ij}(s)$ and assets purchases provide collateral value $(1-h_j(s))\gamma_j(s)P_{ij}(s)$, the bank must raise the difference $h_j(s)\gamma_j(s)P_{ij}(s)$ either by using its existing assets as collateral, or by selling its existing assets. In other words, the modified amount that the bank must finance out of existing assets is $\hat{D}_i = D_i + h_i(s)\gamma_i(s)P_{ii}(s) + \int_j h_j(s)\gamma_j(s)P_{ij}(s)dj$, which now appears on the left hand side of the collateral constraint (equation 3). Thus, the collateral constraint also restricts asset purchases. Final bank consumption is $\hat{c}_i(s) = c_i(s) + F_{ii}(P_{ii}(s),s) - \gamma_i(s)P_{ii}(s) + \int_j[F_{ij}(P_{ij}(s),s) - \gamma_j(s)P_{ij}(s)]dj$, which accounts for gains from asset purchases. Finally, total asset purchases in country i are $L_i^A(s) + P_{ii}(s) + \int_j P_{ji}(s)dj$, which by market clearing must equal liquidations.

From here, Proposition 3 follows the same general form up to two changes. First, the new wedge on asset purchases is $\tau_{ij}^P(s) = -\tau_{ij}^L(s)$, since asset purchases generate the same externality as asset liquidations but with opposite sign. Second, the spillover effect onto bank i' is given by the

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[63]This means that banks never find it optimal to buy assets in country j at the same time that they are selling assets in country j, that is $P_{ij}(s) > 0$ only if $L_{ij}(s) = 0$.

same equation as $\Omega_{i'i}(s)$ in Proposition 3, provided we simply define $L_{i'i}(s) = -P_{i'i}(s)$ for banks that purchase, rather than sell, assets.[64] Thus unsurprisingly, Pigouvian efficiency continues to hold in this setting, absent monopoly rents, which additionally requires $\frac{\partial^2 F_{ij}}{\partial P_{ij}^2} = 0$.

Finally, relating back to the general model, notice that asset purchases appear in the collateral constraint in the same form as asset sales (but with opposite sign), and so satisfy Assumption 9. This reaffirms how Pigouvian efficiency continues to hold in this example.

Example 5: Shadow Banks as a Global Externality. Our results so far have assumed all cross-border agents are regulated. In this example, we show a key limitation to our main result: the presence of "shadow banks," who cannot be regulated by either the global or country planner, can lead to a breakdown of Pigouvian efficiency, even if planners assign welfare weights of zero to shadow banks. One important practical implication of this section is that Pigouvian taxation is an efficient method of regulating previously unregulated intermediaries.

To illustrate the limitation, suppose that instead of local arbitrageurs there is an unregulated global financial intermediary responsible for arbitrage. The global bank has non-separable technology across countries, $\mathcal{F}^G(F^G(s), s)$ with $F^G(s) = \int_i F_i^G(L_i^G(s), s)$.[65], so that its asset demand solves the system of equations $\gamma_i(s) = \frac{\partial \mathcal{F}^G(s)}{\partial F^G(s)} \frac{\partial F_i^G(s)}{\partial L_i^G(s)}$. Non-separable technology means that liquidations in country i affect the price in country k, that is $\frac{\partial \gamma_k(s)}{\partial L_i^G(s)} = \frac{\partial^2 \mathcal{F}^G(s)}{\partial F^G(s)^2} \frac{\partial F_i^G}{\partial L_i^G(s)} \frac{\partial F_k^G}{\partial L_k^G(s)}$. As a result, the globally

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[64]That is to say, for a bank that purchases assets, the spillover is given by

$$\Omega_{i'i}(s) = \underbrace{\frac{\partial \gamma_i(s)}{\partial L_i^A(s)}}_{\text{Price Impact}} \left[\underbrace{-\frac{\lambda_{i'}^1(s)}{\lambda_{i'}^0} P_{i'i}(s)}_{\text{Distributive Externality}} + \underbrace{\frac{\Lambda_{i'}^1(s)}{\lambda_{i'}^0} \left(-h_i(s) P_{i'i}(s) + (1 - h_j(s)) R_i(s) I_{i'i} \right)}_{\text{Collateral Externality}} \right].$$

[65]Note that the results of Example 4 continue to hold in its environment with non-separable technology $\mathcal{F}_i(F_{ii}(P_{ii}(s), s) + \int_j F_{ij}(P_{ij}(s), s), s)$, and that the key difference in this example is regulatory status and not non-separability (non-separable technology does not violate absence of monopoly rents, which requires that $\frac{\partial^2 F_{ij}(s)}{\partial P_{ij}(s)^2} = 0$). Conversely if technology is separable, then this model is equivalent to the baseline model, with local arbitrageurs reinterpreted as a global arbitrageur with separable technology.

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optimal wedges on liquidations (with a welfare weight of 0 on the global arbitrageur) are

$$\tau_{ji}^L(s) = \underbrace{-\Omega_{ii}(s) - \int_{i'}\Omega_{i'i}(s)di'}_{\text{Baseline Model Spillovers}} + \underbrace{\frac{\partial^2\mathcal{F}^G(s)/\partial F^G(s)^2}{\partial\mathcal{F}^G(s)/\partial F^G(s)}\frac{\partial F_i^G}{\partial L_i^G(s)}\int_k\left[-\Omega_{kk}(s) - \int_{k'}\Omega_{k'k}(s)dk'\right]dk}_{\text{Shadow Banking Spillovers}}.$$

Under non-cooperative Pigouvian taxation, country planners account for the set of "Baseline Model Spillovers," but neglect the "Shadow Banking Spillovers" which arise from non-separable technology. Intuitively, although liquidations in country i reduce prices in other countries through shadow banking, the domestic planner does not have taxes for foreign bank activities in foreign countries, and so cannot internalize these spillovers. Conversely, the domestic planner continues to properly internalize the spillovers resulting from the domestic liquidation price, where taxes do apply. To see how this example violates Assumption 9, the value of asset liquidations as $\gamma_j(s)L_{ij}(s)$ is now determined by two functions: (i) $\varphi_{ij}^1(s) = \frac{\partial F_j^G(L_j^G(s),s)}{\partial L_j^G(s)}L_{ij}(s)$, which is the same function employed in the baseline model with local arbitrageurs and satisfies Assumption 9; and, (ii) $\varphi_{ij}^2(s) = F_j^G(L_j^G(s),s)$, which captures non-separability and does not satisfy Assumption 9.[66] Notice that $\varphi_{ij}^2(s)$ is in fact of the same general form as the climate change spillover of Example 3: the failure of efficiency, in that the shadow banking spillover generates spillovers in foreign countries which are not internalized by revenue collection, is actually a close cousin of the climate change example.

This example showcases the difficulty that unregulated agents pose for Pigouvian efficiency, since cross-border spillovers via unregulated agents are not internalized through revenue collection

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[66]For full clarity, in this case we have $\varphi_i^1(s) = \frac{\partial F_i^G(L_i^G(s),s)}{\partial L_i^G(s)}L_{ii}(s) + \int_j\frac{\partial F_j^G(L_j^G(s),s)}{\partial L_j^G(s)}L_{ij}(s)dj$ and $\varphi_i^2(s) = \int_j F_j^G(L_j^G(s),s)dj$, so that we have

$$\frac{\partial\mathcal{F}^G(\varphi_i^2(s),s)}{\partial F^G(s)}\cdot\varphi_i^1(s) = \frac{\partial\mathcal{F}^G(\varphi_i^2(s),s)}{\partial F^G(s)}\frac{\partial F_i^G(L_i^G(s),s)}{\partial L_i^G(s)}L_{ii}(s) + \int_j\frac{\partial\mathcal{F}^G(\varphi_i^2(s),s)}{\partial F^G(s)}\frac{\partial F_j^G(L_j^G(s),s)}{\partial L_j^G(s)}L_{ij}(s)dj$$

$$= \gamma_i(s)L_{ii}(s) + \int_j\gamma_j(s)L_{ij}(s)dj,$$

with similar functions used for defining collateral values.

from these agents.[67] One concrete implication is that Pigouvian taxation provides an efficient method of regulating previously unregulated shadow banks or other cross-border capital flows. Our theory thus suggests a novel synergy between regulation of banks and shadow banks (or unregulated capital flows). By applying Pigouvian taxes to manage unregulated capital flows, the domestic regulator also improves the efficiency of domestic macroprudential regulation, internalizing spillovers to foreign agents through revenue collection.

7   Conclusion

We study a model of cross-border banking, in which endogenous cross-border propagation of fire sales generates international financial stability spillovers. Our main and most surprising normative contribution is to show that non-cooperative national governments using revenue-generating Pigouvian taxes can implement the globally efficient allocation, eliminating the need for international cooperation. The motivation to collect revenues from foreign banks enables the domestic government to internalize the impacts of domestic regulation and domestic fire sales on the value of foreign banks to foreigners, which would otherwise be neglected by the domestic government when designing revenue-neutral regulation. From a policy perspective, this suggests that giving a more prominent role to revenue-generating Pigouvian policies in the macroprudential regime may be desirable. By doing so, policymakers may be able to reduce the need for cooperative regulatory agreements and avoid the inherent difficulties of cooperation.

An important property of our model is that non-cooperative governments employing taxes do not engage in a counterproductive race to the bottom, despite the motivation to collect tax revenue. However, our model focuses on bank externality regulation, and does not address broader motivations for taxation such as financing public good expenditures. Cooperation over taxation for public financing is also an important and ongoing debate. An interesting direction for future

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[67]Importantly, this needs to be caveated. If another multinational agent is subject to externalities but conducts activities that do not generate externalities, Proposition 8 implies this agent is not regulated in equilibrium and the Pigouvian tax set is 0. This draws an important distinction between an agent who is *unregulated in equilibrium* versus one who cannot be regulated.

research would be to study whether the forces identified in this paper also have implications for cooperation in these settings.

References

Acharya, V., D. Pierret, and S. Steffen (2021). Lender of last resort, buyer of last resort, and a fear of fire sales in the sovereign bond market. *Financial Markets, Institutions and Instruments 30*(4), 87–112.

Acharya, V. V. (2003). Is the International Convergence of Capital Adequacy Regulation Desirable? *The Journal of Finance 58*(6), 2745–2781.

Avdjiev, S., B. Hardy, S. Kalemli-Ozcan, and L. Servén (2018, jan). Gross Capital Flows by Banks, Corporates and Sovereigns.

Baker III, J. A., M. Feldstein, T. Halstead, N. G. Mankiw, H. M. Paulson Jr, G. P. Shultz, T. Stephenson, and R. Walton (2017). The Conservative Case For Carbon Dividends.

Beck, T., R. Todorov, and W. Wagner (2013, jan). Supervising cross-border banks: Theory, evidence and policy. *Economic Policy 28*(73), 5–44.

Beck, T. and W. Wagner (2016). Supranational supervision: How much and for whom? *International Journal of Central Banking 12*(2), 221–268.

Bengui, J. (2014). Macro-Prudential Policy Coordination.

Bernheim, B. D. and M. D. Whinston (1986, jul). Common Agency. *Econometrica 54*(4), 923–942.

Bhagwati, J. (1965). *On the equivalence of tariffs and quotas, in: R.E. Baldwin et al., Trade growth and the balance of payments – Essays in honor of G. Haberler (Rand-McNally, Chicago).*

Bhagwati, J. (1968). More on the Equivalence of Tariffs and Quotas. *American Economic Review 58*(1), 142–146.

Bhagwati, J. N. and T. N. Srinivasan (1980). Revenue Seeking: A Generalization of the Theory of Tariffs. *Journal of Political Economy 88*(6), 1069–1087.

Bianchi, J. (2011, dec). Overborrowing and systemic externalities in the business cycle. *American Economic Review 101*(7), 3400–3426.

Bianchi, J. (2016, dec). Efficient bailouts? *American Economic Review 106*(12), 3607–3659.

Bianchi, J., C. Liu, and E. G. Mendoza (2016, mar). Fundamentals news, global liquidity and macroprudential policy. *Journal of International Economics 99*, S2–S15.

Bianchi, J. and E. G. Mendoza (2010). Overborrowing, Financial Crises and 'Macro-prudential' Taxes. *NBER Working Papers*.

Bianchi, J. and E. G. Mendoza (2018, apr). Optimal Time-Consistent Macroprudential Policy. *Journal of Political Economy 126*(2), 588–634.

BIS (2010). Basel III: A global regulatory framework for more resilient banks and banking systems.

Bolton, P. and M. Oehmke (2019, jun). Bank Resolution and the Structure of Global Banks. *The Review of Financial Studies 32*(6), 2384–2421.

Broner, F., T. Didier, A. Erce, and S. L. Schmukler (2013, jan). Gross capital flows: Dynamics and crises. *Journal of Monetary Economics 60*(1), 113–133.

Caballero, R. J., E. Farhi, and P. O. Gourinchas (2008, feb). An equilibrium model of global imbalances and low interest rates. *American Economic Review 98*(1), 358–393.

Caballero, R. J. and A. Krishnamurthy (2001, dec). International and domestic collateral constraints in a model of emerging market crises. *Journal of Monetary Economics 48*(3), 513–548.

Caballero, R. J. and A. Simsek (2018). Reach for Yield and Fickle Capital Flows. *AEA Papers and Proceedings 108*, 493–98.

Caballero, R. J. and A. Simsek (2020). A Model of Fickle Capital Flows and Retrenchment. *Journal of Political Economy 128*(6), 2288–2328.

Calzolari, G., J.-E. Colliard, and G. Lóránth (2019, aug). Multinational Banks and Supranational Supervision. *The Review of Financial Studies 32*(8), 2997–3035.

Cassing, J. H. and A. L. Hillman (1985). Political influence motives and the choice between tariffs and quotas. *Journal of International Economics 19*(3-4), 279–290.

Chari, V. V. and P. J. Kehoe (2016, sep). Bailouts, Time Inconsistency, and Optimal Regulation: A Macroeconomic View. *American Economic Review 106*(9), 2458–2493.

Clayton, C. and A. Schaab (2021). Bail-Ins, Optimal Regulation, and Crisis Resolution.

Cochrane, J. H. (2014). Toward a Run-Free Financial System.

Coeurdacier, N. and H. Rey (2013, mar). Home bias in open economy financial macroeconomics. *Journal of Economic Literature 51*(1), 63–115.

Coppola, A., M. Maggiori, B. Neiman, and J. Schreger (2021). Redrawing the Map of Global Capital Flows: The Role of Cross-Border Financing and Tax Havens. *Quarterly Journal of Economics 136*(3), 1499–1556.

Dasgupta, P. and J. Stiglitz (1977). Tariffs vs. Quotas as Revenue Raising Devices under Uncertainty. *The American Economic Review* (67(5)), 975–981.

Dávila, E. and A. Korinek (2018, jan). Pecuniary Externalities in Economies with Financial Frictions. *The Review of Economic Studies 85*(1), 352–395.

Davis, J. S. and E. Van Wincoop (2018, dec). Globalization and the increasing correlation between capital inflows and outflows. *Journal of Monetary Economics 100*, 83–100.

De Marco, F., M. Macchiavelli, and R. Valchev (2019). International Diversification and Information Heterogeneity.

De Nicoló, G., G. Favara, and L. Ratnovski (2014). Externalities and Macroprudential Policy. *Journal of Financial Perspectives 2*(1).

Dell'Ariccia, G. and R. Marquez (2006, feb). Competition among regulators and credit market integration. *Journal of Financial Economics 79*(2), 401–430.

Dixit, A., G. M. Grossman, and E. Helpman (1997, aug). Common Agency and Coordination: General Theory and Application to Government Policy Making. *Journal of Political Economy 105*(4), 752–769.

ECB (2018). SSM Supervisory Manual.

Erten, B., A. Korinek, and J.-a. Ocampo (2021). Capital Controls: Theory and Evidence. *Journal of Economic Literature 59*(1), 45–89.

European Commision (2015). Proposal for a Regulation of the European Parliament and of the Council amending Regulation (EU) 806/2014 in order to establish a European Deposit Insurance Scheme.

Falvey, R. E. (1978). A note on preferential and illegal trade under quantitative restrictions. *Quarterly Journal of Economics 92*(1), 175–178.

Farhi, E., M. Golosov, and A. Tsyvinski (2009, jul). A theory of liquidity and regulation of financial intermediation. *Review of Economic Studies 76*(3), 973–992.

Farhi, E. and J. Tirole (2012, feb). Collective moral hazard, maturity mismatch, and systemic bailouts. *American Economic Review 102*(1), 60–93.

Farhi, E. and J. Tirole (2018, jul). Deadly embrace: Sovereign and financial balance sheets doom loops. *Review of Economic Studies 85*(3), 1781–1823.

Farhi, E. and I. Werning (2016, sep). A Theory of Macroprudential Policies in the Presence of Nominal Rigidities. *Econometrica 84*(5), 1645–1704.

Farhi, E. and I. Werning (2017, dec). Fiscal unions. *American Economic Review 107*(12), 3788–3834.

Financial Stability Board (2013a). Recovery and Resolution Planning for Systemically Important Financial Institutions: Guidance on Developing Effective Resolution Strategies.

Financial Stability Board (2013b). Strengthening Oversight and Regulation of Shadow Banking: Policy Framework for Strengthening Oversight and Regulation of Shadow Banking Entities.

Fishelson, G. and F. Flatters (1975). The (non)equivalence of optimal tariffs and quotas under uncertainty. *Journal of International Economics 5*(4), 385–393.

Foarta, D. (2018, apr). The limits to partial banking unions: A political economy approach. *American Economic Review 108*(4-5), 1187–1213.

Forbes, K. J. and F. E. Warnock (2012, nov). Capital flow waves: Surges, stops, flight, and retrenchment. *Journal of International Economics 88*(2), 235–251.

Geithner, T. F. (2016). *Are we safer? : the case for strengthening the Bagehot arsenal*. Per Jacobsson Foundation.

Gersbach, H., H. Haller, and S. Papageorgiou (2020). Regulatory competition in banking: Curse or blessing? *Journal of Banking and Finance 121*.

Greenwood, R., J. C. Stein, S. G. Hanson, and A. Sunderam (2017). Strengthening and streamlining bank capital regulation. *Brookings Papers on Economic Activity 2017*(Fall), 479–565.

IMF (2012). The Liberalization and Management of Capital Flows - An Institutional View.

Jeanne, O. and A. Korinek (2020, may). Macroprudential Regulation versus mopping up after the crash. *The Review of Economic Studies 87*(3), 1470–1497.

Kara, G. I. (2016, mar). Systemic risk, international regulation, and the limits of coordination. *Journal of International Economics 99*, 192–222.

Keister, T. (2016, apr). Bailouts and financial fragility. *Review of Economic Studies 83*(2), 704–736.

Kiyotaki, N. and J. Moore (1997, apr). Credit Cycles. *Journal of Political Economy 105*(2), 211–248.

Kocherlakota, N. (2010). Taxing Risk and the Optimal Regulation of Financial Institutions. *Federal Reserve Bank of Minneapolis Economic Policy Paper 10-3*(May), 1–12.

Korinek, A. (2017). Currency Wars or Efficient Spillovers? A General Theory of International Policy Cooperation.

Korinek, A. and A. Simsek (2016, mar). Liquidity trap and excessive leverage. *American Economic Review 106*(3), 699–738.

Lorenzoni, G. (2008). Inefficient credit booms. *Review of Economic Studies 75*(3), 809–833.

Magee, S. P. (1972). The Welfare Effects of Restrictions on U.S. Trade. *Brookings Papers on Economic Activity, 3*.

Maggiori, M. (2017, oct). Financial intermediation, international risk sharing, and reserve currencies. *American Economic Review 107*(10), 3038–3071.

Maggiori, M., B. Neiman, and J. Schreger (2020). International currencies and capital allocation.

Journal of Political Economy 128(6), 2019–2066.

Mankiw, N. G. (2009, jan). Smart taxes: An open invitation to join the pigou club. *Eastern Economic Journal 35*(1), 14–23.

Masur, J. S. and E. A. Posner (2015, jan). Toward a Pigouvian state. *University of Pennsylvania Law Review 164*(1), 93–147.

Mendoza, E. G., V. Quadrini, and J. V. Ríos-Rull (2009, jun). Financial integration, financial development, and global imbalances. *Journal of Political Economy 117*(3), 371–416.

Milesi-Ferretti, G. M. and C. Tille (2011, apr). The great retrenchment: International capital flows during the global financial crisis. *Economic Policy 26*(66), 285–342.

Morrison, A. D. and L. White (2009). Level playing fields in international financial regulation. *Journal of Finance 64*(3), 1099–1142.

Niepmann, F. (2015, jul). Banking across borders. *Journal of International Economics 96*(2), 244–265.

Niepmann, F. and T. Schmidt-Eisenlohr (2013, nov). Bank bailouts, international linkages, and cooperation. *American Economic Journal: Economic Policy 5*(4), 270–305.

Nolle, D. E. (2012, dec). Foreign Banks in the U.S.: A Reference Document.

Pelcovits, M. D. (1976). Quotas versus tariffs. *Journal of International Economics 6*(4), 363–370.

Perotti, E. and J. Suarez (2011). A Pigovian Approach to Liquidity Regulation. *International Journal of Central Banking 7*(4), 3–41.

Roberts, M. J. and M. Spence (1976). Effluent Charges and Licenses Under Uncertainty. *Journal of Public Economics 5*, 193–208.

Schmitt-Grohé, S. and M. Uribe (2016, oct). Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment. *Journal of Political Economy 124*(5), 1466–1514.

Segura, A. and S. Vicente (2019). Bank Resolution and Public Backstop in An Asymmetric Banking Union.

Shen, L. S. (2019). Global vs. Local Banking: Firm Financing in a Globalized Financial System.

Shibata, H. (1968). A note on the equivalence of tariffs and quotas. *American Economic Review 58*(1), 137–142.

Spence, M. (1977). Nonlinear prices and welfare. *Journal of Public Economics 8*(1), 1–18.

Stein, J. C. (2012, feb). Monetary Policy as Financial Stability Regulation. *The Quarterly Journal of Economics 127*(1), 57–95.

Szentes, B. (2015, jan). Contractible Contracts in Common Agency Problems. *The Review of Economic Studies 82*(1), 391–422.

Tucker, P. (2014). Regulatory reform, stability and central banking.

Tucker, P. (2016, sep). The Design and Governance of Financial Stability Regimes: A Common-resource Problem that Challenges Technical Know-How, Democratic Accountability and International Coordination. *Essays on International Finance 3*, Centre for International Governance Innovation.

Tullock, G. (1967, jun). Excess benefit. *Water Resources Research 3*(2), 643–644.

US Department of Treasury (2018). Orderly Liquidation Authority and Bankruptcy Reform.

Weitzman, M. L. (1974, oct). Prices vs. Quantities. *The Review of Economic Studies 41*(4), 477–491.

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A   Proofs

Proofs from the baseline model, the macroprudential regulation model, and the general model are contained in this appendix. It is worthwhile to note that the baseline model (and the macropru section) is an application of the model of Section 6. However, for transparency we present direct proofs for these results.

A.1   Competitive Equilibrium FOCs (Section 2.3)

The bank Lagrangian is (without loss of generality, multiplying utility by a weight $\omega_i > 0$, in anticipation of the planning problem)

$$
\begin{aligned}
\mathcal{L}_i = {} & \omega_i \int_s c_i(s) f(s) ds + \lambda_i^0 \left[A_i + D_i - \Phi_{ii}(I_{ii}) - \int_j \Phi_{ij}(I_{ij}) dj \right] \\
& + \int_s \lambda_i^1(s) \left[\gamma_i(s) L_{ii}(s) + (1 + r_{ii})(R_i(s) I_{ii} - L_{ii}(s)) \right. \\
& \qquad\qquad \left. + \int_j \left[\gamma_j(s) L_{ij}(s) + (1 + r_{ij})(R_j(s) I_{ij} - L_{ij}(s)) \right] dj - c_i(s) - D_i \right] f(s) ds \\
& + \int_s \Lambda_i^1(s) \left[-D_i + \gamma_i(s) L_{ii}(s) + \int_j \gamma_j(s) L_{ij}(s) dj + (1 - h_i(s)) \mathcal{C}_{ii}(s) + \int_j (1 - h_j(s)) \mathcal{C}_{ij}(s) dj \right] f(s) ds \\
& + \int_s \left[\underline{\xi}_{ii}(s) L_{ij}(s) + \overline{\xi}_{ii}(s)(R_i I_{ii} - L_{ii}(s)) + \int_j \left(\underline{\xi}_{ii}(s) L_{ij}(s) + \overline{\xi}_{ij}(s)(R_j I_{ij} - L_{ij}(s)) \right) \right] f(s) ds
\end{aligned}
$$

where we recall that $\mathcal{C}_{ij}(s) = \gamma_j(s) \left[R_j(s) I_{ij} - L_{ij}(s) \right]$.

FOC for I_{ij}.   Taking the first order condition in I_{ij}, we obtain

$$
0 \geq -\lambda_i^0 \frac{\partial \Phi_{ij}}{\partial I_{ij}} + E\left[\lambda_i^1(1 + r_{ij}) R_j \right] + E\left[\Lambda_i^1(1 - h_j) \gamma_j R_j \right] + E\left[\overline{\xi}_{ij} R_j \right].
$$

Expanding the first expectation, we obtain the result.

FOC for $L_{ij}(s)$. Taking the first order condition in $L_{ij}(s)$, we obtain

$$0 = \lambda_i^1(s)(\gamma_j(s) - (1 + r_{ij}))f(s) + \Lambda_i^1(s)(\gamma_j(s) - (1 - h_j(s))\gamma_j(s))f(s) + \underline{\xi}_{ij}(s)f(s) - \overline{\xi}_{ij}(s)f(s)$$

which simplifies to the result.

A.2 Proof of Proposition 1

Give the definition of the private bank Lagrangian in Section A.1, we can define the Lagrangian of the global planner as (recall that we have already incorporated the welfare weights ω_i into the private Lagrangians)

$$\mathcal{L}^G = \int_{i'} \mathcal{L}_{i'} di - \lambda^0 \int_{i'} \mathcal{T}_{i'} di',$$

where $\int_{i'} \mathcal{T}_{i'} di' = 0$ are inter-country transfers (so that $\mathcal{T}_i$ increases A_i). First, optimal transfers satisfy

$$0 = \lambda_i^0 - \lambda^0$$

so that $\lambda_i^0 = \lambda^0$ for all i (date 0 weighted marginal value of wealth is equalized across countries). Now, consider the social optimality for liquidations $L_{ji}(s)$ for either $i = j$ or $i \neq j$. The social optimality condition is

$$0 = \frac{\partial \mathcal{L}_j}{\partial L_{ji}(s)} + \frac{\partial}{\partial L_i^A(s)} \int_{i'} \mathcal{L}_{i'} di'$$

Next, consider the decision rule of private banks, who are subject to wedges τ. Their decision rule for liquidations is

$$0 = \frac{\partial \mathcal{L}_j}{\partial L_{ji}(s)} - \lambda_i^0 \tau_{ji}^L(s)f(s),$$

accounting for the effect of the wedge. Combining these equations, we obtain

$$\tau_{ji}^L(s) = -\frac{1}{\lambda_j^0} \frac{1}{f(s)} \frac{\partial}{\partial L_i^A(s)} \int_{i'} \mathcal{L}_{i'} di'.$$

It remains now to evaluate the derivative. From the private Lagrangian definition, we have for $i' \neq i$

$$\frac{\partial \mathcal{L}_{i'}}{\partial L_i^A(s)} = \lambda_{i'}^1(s) \frac{\partial \gamma_i(s)}{\partial L_i^A(s)} L_{i'i}(s)f(s) + \Lambda_{i'}^1(s) \left[\frac{\partial \gamma_i(s)}{\partial L_i^A(s)} L_{i'i}(s) + (1 - h_i(s)) \frac{\partial C_{i'i}(s)}{\partial L_i^A(s)} \right] f(s)$$

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while for $i' = i$, we have $\frac{\partial \mathcal{L}_i}{\partial L_i^A(s)} di$ equal to the same expression (due to home bias). From the definition, we have $\frac{\partial \mathcal{C}_{i'i}(s)}{\partial L_i^A(s)} = \frac{\partial \gamma_i(s)}{\partial L_i^A(s)} \left[R_i(s)I_{i'i} - L_{i'i}(s) \right]$, so that we have

$$\frac{\partial \mathcal{L}_{i'}}{\partial L_i^A(s)} = \lambda_{i'}^0 \frac{\partial \gamma_i(s)}{\partial L_i^A(s)} \left[\frac{\lambda_{i'}^1(s)}{\lambda_{i'}^0} L_{i'i}(s) + \frac{\Lambda_{i'}^1(s)}{\lambda_{i'}^0} \left[L_{i'i}(s) + (1 - h_i(s)) \left[R_i(s)I_{i'i} - L_{i'i}(s) \right] \right] \right] f(s) = \lambda_{i'}^0 \Omega_{i'i}(s) f(s)$$

under the definition of Ω given in the statement of the proposition. Finally, substituting into the integral and using that $\lambda_{i'}^0 = \lambda^0$ for all i',

$$\frac{\partial}{\partial L_i^A(s)} \int_{i'} \mathcal{L}_{i'} di' = \frac{\partial \mathcal{L}_i}{\partial L_i^A(s)} + \int_{i' \neq i} \frac{\partial \mathcal{L}_{i'}}{\partial L_i^A(s)} di' = \lambda^0 \left[\Omega_{ii}(s) + \int_{i'} \Omega_{i'i}(s) di' \right] f(s).$$

Finally, substituting into the wedge formula and using $\lambda_j^0 = \lambda^0$, we obtain

$$\tau_{ji}^L(s) = -\Omega_{ii}(s) - \int_{i'} \Omega_{i'i}(s) di',$$

giving the result. This derivation did not rely on the identity of country j, and so is valid for all j.

Finally, for all other choice variables (I, c, D), these choice variables do not directly impact the liquidation price, so that the private and social optimality conditions coincide. As a result, $\tau_i^c = 0$, $\tau_i^I = 0$, and $\tau_i^D = 0$ for all i.

A.3 Proof of Lemma 2

The first part of implementability, that the country i planner can directly choose domestic bank allocations (c_i, D_i, I_i, L_i) subject to constraints, is standard given complete wedges (a standard constrained efficient planning problem). Consider the domestic allocations I_{ji} and L_{ji} of foreign bank j. At interior solutions,[68] given taxes τ_i and τ_j on foreign bank j we have the first order conditions for liquidations given by

$$0 = -\lambda_j^0 \left[\tau_{i,ji}^L(s) + \tau_{j,ji}^L(s) \right] + \frac{\partial \mathcal{L}_j(s)}{\partial L_{ji}(s)},$$

---

[68]Appendix E.2 shows that this argument generalizes under corner solutions, because it is optimal for country planner i to ensure that even at a corner solution, the foreign bank is on its FOC even at the corner solution.

and substituting in the competitive FOC at an interior solution ($\xi_{ji} = 0$), we have

$$0 = -\lambda_j^0 \left[\tau_{i,ji}^L(s) + \tau_{j,ji}^L(s) \right] + \lambda_j^1(s) \left(\gamma_i(s) - (1 + r_{ji}) \right) f(s) + \Lambda_j^1(s) \left(\gamma_j(s) - (1 - h_i(s)) \gamma_i(s) \right) f(s)$$

which rearranges to the result for the liquidation wedges. The same steps give us

$$0 = -\lambda_j^0 \left[\tau_{i,ji}^I + \tau_{j,ji}^I \right] - \lambda_j^0 \frac{\partial \Phi_{ji}}{\partial I_{ji}} + E \left[\lambda_j^1 (1 + r_{ji}) R_i \right] + E \left[\Lambda_j^1 (1 - h_i) \gamma_i R_i \right]$$

which rearranges to the result for the investment wedges. Observe that because country $j \neq i$ only maintains a marginal (density) activities presence in country i, the country planner i takes as given the marginal values of wealth λ_j^0, λ_j^1 and the collateral constraint Lagrange multiplier Λ_j^1. As a result, the FOC for liqudiations is linear, and the country i planner can choose any interior value of liquidations $L_{ji}(s)$ by setting the wedge $\tau_{ji}^L(s)$ to clear this equation. Finally, the investment equation gives a demand function relating $\frac{\partial \Phi_{ji}}{\partial I_{ji}}$ to $\tau_{i,ji}^I$, so that again the social planner can enforce any demand I_{ji} by setting $\tau_{i,ji}^I$ to clear the first order condition. As such, the social planner can solve the problem by directly choosing foreign allocations (I_{ji}, L_{ji}) by using the wedges described above.

A.4   Proof of Proposition 3

The objective of country planner i under quantity regulation is to maximize domestic welfare by choosing feasible allocations $(c_i, I_i, L_i, D_i, \{I_{ji}, L_{ji}\})$ and wedges $\{\tau_{i,ji}^I, \tau_{i,ji}^L\}$ on foreign banks, taking as given foreign revenue remissions and that the implementing wedges for foreign banks must be given as in Lemma 2. However, given that revenue remissions are taken as given, the social planner's problem in country i can be represented from the Lagrangian $\mathcal{L}_i$, internalizing the liquidation function γ_i, combined with the implementability conditions of Lemma 2. However, because the implementing wedges $\{\tau_{i,ji}^I, \tau_{i,ji}^L\}$ do not appear in the Lagrangian $\mathscr{L}_i$, the social planner's Lagrange multipliers on the implementability conditions of Lemma 2 are 0. As such, we can represent country planner i's Lagrangian as $\mathcal{L}_i$ (internalizing γ_i), with the choice variables being allocations $(c_i, I_i, L_i, D_i, \{I_{ji}, L_{ji}\})$. Implementability simply gives the method of implementing the foreign allocations.

Now that we have the domestic planner's problem, we have the social optimality condition for

domestic liquidations by domestic banks, $L_{ii}(s)$, given by

$$0 = \frac{\partial \mathcal{L}_i}{\partial L_{ii}(s)} + \frac{\partial \mathcal{L}_i}{\partial L_i^A(s)}.$$

Using the same steps and definitions as in the proof of Proposition 1, we obtain

$$\tau_{i,ii}^L(s) = -\Omega_{ii}(s),$$

giving the first result.

Next, the social optimality condition for domestic liquidations by foreign banks, $L_{ji}(s)$, is given by

$$0 \geq \frac{\partial \mathcal{L}_i}{\partial L_i^A(s)},$$

so that we have an allocation rule $0 = L_{ji}(s)\Omega_{ii}(s)$, with $L_{ji}(s) = 0$ if $\Omega_{ii}(s) < 0$.

Finally, for all other domestic choices (c_i, I_i, D_i), the private and social FOCs align, and we have $\tau_i^c = 0$, $\tau_i^I = 0$, and $\tau_i^D = 0$. Lastly, domestic investment by foreign banks has no welfare impact whatsoever, so by convention we set $\tau_{ji}^I = 0$.

A.5 Proof of Proposition 4

The objective of country planner i is the same as in Proposition 3, except for the internalized revenue remission. Total revenues collected by country planner i are given by

$$\Pi_i = \int_{i'} \left[\tau_{i,i'i}^I I_{i'i} + \tau_{i,i'i}^L L_{i'i} \right] di'.$$

This revenue is remitted lump sum to domestic banks into their budget constraint at date 0, and so is valued by the Lagrange multiplier λ_i^0. In other words, we can represent the Lagrangian of social planner i, internalizing both the liquidation function γ_i and the wedge formulas $\tau_{i,i'i}$, by

$$\mathcal{L}_i^\Pi = \mathcal{L}_i + \lambda_i^0 \Pi_i,$$

where as before the choice variables are allocations. Now, consider the impact of a change in price γ_i on revenues collected. Here, we have

$$\frac{\partial \Pi_i}{\partial \gamma_i(s)} = \int_{i'} \left[\frac{\partial \tau^I_{i,i'i}}{\partial \gamma_i(s)} I_{i'i} + \frac{\partial \tau^L_{i,i'i}}{\partial \gamma_i} L_{i'i} \right] di'.$$

From Lemma 2, we have

$$\frac{\partial \tau^L_{i,ji}(s)}{\partial \gamma_i(s)} = \frac{\lambda^1_j(s)}{\lambda^0_j} + \frac{1}{\lambda^0_j} \Lambda^1_j(s) h_i(s)$$

$$\frac{\partial \tau^I_{i,ji}}{\partial \gamma_i(s)} = \frac{1}{\lambda^0_j} \Lambda^1_j (1 - h_i) R_i f(s) ds$$

From here, we have

$$\frac{\partial \tau^I_{i,i'i}}{\partial \gamma_i(s)} I_{i'i} + \frac{\partial \tau^L_{i,i'i}}{\partial \gamma_i} L_{i'i} = \left[\frac{\lambda^1_{i'}(s)}{\lambda^0_{i'}} + \frac{1}{\lambda^0_{i'}} \Lambda^1_{i'}(s) h_i(s) \right] L_{i'i}(s) f(s) + \frac{1}{\lambda^0_{i'}} \Lambda^1_{i'} (1 - h_i) R_i I_{i'i} f(s)$$

And so adding and subtracting $\frac{1}{\lambda^0_{i'}} \Lambda^1_{i'}(s) h_i(s)$, we obtain

$$\frac{\partial \tau^I_{i,i'i}}{\partial \gamma_i(s)} I_{i'i} + \frac{\partial \tau^L_{i,i'i}}{\partial \gamma_i} L_{i'i} = \left[\frac{\lambda^1_{i'}(s)}{\lambda^0_{i'}} L_{i'i}(s) + \frac{\Lambda^1_{i'}(s)}{\lambda^0_{i'}} \left[L_{i'i}(s) + (1 - h_i(s)) \left[R_i I_{i'i} - L_{i'i}(s) \right] \right] \right] f(s) = \frac{\Omega_{i'i}(s)}{\partial \gamma_i(s) / \partial L^A_i(s)} f(s)$$

where we have substituted in the definition of $\Omega_{i'i}(s)$ from Proposition 1. As a result, we have

$$\frac{\partial \Pi_i}{\partial \gamma_i(s)} = \frac{f(s)}{\partial \gamma_i(s) / \partial L^A_i(s)} \int_{i'} \Omega_{i'i}(s) di'.$$

From here, we can find the social first order conditions. For domestic liquidations $L_{ii}(s)$, we have

$$0 = \frac{\partial \mathcal{L}_i}{\partial L_{ii}(s)} + \frac{\partial \mathcal{L}_i}{\partial L^A_i(s)} + \lambda^0_i \frac{\partial \Pi_i}{\partial L^A_i(s)},$$

which using the results from the proof of Proposition 3 yields

$$0 = \lambda^0_i \tau^L_{i,ii}(s) f(s) + \lambda^0_i \Omega^L_{ii}(s) f(s) + \lambda^0_i \frac{\partial \gamma_i(s)}{\partial L^A_i(s)} \frac{\partial \Pi_i}{\partial \gamma_i(s)}.$$

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Substituting in the deriative from above and rearranging, we obtain

$$\tau_{i,ii}^L(s) = -\Omega_{ii}^L(s) - \int_{i'} \Omega_{i'i}(s) di',$$

yielding the tax formula for $\tau_{i,ii}^L(s)$.

Consider next the first order condition for $L_{ji}(s)$ for $j \neq i$. Now, we have

$$0 = \frac{\partial \mathcal{L}_i}{\partial L_i^A(s)} + \lambda_i^0 \left[\frac{\partial \Pi_i}{\partial L_i^A(s)} + \frac{\partial \Pi_i}{\partial L_{ji}(s)} \right],$$

where $\frac{\partial \Pi_i}{\partial L_{ji}(s)}$ is the direct effect of the change in liquidations on tax revenue (holding prices fixed). Using the same steps, this rearranges to

$$\frac{\partial \Pi_i}{\partial L_{ji}(s)} = -\Omega_{ii}^L(s) f(s) - \int_{i'} \Omega_{i'i}(s) di' f(s).$$

Finally, from the tax formula, we have

$$\frac{\partial \Pi_i}{\partial L_{ji}(s)} = \tau_{i,ji}^L(s) f(s) + \frac{\partial \tau_{i,ji}^L(s)}{\partial L_{ji}(s)} L_{ji}(s) f(s) + \frac{\partial \tau_{i,ji}^I}{\partial L_{ji}(s)} I_{ji} = \tau_{i,ji}^L(s) f(s),$$

since $L_{ji}(s)$ does not appear directly in the tax formulas of Lemma 2. Substituting back in, we obtain

$$\tau_{i,ji}^L(s) = -\Omega_{ii}^L(s) - \int_{i'} \Omega_{i'i}(s) di',$$

yielding the tax formula for $\tau_{i,ji}^L(s)$.

Consider next the first order condition for I_{ji} for $j \neq i$. We have

$$0 = \lambda_i^0 \frac{\partial \Pi_i}{\partial I_{ji}},$$

given that I_{ji} has no other welfare impact. From here, we have (since $\tau_{i,ji}^L$ does not depend directly of I_{ji})

$$\frac{\partial \Pi_i}{\partial I_{ji}} = \tau_{i,ji}^I + \frac{\partial \tau_{i,ji}^I}{\partial I_{ji}} I_{ji} = \tau_{i,ji}^I - \frac{\partial^2 \Phi_{ji}}{\partial I_{ji}^2}.$$

Substituting back in, we obtain

$$\tau^I_{i,ji} = \frac{\partial^2 \Phi_{ji}}{\partial I^2_{ji}} I_{ji},$$

giving the result for $\tau^I_{i,ji}$.

Finally, for all other domestic allocations (c_i, D_i, I_i), the private and social first order conditions align, and so we have $\tau^c_i = 0$, $\tau^D_i = 0$, and $\tau^I_{i,ii} = 0$.

A.6 Proof of Proposition 5

The proof follows immediately from Proposition 4: when $\frac{\partial^2 \Phi_{ji}}{\partial I^2_{ji}} = 0$ for all i and $j \neq i$, each country planner implements the same wedges as the global planner, that is to say $\tau^L_{i,ji}(s) = \tau^L_{ji}(s)$ for all i and j, while all other wedges are zero.

A.7 Proof of Proposition 6

The proof follows the same general steps as the proof of Proposition 1. The wedge on debt D_{ji} is thus given by[69]

$$\tau^D_{ji} = -\frac{1}{\lambda^0_j} \int_{s \in S^D_{ji}} \left[\frac{\partial L_{ji}(s)}{\partial D_{ji}(s)} \frac{\partial}{\partial L^A_i(s)} \int_{i'} \mathcal{L}_{i'} di' \right] f(s) ds.$$

Note that we have $\frac{\partial L_{ji}(s)}{\partial D_{ji}(s)} = \frac{1}{h_i(s)\gamma_i(s)}$ when $s \in S^D_{ji}$. It remains to characterize the spillover. Given the welfare function of country i' banks, we have

$$\frac{\partial \mathcal{L}_{i'}}{\partial L^A_i(s)} = \frac{d\gamma_i(s)}{dL^A_i(s)} \left[(\gamma_i(s) - 1) L_{i'i}(s) f(s) \right] = \frac{d\gamma_i(s)}{dL^A_i(s)} \left[L_{i'i}(s) + (\gamma_i(s) - 1) \frac{\partial L_{i'i}(s)}{\partial \gamma_i(s)} \right] f(s).$$

 From here, the result follows given that

$$\frac{dL_{i'i}(s)}{d\gamma_i(s)} = \frac{-\frac{\partial d^*_i(s)}{\partial \gamma_i(s)} h_i(s)\gamma_j(s) - (D_{i'i} - d^*_i(s)I_{i'i})h_i(s)}{(h_i(s)\gamma_i(s))^2} = \frac{-h_i(s)d^*_i(s)I_{i'i} - (D_{i'i} - d^*_i(s)I_{i'i})h_i(s)}{h_i(s)\gamma_i(s)} = \frac{-D_{i'i}}{h_i(s)\gamma_i(s)}.$$

All that remains is to characterize the total derivative of the equilibrium price in liquidations, which we had denoted $\frac{d\gamma_i(s)}{d\varepsilon}$. This total derivative is non-trivial because liquidations are now endogenous to the price via the collateral constraint. In particular, we can characterize this impact by defining

---

[69]Note that the boundary term from Leibniz rule is zero since liquidations are zero at the boundary.

$L_i^{A,\varepsilon}(s) = L_{ii}(s) + \int_j L_{ij}(s)ds + \varepsilon$ and by totally differentiating the equilibrium price relationship $\gamma_i(s) = \frac{\partial \mathcal{F}(L_i^A(s),s)}{\partial L_i^A(s)}$ in ε around $\varepsilon = 0$. Evaluating this total derivative, we obtain

$$\frac{d\gamma_i(s)}{d\varepsilon} = \frac{\frac{\partial^2 \mathcal{F}_i(s)}{\partial L_i(s)^2}}{1 - \frac{\partial^2 \mathcal{F}_i(s)}{\partial L_i(s)^2}\left[\frac{\partial L_{ii}(s)}{\partial \gamma_i(s)} + \int_j \frac{\partial L_{ij}(s)}{\partial \gamma_i(s)}dj\right]} = \frac{\frac{\partial^2 \mathcal{F}_i(s)}{\partial L_i(s)^2}}{1 - \frac{\partial^2 \mathcal{F}_i(s)}{\partial L_i(s)^2}D_i^A(s)}.$$

where the last line follows from recalling that $\frac{dL_{i'i}(s)}{d\gamma_i(s)} = \frac{-D_{i'i}}{h_i(s)\gamma_i(s)}$.

The result for τ_{ji}^I then follows in the same manner, with the only difference being the liquidation impact is instead $\frac{\partial L_{ji}(s)}{\partial I_{ji}} = \frac{-d_i^*(s)}{h_i(s)\gamma_i(s)}$ across states $s \in S_{ji}^D$.

A.8 Proof of Proposition 7

To begin with, starting from the Lagrangian of bank j, we have wedges given by

$$\tau_{i,ji}^D = -\tau_{j,ji}^D + \frac{1}{\lambda_j}\left[-1 + \mathbb{E}\left[(\gamma_i(s) - 1)\frac{\partial L_{ji}(s)}{\partial D_{ji}(s)}\right]\right]$$

$$\tau_{i,ji}^I = -\tau_{j,ji}^I + \frac{1}{\lambda_j}\left[E[R_i] + \mathbb{E}\left[(\gamma_i(s) - 1)\frac{\partial L_{ji}(s)}{\partial I_{ji}}\right] - \lambda_j \frac{\partial \Phi_{ji}}{\partial I_{ji}}\right]$$

Once again, starting from a desired allocation (D_{ji}, I_{ji}), the planner of country i sets wedges according to the above in order to implement this allocation. Note that planner i takes as given the Lagrange multipliers. Now that we know the implementing wedges, we can solve the problem of country planner i as before. As in the proof of Proposition 4, we can internalize both the liquidation price and wedges to obtain a Lagrangian $\mathcal{L}_i + \lambda_i^0 \Pi_i$, where

$$\Pi_i = \int_{i'}\left[\tau_{i,i'i}^D D_{i'i} + \tau_{i,i'i}^I I_{i'i}\right].$$

The steps are the same as in the proof of Proposition 4. Starting by characterizing the revenue derivative from a change in the equilibrium price, we have

$$\frac{\partial \Pi_i}{\partial \gamma_i(s)} = \int_{i'} \frac{\partial\left[\tau_{i,i'i}^D D_{i'i} + \tau_{i,i'i}^I I_{i'i}\right]}{\partial \gamma_i(s)}di'.$$

Taking the derivative, we have

$$
\begin{aligned}
\frac{\partial[\tau^D_{i,ji}D_{ji}+\tau^I_{i,ji}I_{ji}]}{\partial\gamma_i(s)} &= \frac{\partial}{\partial\gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\left[(\gamma_i(s)-1)\frac{\partial L_{i'i}(s)}{\partial D_{i'i}(s)}D_{i'i}+(\gamma_i(s)-1)\frac{\partial L_{i'i}(s)}{\partial I_{i'i}(s)}I_{i'i}\right] \\
&= \frac{\partial}{\partial\gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\left[(\gamma_i(s)-1)\left(\frac{1}{h_i(s)\gamma_i(s)}D_{i'i}-\frac{d_i^*(s)}{h_i(s)\gamma_i(s)}I_{i'i}\right)\mathbf{1}_{s\in S^D_{i'i}}\right] \\
&= \frac{\partial}{\partial\gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\left[(\gamma_i(s)-1)L_{i'i}(s)\right]
\end{aligned}
$$

where the last line follows from the collateral constraint and from noting that $L_{i'i}=0$ for $s\notin S^D_{i'i}$. As such, again noting that the Leibniz boundary term drops out because $L_{i'i}=0$ at the boundary, we have

$$
\frac{\partial[\tau^D_{i,ji}D_{ji}+\tau^I_{i,ji}I_{ji}]}{\partial\gamma_i(s)} = \frac{1}{\lambda_{i'}}\mathbb{E}\left[\frac{\partial}{\partial\gamma_i(s)}\left[(\gamma_i(s)-1)L_{i'i}(s)\right]\right]
$$

which is the spillover term from above.

From here, efficient setting of domestic wedges on domestic banks follows as in the baseline model. What remains is wedge setting on foreign banks. The first order condition for choice $D_{i'i}$ of debt by foreign banks is given by

$$
0=\mathbb{E}\left[\frac{\partial\mathcal{L}_i}{\partial\gamma_i(s)}\frac{d\gamma_i(s)}{dD_{i'i}}+\lambda_i^0\frac{d\Pi_i}{d\gamma_i(s)}\frac{d\gamma_i(s)}{dD_{i'i}}\right]+\lambda_i^0\frac{\partial[\tau^D_{i,i'i}D_{i'i}+\tau^I_{i,i'i}I_{i'i}]}{\partial D_{i'i}}.
$$

As just shown above, the revenue derivative captured foreign spillovers. Similar to the steps above, the direct derivative of revenue from bank i' is

$$
\frac{\partial[\tau^D_{i,i'i}D_{i'i}+\tau^I_{i,i'i}I_{i'i}]}{\partial D_{i'i}} = \frac{1}{\lambda_{i'}}\frac{\partial}{\partial D_{i'i}}\left[-D_{i'i}+\mathbb{E}[(\gamma_i(s)-1)L_{i'i}(s)]\right]=\frac{1}{\lambda_{i'}}\left[-1+\mathbb{E}[(\gamma_i(s)-1)\frac{\partial L_{i'i}(s)}{\partial D_{i'i}}]\right]=\tau^D_{i,i'i},
$$

giving that the wedge on debt is set efficiently.

Finally, we need to characterize the wedge on investment. From the same steps,

$$
0=\mathbb{E}\left[\frac{\partial\mathcal{L}_i}{\partial\gamma_i(s)}\frac{d\gamma_i(s)}{dI_{i'i}}+\lambda_i^0\frac{d\Pi_i}{d\gamma_i(s)}\frac{d\gamma_i(s)}{dI_{i'i}}\right]+\lambda_i^0\frac{\partial[\tau^D_{i,i'i}D_{i'i}+\tau^I_{i,i'i}I_{i'i}]}{\partial I_{i'i}},
$$

and from here we have

$$\frac{\partial[\tau_{i,i'i}^D D_{i'i} + \tau_{i,i'i}^I I_{i'i}]}{\partial I_{i'i}} = \frac{1}{\lambda_{i'}} \frac{\partial}{I_{i'i}} \left[E[R_i]I_{i'i} + \mathbb{E}[(\gamma_i(s) - 1)L_{i'i}(s)] - \lambda_{i'} \frac{\partial \Phi_{i'i}}{\partial I_{i'i}} I_{i'i} \right]$$

$$= \frac{1}{\lambda_{i'}} \left[E[R_i] + \mathbb{E}[(\gamma_i(s) - 1)\frac{\partial L_{i'i}(s)}{\partial I_{i'i}}] - \lambda_{i'} \frac{\partial \Phi_{i'i}}{\partial I_{i'i}} \right] - \frac{\partial^2 \Phi_{i'i}}{\partial I_{i'i}^2} I_{i'i}$$

$$= \tau_{i,i'i}^I - \frac{\partial^2 \Phi_{i'i}}{\partial I_{i'i}^2} I_{i'i}$$

so that once again, efficiency requires $\frac{\partial^2 \Phi_{i'i}}{\partial I_{i'i}^2} = 0$.

A.9   Proof of Proposition 8

The Lagrangian of the global planner is

$$\mathcal{L}^G = \int_i \left[\omega_i U_i \left(u_i(a_i), u_i^A(a_i, a^A)) \right) + \Lambda_i \Gamma_i \left(A_i + \mathcal{T}_i, \phi_i(a_i), \phi_i^A(a_i, a^A) \right) \right] di - \lambda^0 \int_i \mathcal{T}_i di.$$

From here, we have

$$\frac{d\mathcal{L}^G}{da_{ij}(m)} = \frac{\partial \mathcal{L}_i}{\partial a_{ij}(m)} + \frac{\partial \mathcal{L}_j}{\partial a_j^A(m)} + \int_{i'} \frac{\partial \mathcal{L}_{i'}}{\partial a_j^A(m)} di'$$

so that we obtain the required wedge

$$\tau_{ij}(m) = -\frac{1}{\lambda_i^0} \left[\frac{\partial \mathcal{L}_j}{\partial a_j^A(m)} + \int_{i'} \frac{\partial \mathcal{L}_{i'}}{\partial a_j^A(m)} di' \right]$$

where we define $\lambda_i^0 \equiv \Lambda_i \frac{\partial \Gamma_i}{\partial W_i}$. Next, we can characterize the derivative

$$\frac{\partial \mathcal{L}_i}{\partial a_j^A(m)} = \omega_i \frac{\partial U_i}{\partial u_i^A} \frac{\partial u_{ij}^A}{\partial a_j^A(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A} \frac{\partial \phi_{ij}^A}{\partial a_j^A(m)}.$$

Finally, defining $\Omega_{i,j}(m) = \frac{1}{\lambda_i^0} \frac{\partial \mathcal{L}_i}{\partial a_j^A(m)}$ and using that $\lambda^0 = \lambda_i^0$ (from the FOC for $\mathcal{T}_i$), we obtain

$$\tau_{ij}(m) = -\Omega_{j,j}(m) - \int_{i'} \Omega_{i',j}(m) di'$$

giving the result.

B   Section 5 Appendix

This appendix presents formal results underlying results and discussion in Section 5 that were not presented in the main text.

B.1   Baseline Model

In Section 5, we presented the global optimum (Proposition 6) as well as the Pigouvian efficiency result (Proposition 7). The following result characterizes the non-cooperative equilibrium under quantity regulation.

Proposition 11. *The non-cooperative equilibrium under quantity regulation has the following features.*

1. *Domestic regulation of domestic banks is given by*

$$\tau_{ii}^D = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\Omega_{ii}(s) \frac{1}{h_i(s)\gamma_i(s)} \bigg| s \in S_{ji}^D\right]$$

$$\tau_{ii}^I = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\Omega_{ii}(s) \frac{-d_i^*(s)}{h_i(s)\gamma_i(s)} \bigg| s \in S_{ji}^D\right]$$

where the domestic cost of liquidations $\Omega_{ii}(s)$ is

$$\Omega_{ii}(s) = \left|\frac{\partial \gamma_i(s)}{\partial \varepsilon}\right| \cdot \frac{1}{h_i(s)\gamma_i(s)} \left[\frac{1}{\gamma_i(s)} D_{ii} - d_i^*(s)I_{ii}\right] \cdot \mathbf{1}_{s \in S_{ii}^D}$$

2. *Domestic regulation of foreign banks achieves an allocation rule $D_{ji} \leq d_i^* I_{ij}$, that is domestic regulation of foreign banks prevents foreign banks from liquidating the domestic asset.*

B.1.1   Proof of Proposition 11

The proof follows the same steps as the proof of Proposition 7, except that optimal allocations no longer include terms related to revenue derivatives. Hence, optimal rules are simply those without revenue derivatives. Domestic regulation of domestic banks only accounts for domestic spillovers (omitting foreign spillovers, which arise from the revenue derivative). Similarly, the allocation rule

for foreign banks only features the domestic spillover that arises from foreign subsidiary distress, and hence sets $D_{ji} \leq d_i^* I_{ji}$.

B.2 Liquidity Regulation

We now present the formal results for cooperative and non-cooperative policies in Section 5.3, where we have incorporated liquid assets and liquidity regulation. Recall the modified definition of the distress region.

Proposition 12. *In the model with liquidity regulation,*

1. *The globally efficient allocation can be decentralized using wedges*

$$\tau_{ji}^D = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\tau_i^L(s) \frac{1}{h_i(s)\gamma_i(s)} \middle| s \in S_{ji}^D\right]$$

$$\tau_{ji}^T = -\tau_{ji}^D$$

$$\tau_{ji}^I = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\tau_i^L(s) \frac{-d_i^*(s)}{h_i(s)\gamma_i(s)} \middle| s \in S_{ji}^D\right]$$

where the total social cost $\tau_i^L(s) \geq 0$ of liquidations in country i in state s is

$$\tau_i^L(s) = \left| \frac{d\gamma_i(s)}{dL_i^A(s)} \right| \cdot \frac{1}{h_i(s)\gamma_i(s)} \left[\frac{1}{\gamma_i(s)} \left(D_i^A(s) - T_i^A(s) \right) - d_i^*(s)I_i^A(s) \right],$$

where the total price impact $\frac{d\gamma_i(s)}{dL_i^A(s)}$ is defined in the proof.

2. *Under non-cooperative quantity regulation,*

   (a) *Domestic regulation of domestic banks is given by*

   $$\tau_{ii}^D = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\Omega_{ii}(s) \frac{1}{h_i(s)\gamma_i(s)} \middle| s \in S_{ji}^D\right]$$

   $$\tau_{ii}^T = -\tau_{ii}^D$$

   $$\tau_{ii}^I = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\Omega_{ii}(s) \frac{-d_i^*(s)}{h_i(s)\gamma_i(s)} \middle| s \in S_{ji}^D\right]$$

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where the domestic cost of liquidations $\Omega_{ii}(s)$ is

$$\Omega_{ii}(s) = \left| \frac{\partial \gamma_i(s)}{\partial \varepsilon} \right| \cdot \frac{1}{h_i(s)\gamma_i(s)} \left[\frac{1}{\gamma_i(s)}[D_{ii} - T_{ii}] - d_i^*(s)I_{ii} \right] \cdot \mathbf{1}_{s \in S_{ii}^D}$$

(b) *Domestic regulation of foreign banks achieves an allocation rule $D_{ji} \leq d_i^* I_{ij} + T_{ij}$, that is domestic regulation of foreign banks prevents foreign banks from liquidating the domestic asset.*

3. *Suppose that for all i and $j \neq i$, $\frac{\partial^2 \Phi_{ij}}{\partial I_{ij}^2} = \frac{\partial^2 \Phi_{ij}}{\partial T_{ij}^2} = \frac{\partial^2 \Phi_{ij}}{\partial T_{ij} \partial I_{ij}} = 0$. Then, the non-cooperative equilibrium under Pigouvian taxation is globally efficient. There is no scope for cooperation.*

B.2.1 Proof of Propositions 12

The proofs follow the same steps as the proofs of Propositions 6, 7, and 11, and we highlight here only the differences. First, in evaluating the spillover effect (Proposition 6, we now have instead

$$\frac{dL_{i'i}(s)}{d\gamma_i(s)} = \frac{-\frac{\partial d_i^*(s)}{\partial \gamma_i(s)} h_i(s)\gamma_j(s) - (D_{i'i} - T_{i'i} - d_i^*(s)I_{i'i})h_i(s)}{(h_i(s)\gamma_i(s))^2} = \frac{-D_{i'i} - T_{i'i}}{h_i(s)\gamma_i(s)},$$

from which the social cost $\tau_i^L(s)$ follows. From here, the globally optimal wedges τ_{ji}^D and τ_{ji}^I follow exactly as before, which the liquidation wedge

$$\tau_{ji}^I = -\frac{1}{\lambda_j^0} \int_{s \in S_{ji}^D} \left[\frac{\partial L_{ji}(s)}{\partial T_{ji}(s)} \frac{\partial}{\partial L_i^A(s)} \int_{i'} \mathcal{L}_{i'} di' \right] f(s) ds = \frac{1}{\lambda_j^0} \int_{s \in S_{ji}^D} \left[\frac{\partial L_{ji}(s)}{\partial D_{ji}(s)} \frac{\partial}{\partial L_i^A(s)} \int_{i'} \mathcal{L}_{i'} di' \right] f(s) ds = -\tau_{ji}^D.$$

Next, turning to the non-cooperative problem, we have the same implementability conditions for debt and illiquid investment, as well as the additional implementability condition for liquid investment

$$\tau_{i,ji}^T = -\tau_{j,ji}^T + \frac{1}{\lambda_j} \left[1 + \mathbb{E} \left[(\gamma_i(s) - 1) \frac{\partial L_{ji}(s)}{\partial T_{ji}} \right] - \lambda_j \frac{\partial \Phi_{ji}}{\partial T_{ji}} \right]$$

Thus, taking the derivative we have

$$
\begin{aligned}
\frac{\partial[\tau_{i,ji}^D D_{ji} + \tau_{i,ji}^I I_{ji} + \tau_{i,ji}^T T_{ji}]}{\partial \gamma_i(s)} &= \frac{\partial}{\partial \gamma_i(s)} \frac{1}{\lambda_{i'}} \mathbb{E}\left[(\gamma_i(s) - 1) \left(\frac{\partial L_{i'i}(s)}{\partial D_{i'i}(s)} D_{i'i} + \frac{\partial L_{i'i}(s)}{\partial I_{i'i}(s)} I_{i'i} + \frac{\partial L_{i'i}(s)}{\partial T_{i'i}(s)} T_{i'i} \right) \right] \\
&= \frac{\partial}{\partial \gamma_i(s)} \frac{1}{\lambda_{i'}} \mathbb{E}\left[(\gamma_i(s) - 1) \left(\frac{1}{h_i(s)\gamma_i(s)} [D_{i'i} - T_{i'i}] - \frac{d_i^*(s)}{h_i(s)\gamma_i(s)} I_{i'i} \right) \mathbf{1}_{s \in S_{i'i}^D} \right] \\
&= \frac{\partial}{\partial \gamma_i(s)} \frac{1}{\lambda_{i'}} \mathbb{E}\left[(\gamma_i(s) - 1) L_{i'i}(s) \right]
\end{aligned}
$$

so that the revenue derivative results in internalizing foreign spillovers. From here, the remainder of the proof proceeds as before.

B.3 Cross-Border Support and Resolution

We now present the formal results for cooperative and non-cooperative policies in Section 5.4, where we have incorporated cross-border subsidiary support.

Proposition 13. *In the model with cross-border support,*

1. *The globally efficient allocation can be decentralized using wedges*

$$
\tau_{ji}^D = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\tau_i^L(s) \frac{1}{h_i(s)\gamma_i(s)} \,\middle|\, s \in S_{ji}^D \right]
$$

$$
\tau_{ji}^G(s) = -\tau_i^L(s) \frac{1}{h_i(s)\gamma_i(s)} \mathbf{1}_{s \in S_{ji}^D}
$$

$$
\tau_{ji}^I = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\tau_i^L(s) \frac{-d_i^*(s)}{h_i(s)\gamma_i(s)} \,\middle|\, s \in S_{ji}^D \right]
$$

where the total social cost $\tau_i^L(s) \geq 0$ of liquidations in country i in state s is

$$
\tau_i^L(s) = \left| \frac{d\gamma_i(s)}{dL_i^A(s)} \right| \cdot \frac{1}{h_i(s)\gamma_i(s)} \left[\frac{1}{\gamma_i(s)} \left(D_i^A(s) - G_i^A(s) \right) - d_i^*(s) I_i^A(s) \right],
$$

where the total price impact $\frac{d\gamma_i(s)}{dL_i^A(s)}$ is defined in the proof.

2. *Under non-cooperative quantity regulation,*

(a) Domestic regulation of domestic banks is given by

$$\tau_{ii}^D = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\Omega_{ii}(s)\frac{1}{h_i(s)\gamma_i(s)}\,\middle|\,s \in S_{ji}^D\right]$$

$$\tau_{ii}^G(s) = -\Omega_{ii}(s)\frac{1}{h_i(s)\gamma_i(s)}\mathbf{I}_{s \in S_{ji}^D}$$

$$\tau_{ii}^I = \Pr(s \in S_{ji}^D) \cdot \mathbb{E}\left[\Omega_{ii}(s)\frac{-d_i^*(s)}{h_i(s)\gamma_i(s)}\,\middle|\,s \in S_{ji}^D\right]$$

where the domestic cost of liquidations $\Omega_{ii}(s)$ is

$$\Omega_{ii}(s) = \left|\frac{\partial \gamma_i(s)}{\partial \varepsilon}\right| \cdot \frac{1}{h_i(s)\gamma_i(s)}\left[\frac{1}{\gamma_i(s)}\left(D_{ii} - G_{ii}(s)\right) - d_i^*(s)I_{ii}\right] \cdot \mathbf{1}_{s \in S_{ii}^D}$$

(b) Domestic regulation of foreign banks achieves an allocation rule $D_{ji} \le \inf_{s \in S}\{d_i^*(s)I_{ji} + G_{ji}(s)\}$, that is domestic regulation of foreign banks prevents foreign banks from liquidating the domestic asset.

3. *Suppose that for all i and $j \ne i$, $\frac{\partial^2 \Phi_{ij}}{\partial I_{ij}^2} = 0$. Then, the non-cooperative equilibrium under Pigouvian taxation is globally efficient. There is no scope for cooperation.*

B.3.1 Proof of Propositions 13

The proofs follow the same steps as the proofs of Propositions 6, 7, and 11. We again highlight the key differences. Evaluating the spillover effect (Proposition 6, we now have instead

$$\frac{dL_{i'i}(s)}{d\gamma_i(s)} = \frac{-D_{i'i} - G_{i'i}(s)}{h_i(s)\gamma_i(s)},$$

from which the social cost $\tau_i^L(s)$ follows. From here, the globally optimal wedges τ_{ji}^D and τ_{ji}^I follow exactly as before, which the liquidation wedge

$$\tau_{ji}^I = -\frac{1}{\lambda_j^0}\frac{\partial L_{ji}(s)}{\partial G_{ji}(s)}\frac{\partial}{\partial L_i^A(s)}\int_{i'} \mathcal{L}_{i'}di' = -\tau_i^L(s)\frac{1}{h_i(s)\gamma_i(s)}\mathbf{1}_{s \in S_{ji}^D}.$$

Next, turning to the non-cooperative problem, we have the same implementability conditions

for debt and illiquid investment, as well as the additional implementability condition for cross-border support

$$\tau_{i,ji}^G(s) = -\tau_{j,ji}^G(s) + \frac{1}{\lambda_j}\left[1 + (\gamma_i(s) - 1)\frac{\partial L_{ji}(s)}{\partial G_{ji}(s)}\right] - \mu_j(s)\right]$$

where $\mu_j(s)$ is the Lagrange multiplier on the subsidiary support budget constraint. Thus, taking the derivative we have

$$\frac{\partial[\tau_{i,ji}^D D_{ji} + \tau_{i,ji}^I I_{ji} + \tau_{i,ji}^T T_{ji}]}{\partial \gamma_i(s)} = \frac{\partial}{\partial \gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\left[(\gamma_i(s) - 1)\left(\frac{\partial L_{i'i}(s)}{\partial D_{i'i}(s)}D_{i'i} + \frac{\partial L_{i'i}(s)}{\partial I_{i'i}(s)}I_{i'i} + \frac{\partial L_{i'i}(s)}{\partial G_{i'i}(s)}G_{i'i(s)}\right)\right]$$

$$= \frac{\partial}{\partial \gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\left[(\gamma_i(s) - 1)\left(\frac{1}{h_i(s)\gamma_i(s)}[D_{i'i} - G_{i'i}(s)] - \frac{d_i^*(s)}{h_i(s)\gamma_i(s)}I_{i'i}\right)\mathbf{1}_{s \in S_{i'i}^D}\right]$$

$$= \frac{\partial}{\partial \gamma_i(s)}\frac{1}{\lambda_{i'}}\mathbb{E}\left[(\gamma_i(s) - 1)L_{i'i}(s)\right]$$

so that the revenue derivative results in internalizing foreign spillovers. From here, the remainder of the proof proceeds as before.

B.4 Bailouts and Fiscal Backstops

We incorporate bailouts into the baseline macroprudential regulatory model of Section 5. We model bailouts as ex ante lump sum transfer commitments $T_{ij}^1(s) \geq 0$, which provides a tractable way of representing the various possible bailout instruments.[70] At date 1, the country-level liquid net worth of a bank is therefore $A_{ij}^1(s) = -D_{ij} + T_{ij}^1(s)$, which may be negative. Notice that because bailouts are state contingent whereas debt is non-contingent, macroprudential regulation that reduces debt issuance is not a perfect substitute for bailouts. The country level collateral constraint now generates a liquidation rule $L_{ij}(s) = \frac{1}{h_j(s)\gamma_j(s)}\max\left\{D_{ij} - T_{ij}^1(s) - d_j^*(s)I_{ij}, 0\right\}$. Bailouts are financed by domestic taxpayers, with a utility cost $V_i^T(\mathcal{T}_i)$ of tax revenue collections.[71] Country planners trade state-contingent claims on taxpayer revenue, yielding a tax-bailout budget constraint

$$\int_s \left[T_{i,ii}^1(s) + \int_j T_{i,ji}^1(s) + \int_j T_{i,ij}^1(s)\right]f(s)ds \leq G_i + \mathcal{T}_i \tag{19}$$

---

[70]Although in theory fiscal backstops such as deposit insurance and LOLR rule out bad equilibria without being used on the equilibrium path, in practice these measures often are associated with undesirable transfers and moral hazard.

[71]See Appendix B.5.1 for a foundation.

where $T_{i,ii}^1(s) + \int_j T_{i,ji}^1(s) + \int_j T_{i,ij}^1(s)$ is required revenue for bailouts in state s. G_i is an existing inter-country tax revenue claim, with $\int_i G_i di = 0$, which we use in decentralizing the cooperative outcome. The ability for country planners to trade contingent bailout claims means that they could in principle implement a "common fiscal backstop" via trading of claims in a decentralized manner, that is they have the same set of tools that a common fiscal authority would have.

B.4.1 Globally Efficient Policies

We characterize the globally efficient bailout policies, and discuss the non-cooperative bailout rules. The formal characterization of globally efficient regulation and non-cooperative policies are contained in Appendix B.5.

Globally Efficient Bailouts. The global planning problem places welfare weights ω_i on countries and relative welfare weights ω_i^T on taxpayers. The following proposition characterizes the globally efficient bailout rule.[72]

Proposition 14. *The globally efficient bailout rule for* $T_{ij}^1(s)$ *is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B_{ij}^1(s)}_{\text{Bank Benefit}} + \underbrace{\Omega_{j,j}^B(s) \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)}}_{\text{Domestic Spillovers}} + \underbrace{\int_{i'} \Omega_{i',j}^B(s) di' \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)}}_{\text{Foreign Spillovers}} \tag{20}$$

where the terms B_{ij}^1, $\Omega_{j,j}^B$, *and* $\Omega_{i',j}$ *are defined in the proof.*

The globally efficient bailout rules trade off the marginal cost of the bailout to taxpayers against both the direct benefit to the bank receiving the bailout, and the spillover benefits from reduced liquidations and fire sales. As in the baseline regulatory problem, globally efficient policy considers the complete set of spillovers when designing bailouts. There is equal bailout treatment in the sense that domestic and foreign banks that have the same benefit $B_{ij}^1(s)$ and the liquidation responsiveness $\frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)}$ from the bailout face the same marginal bailout rule.

Discussion.

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[72]The Appendix also characterizes cooperative regulation, optimal tax collection, and an irrelevance result for bailout sharing rules.

Bailout Home Bias.   Our model predicts that country planners provide stronger backstops for domestic banks and domestic operations. There are several examples of home bias in deposit insurance, including: US deposit insurance not applying to foreign branches of US banks; Iceland's decision not to honor deposit guarantee obligations to UK depositors after its despoit guarantee scheme was breached; and EU policies against deposit insurance discrimination by nationality.[73]

Common LOLR and Common Deposit Insurance in the EU.   Our model predicts overly weak fiscal backstops. This coincides with the EU motivation for Common Deposit Insurance, whose purpose is to "increase the resilience of the Banking Union against future crises" (European Commision (2015)). It further coincides the ECB acting as a common LOLR to the European Union.

Asymmetric Contributions to Backstops.   Concerns may arise about sharing a fiscal backstop if countries benefit asymmetrically from it, for example if some countries are net contributors while others are net recipients.[74] Proposition 14 implies that asymmetric bailouts can be optimal if it mitigates domestic fire sales and so promotes cross-border financial integration.

B.4.2   Non-Cooperative Quantity Regulation

We now characterize the optimal bailout rules that arise in under non-cooperative quantity regulation. Regulatory policy under non-cooperative quantity regulation in fact takes the same form as Proposition 11 (up to the modified definition of the distress region), which is shown formally in the proof.

Proposition 15. *The bailout rules in the non-cooperative equilibrium under quantity regulation are as follows.*

---

[73]See 78 FR 56583; "Iceland Triumphs in Icesave court battle," *Financial Times*, January 28, 2013; and European Commision (2015)

[74]For example, Iceland had difficulty servicing its backstop because its banking system was large relative to taxpayer basis.

1. *The optimal bailout rule for the domestic operations of a domestic bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B_{ii}^1(s)}_{\text{Bank Benefit}} + \underbrace{\Omega_{i,i}^B(s) \frac{\partial L_{ii}(s)}{\partial A_{ii}^1(s)}}_{\text{Domestic Spillovers}} \tag{21}$$

2. *The optimal bailout rule for the foreign operations of a domestic bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B_{ij}^1(s)}_{\text{Bank Benefit}} \tag{22}$$

3. *The optimal bailout rule for the domestic operations of a foreign bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{\Omega_{i,i}^B(s) \frac{\partial L_{ji}(s)}{\partial A_{ji}^1(s)}}_{\text{Domestic Spillovers}} \tag{23}$$

Three factors govern the non-cooperative bailout rules: the social cost of taxes, the direct benefit of bailouts to banks, and the domestic fire sale spillover. When choosing bailouts of domestic activities of domestic banks, the domestic planner considers all three factors, but neglects spillover costs to foreign banks. Moreover, the domestic planner neglects the benefits of alleviating foreign fire sales when choosing bailouts of foreign activities of domestic banks, and neglects the benefits of the bailout transfer when choosing bailouts of domestic activities of foreign banks. Country planners are *home biased* in their bailout decisions, generally preferring to bail out domestic activities of domestic banks.

Relative to the globally efficient bailout rule, non-cooperative planners under-value all bailout activities, including bailouts of domestic activities of domestic banks, not accounting for either benefits or spillovers to foreign banks. The cooperative agreement increases bailouts of both domestic and foreign banks. Multilateral fire sale spillovers imply the need for multilateral bailout cooperation.

B.4.3 Non-Cooperative Pigouvian Taxation

We next consider non-cooperative taxation in the model. We first show how efficiency breaks down even under Pigouvian taxation, and then we show how efficiency can be restored by taxing banks for the bailouts they expect to receive.

Proposition 16. *Suppose that the monopolist distortion is 0. Then, non-cooperative optimal taxation is as follows.*

1. *Domestic taxes on domestic banks' domestic activities are*

$$\tau^D_{i,ii} = E\left[\left[\Omega^B_{i,i}(s) + \int_{i'}\Omega^B_{i',i}(s)di' + \int_{i'}\Delta^T_{i'i}(s)T^1_{i'i}(s)di'\right]\frac{\partial L_{ii}(s)}{\partial A^1_{ii}(s)}\right] \tag{24}$$

$$\tau^I_{i,ii} = -E\left[\left[\Omega^B_{i,i}(s) + \int_{i'}\Omega^B_{i',i}(s)di' + \int_{i'}\Delta_{i'i}(s)T^1_{i'i}(s)di'\right]\frac{\partial L_{ii}(s)}{\partial I_{ii}}\right] \tag{25}$$

   where $\Delta^T_{ij}(s)$ is defined in the proof.

2. *Domestic taxes on foreign banks' domestic activities are*

$$\tau^D_{i,ji} = E\left[\left[\Omega^B_{i,i}(s) + \int_{i'}\Omega^B_{i',i}(s)di' + \int_{i'}\Delta^T_{i'i}(s)T^1_{i'i}(s)di'\right]\frac{\partial L_{ji}(s)}{\partial A^1_{ji}(s)}\right] \tag{26}$$

$$\tau^I_{i,ji} = -E\left[\left[\Omega^B_{i,i}(s) + \int_{i'}\Omega^B_{i',i}(s)di' + \int_{i'}\Delta^T_{i'i}(s)T^1_{i'i}(s)di'\right]\frac{\partial L_{ji}(s)}{\partial I_{ji}}\right] \tag{27}$$

Moreover, the optimal bailout rules for banks are the same as in Proposition 19, but with the spillover effects defined above.

Although the result here appears largely as in the baseline model, there is one substantive difference: the additional terms $\Delta_{i'i}(s)T^1_{i'i}(s)$ that arise in the revenue derivatives. These terms arise whenever there are bailouts by *some country* (not necessarily i) of domestic activities of foreign banks. This effect arises because bailout revenue is not a choice variable of private agents, but rather is an untaxed and unpriced action of governments. In absence of bailouts ($T^1_{ji}(s) = 0$), this term disappears and we revert to the effective characterizations in the first half of this paper. In other words, bailouts

lead to a violation of Assumption 9.[75]

Finally, we could consider the bailout rule for banks. The bailout rule for domestic banks is of the same form as in Proposition 19, except for the change in the spillover. Importantly, however, it is immediate to observe that the bailout rule for foreign banks does not consider the direct revenue benefit to banks from bailout revenue, because there is no tax on bailouts (i.e. it is not a private choice variable). As a result, cooperation is likely to be required over bailouts of cross-border banks even if non-cooperative Pigouvian taxation is able to achieve close-to-optimal internalization of spillovers. However, it is worthwhile to note that if the terms Δ_{ij}^T are close to zero, then Pigouvian taxation transforms the bailout problem to a *bilateral* problem, where the domestic planner simply neglects the benefit to foreign banks of receiving a bailout. Transforming the problem into a bilateral surplus problem, rather than a multilateral problem, may simplify cooperation over bailouts. For example, it may allow for simple agreements such as reciprocity on provision of deposit insurance and access to LOLR facilities.

B.4.4 Restoring Non-Cooperative Optimality With Bailout Levies

The above results imply that the existence of bailouts limits the ability for non-cooperative Pigouvian taxation to generate efficient policies. This failure arises because bailouts are not priced or otherwise optimally chosen by private banks. This implies that if bailouts *were* chosen by private banks, either explicitly or implicitly, we could restore efficiency.

Suppose that banks can in fact purchase bailout claims from the government, or alternatively that banks are charged ex ante for the bailout claims they will receive. In partciular, banks can purchase claims $T_{ij}^1(s) \geq 0$ at date 0, at a cost $\overline{q} > 1$ (i.e. the marginal cost of taxpayer funds). The first-order condition for bailout claim purchases in state s is

$$\tau_{j,ij}^T(s) = -\tau_{i,ij}^T(s) - \overline{q} + \frac{\lambda_i^1(s)}{\lambda_i^0}\left(1 + \left(\gamma_j(s) - 1\right)\frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)}\right). \tag{28}$$

Following the logic of previous sections, we have $\tau_{i,ij}^T = 0$, since banks now purchase bail out claims and since country i does not internalize impacts on foreign fire sales. As a result, domestic

---

[75]Note that the bailouts model features the nonlinear aggregates property of Appendix E.4, but that Assumption 9 is still the relevant assumption in that section.

planners never force banks to increase their backstop for foreign activities. On the other hand, the revenue that country planner j raises at date 0 from taxing the bailout purchases is of country i banks is $\tau^T_{j,ij}(s)T^1_{ij}(s)$. From here, it is easy to see that the efficiency results of the baseline model are restored. Planner j accounts for the direct benefits of bailouts, and also for the spillover costs.

The results of this section imply that *bailout* cooperation is also not necessary if it can be given a "market mechanism" and taxed. In practice, we could think about these taxes as corresponding to levies on banks for deposit insurance or access to lender of last resort, with the levies calibrated based on how much the bank expects to receive from them. Such levies are consistent with the fact that the Single Resolution Fund in the EU is funded by bank levies, and the Orderly Liquidity Fund in the US is designed to recoup expenditures from either the resolved bank or from other large financial institutions.[76]

The framework suggests that bailout policies are most naturally delegated to the *host* country, who can internalize the benefits and spillovers to foreign banks when using Pigouvian regulation combined with a market mechanism for bailouts. For example, this could correspond to a host country insuring the deposits of the local subsidiary of a foreign bank. This synergizes with other possible considerations, such as benefits to domestic depositors of deposit insurance, that might help to ensure that bailout policies are time consistent.

Time Consistency and Bailout Sharing. The results of this section assume that bailouts and bailout sharing rules are chosen ex ante with commitment. In practice, a key concern may be time consistency problems, where countries that ex post are obliged to send bailout funds to foreign countries renege on their international claims. If there are time consistency problems that prevent non-cooperative sharing of taxpayer funds, there may be a role for cooperation to enforce risk sharing agreements. However, the results of this section imply that the role of cooperation would be limited to enforcement of risk sharing, and would not need to specify the level of risk sharing.

Under Pigouvian taxation, bailout rules are still not efficient. The reason is that bailouts are chosen by governments, not by banks, so that there is not an equilibrium tax rate associated with them. This problem can be fixed if there is a mechanism in place to charge banks for the bailouts they expect to receive. Such a mechanism, which is effectively a Pigouvian tax on bailouts, restores

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[76]See https://srb.europa.eu/en/content/single-resolution-fund for the former, and US Department of Treasury (2018) for the latter.

efficiency, including over bailout rules. One example of such a mechanism would be a deposit insurance levy.

B.5 Bailouts Model: Additional Results

In this appendix, we provide the additional results from the bailouts section, including the characterization of taxpayers, the relevant implementability conditions, characterization of regulatory policy, and characterization of non-cooperative taxation.

B.5.1 Taxpayers

We provide a foundation for the reduced-form indirect utility function $V_i(\mathcal{T}_i)$ of tax revenue collections from taxpayers, and show a tax smoothing result.

A unit continuum of domestic taxpayers are born at date 1 with an endowment $\overline{T}_i^1(s)$ of the consumption good. Given tax collections $T_i^1(s) \leq \overline{T}_i^1(s)$, taxpayers enjoy consumption utility $u_i^T\left(\overline{T}_i^1(s) - T_i^1(s), s\right)$.[77] These tax collections generate total bailout revenue $\mathcal{T}_i = \int_s q(s)T_i^1(s)f(s)ds$.[78] We characterize the optimal tax collection problem of country planner i, who has decided to collect a total $\mathcal{T}_i$ in tax revenue for use in bailouts.

Lemma 17. *Taxpayer utility can be represented by the indirect utility function*

$$V_i^T(\mathcal{T}_i) = \int_s u_i\left(\overline{T}_i^1(s) - T_i^1(\mathcal{T}_i, s), s\right) f(s)ds \tag{29}$$

where $T_i^1(\mathcal{T}_i, s)$ is given by the tax smoothing condition

$$\frac{1}{q(s)}u_i'\left(\overline{T}_i^1(s) - T_i^1(s), s\right) = \frac{1}{q(s')}u_i'\left(\overline{T}_i^1(s') - T_i^1(s'), s'\right) \quad \forall s, s'.$$

Lemma 17 allows us to directly incorporate the indirect utility function $V_i^T(\mathcal{T}_i)$ into planner i preferences, and to use total revenue collected $\mathcal{T}_i$ as the choice variable. It implies that countries

---

[77]We impose $u_i^{T\prime}(0,s) = +\infty$. We think of u_i^T as incorporating both consumption preferences and distortionary effects of taxation.

[78]We could assume that the government pays a different price vector $\overline{q}$ for bailout claims, with derivations largely unchanged.

engage in tax smoothing without cooperation, but does not guarantee that they engage in the globally efficient level of bailouts.

Proof of Lemma 17. The optimization problem is

$$\max \omega_i^T \int_s u_i \left(\overline{T}_i^1(s) - T_i^1(s), s \right) f(s) ds \quad \text{s.t.} \quad \int_s q(s) T_i^1(s) f(s) ds \geq \mathcal{T}_i$$

The FOCs are

$$\omega_i^T \frac{\partial u_i \left(\overline{T}_i^1(s) - T_i^1(s), s \right)}{\partial c_i^T(s)} f(s) - \mu q(s) f(s) = 0$$

Combining the FOCs across states, we obtain the result.

B.5.2 Implementability Conditions

We characterize the implementability conditions for domestic allocations of foreign banks, in a manner analogous to the characterization in Lemma 2. Note that now, the domestic choice variables of foreign banks are (D_{ij}, I_{ij}).

Lemma 18. *Country planner j can directly choose all domestic allocations of foreign banks, with implementing wedges*

$$\tau_{j,ij}^I = -\tau_{i,ij}^I - \frac{\partial \Phi_{ij}}{\partial I_{ij}} + \frac{1}{\lambda_i^0} E \left[\lambda_i^1(s) \left((\gamma_j(s) - 1) \frac{\partial L_{ij}(s)}{\partial I_{ij}} + R_{ij}(s) \right) \right] \tag{30}$$

$$\tau_{j,ij}^D = -\tau_{i,ij}^D - E \left[\frac{\lambda_i^1(s)}{\lambda_i^0} \left(1 + (\gamma_j(s) - 1) \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} \right) \right] \tag{31}$$

Using Lemma 18, we can characterize the non-cooperative equilibrium in the same manner as the baseline model. In particular, we isolate the decision problem of the country i planner, who optimizes domestic bank welfare choosing domestic and foreign allocations, subject to domestic bank constraints and to the implementability conditions of Lemma 18, taking as given foreign planner wedges and foreign bailouts.

B.5.3 Non-Cooperative and Cooperative Regulation

We now characterize optimal non-cooperative regulation.

In the non-cooperative equilibrium, country planners choose both the wedges and the bailouts $T^1_{i,ij}(s)$, taking as given the wedges and bailouts of other countries, to maximize domestic social welfare

$$V^P_i = \omega_i \left[\int_s c_i(s) f(s) ds + \omega^T_i V^T_i (\mathfrak{T}_i) \right].$$

The following proposition describes optimal bailout policy under regulation.

Proposition 19. *The bailout rules in the non-cooperative equilibrium under quantity regulation are as follows.*

1. *The optimal bailout rule for the domestic operations of a domestic bank is*

$$\underbrace{\frac{\omega_i \omega^T_i}{\lambda^0_i} \left| \frac{\partial V^T_i}{\partial \mathfrak{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B^1_{ii}(s)}_{\text{Bank Benefit}} + \underbrace{\Omega^B_{i,i}(s) \frac{\partial L_{ii}(s)}{\partial A^1_{ii}(s)}}_{\text{Domestic Spillovers}} \tag{32}$$

2. *The optimal bailout rule for the foreign operations of a domestic bank is*

$$\underbrace{\frac{\omega_i \omega^T_i}{\lambda^0_i} \left| \frac{\partial V^T_i}{\partial \mathfrak{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B^1_{ij}(s)}_{\text{Bank Benefit}} \tag{33}$$

3. *The optimal bailout rule for the domestic operations of a foreign bank is*

$$\underbrace{\frac{\omega_i \omega^T_i}{\lambda^0_i} \left| \frac{\partial V^T_i}{\partial \mathfrak{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{\Omega^B_{i,i}(s) \frac{\partial L_{ji}(s)}{\partial A^1_{ji}(s)}}_{\text{Domestic Spillovers}} \tag{34}$$

Three factors govern the non-cooperative bailout rules: the social cost of taxes, the direct benefit of bailouts to banks, and the domestic fire sale spillover. When choosing bailouts of domestic activities of domestic banks, the domestic planner considers all three factors, but neglects spillover costs to foreign banks. Moreover, the domestic planner neglects the benefits of alleviating foreign fire sales

when choosing bailouts of foreign activities of domestic banks, and neglects the benefits of the bailout transfer when choosing bailouts of domestic activities of foreign banks. Country planners are *home biased* in their bailout decisions, generally preferring to bail out domestic activities of domestic banks.

Relative to the globally efficient bailout rule, non-cooperative planners under-value all bailout activities, including bailouts of domestic activities of domestic banks, not accounting for either benefits or spillovers to foreign banks. The cooperative agreement increases bailouts of both domestic and foreign banks. Multilateral fire sale spillovers imply the need for multilateral bailout cooperation.

Proposition 20. *Optimal non-cooperative regulation is given as follows.*

1. *Domestic taxes on domestic banks' domestic activities are given by*

$$\tau_{i,ii}^D = E\left[\Omega_{i,i}^B(s)\frac{\partial L_{ii}(s)}{\partial A_{ii}^1(s)}\right] \tag{35}$$

$$\tau_{i,ii}^I = -E\left[\Omega_{i,i}^B(s)\frac{\partial L_{ii}(s)}{\partial I_{ii}}\right] \tag{36}$$

   while other domestic taxes on domestic banks are zero. $\Omega_{i,i}^B(s)$ is defined in the proof.

2. *If there is an adverse price spillover $-\Omega_{i,i}^B(s) > 0$, then regulation of foreign banks is equivalent to a ban on foreign liquidations.*

To understand Proposition 20, the fact that liquidations are now determined indirectly, rather than directly, implies that the spillovers $\Omega_{i,i}^B(s)$ now form a basis to price the cost of policies that increase liquidations. This is reflected in the optimal tax rates.

At the same time, the domestic planner prefers sufficiently stringent regulation to prevent foreign banks from contributing to domestic fire sales. This is equivalent to requiring foreign banks to maintain domestic allocations that set $L_{ji}^L = 0$.

Next, we can characterize regulatory policy under the optimal cooperative agreement (global planning).

Proposition 21. *Optimal cooperative policy consists of taxes on investment scale and debt, given by*

$$\tau_{ij}^D = E\left[\left[\Omega_{j,j}^B(s) + \int_{i'}\Omega_{i',j}^B(s)di'\right]\frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)}\right] \tag{37}$$

$$\tau_{ij}^I = -E\left[\left[\Omega_{j,j}^B(s) + \int_{i'}\Omega_{i',j}^B(s)\right]\frac{\partial L_{ij}(s)}{\partial I_{ij}}\right] \tag{38}$$

The intuition of Proposition 21 is analogous to the intuition of Proposition 1. Globally optimal policy accounts for the full set of spillovers. Note that cooperative policy no longer features equal treatment in tax rates, to the extent that the responses of different banks' liquidation rules are different on the margin. There is equal treatment in the sense that the basis of spillover effects $\Omega_{i,j}(s)$ are the same, independent of which country generates the spillover.

Finally, we can characterize the optimal tax collection and bailout sharing rules of the cooperative agreement.

Proposition 22. *Globally optimal tax collection and bailout sharing are as follows.*

1. *Optimal cross-country bailout sharing is given by*

$$\omega_i \omega_i^T \frac{\partial V_i^T(\mathfrak{T}_i)}{\partial \mathfrak{T}_i} = \omega_j \omega_j^T \frac{\partial V_j^T(\mathfrak{T}_j)}{\partial \mathfrak{T}_j} \quad \forall i,j \tag{39}$$

2. *Any bailout sharing rule $\left(T_{i,ij}^1(s), T_{j,ij}^1(s)\right)$ satisfying $T_{ij}^1(s) = T_{i,ij}^1(s) + T_{j,ij}^1(s)$ can be used to implement the globally optimal allocation. Different bailout sharing rules differ in the initial distribution of tax revenue claims G_i. We can set $T_{i,i'j}^1(s) = 0$ whenever $i \notin \{i', j\}$ without loss of generality.*

The bailout sharing rule (39) implies that tax burdens of bailouts are smoothed across countries in an average sense but not state-by-state at date 1, so that some countries may be net contributors or recepients of bailouts in any given state s.[79] Bailout sharing rule irrelevance describes equivalent set of bailout sharing rules, and implies that in principle bailout obligations can be delegated entirely to one country (or to one international organization).[80]

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[79]For example, if countries have the same indirect utility functions and are equally weighted globally, then expected tax burdens are the same across countries.

[80]For example, the responsibility for deposit insurance can be entirely vested in a single entity (the host country,

B.6   Bailout Proofs

B.6.1   Proof of Lemma 18

The Lagrangian of the country i bank problem is given by

$$
\mathcal{L}_i = \int_s c_i(s)f(s)ds + \lambda_i^0 \left[A_i + D_i - T_i - \Phi_{ii}(I_{ii}) - \int_j \Phi_{ij}(I_{ij})dj \right]
$$

$$
+ \int_s \lambda_i^1(s) \left[A_i^1(s) + (\gamma_i(s) - 1)L_{ii}(s) + R_i(s)I_i + \int_j \left((\gamma_j(s) - 1)L_{ij}(s) + R_j(s)I_j \right) dj - c_i(s) \right] f(s)ds
$$

where we have implicitly internalized the demand liquidation function $L_{ij}(s) = \max\{0, -\frac{1}{h_j(s)\gamma_j(s)}A_{ij}^1(s) - \frac{(1-h_j(s))}{h_j(s)}R_j(s)I_{ij}(s)\}$. Taking the FOC is I_{ij} and rearranging, we obtain

$$
\tau_{j,ij}^I = -\tau_{i,ij}^I - \frac{\partial \Phi_{ij}}{\partial I_{ij}} + \frac{1}{\lambda_i^0}E\left[\lambda_i^1(s)\left((\gamma_j(s) - 1)\frac{\partial L_{ij}(s)}{\partial I_{ij}} + R_{ij}(s) \right) \right].
$$

Similarly, taking the FOC for $x_{ij}(s)$ and rearranging, we obtain

$$
\tau_{j,ij}^D = -\tau_{i,ij}^D - E\left[\frac{1}{\lambda_i^0}\lambda_i^1(s)\left(1 + (\gamma_j(s) - 1)\frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} \right) \right].
$$

B.6.2   Proof of Propositions 19 and 20

As in the baseline model, the implementing tax rates of Lemma 18 do not otherwise appear in the country i planning problem. These constraints simply determine these tax rates, for the chosen allocation.

Now, consider the decision problem of the country i planner. The only twist is that the liquidation discount is now given by the equation

$$
\gamma_i(s) = \gamma_i \left(L_{ii}(s) + \int_j L_{ji}(s)dj, s \right),
$$

where we have adopted the shorthand $\gamma_i = \frac{\partial \mathcal{F}_i}{\partial I_i^A}$. From here, we characterize the response of the liquidation price to an increase ε in total liquidations. Totally differentiating the above equation in

---

the home country, or an international deposit guarantee scheme). Once the bailout authority has been delegated to a single entity, the goal of the global planner will be to ensure that that entity chooses bailouts optimally. In practice, imperfectly controllable political economy distortions may lead to bailout funds being misused. See Foarta (2018).

total liquidations, we have

$$\frac{\partial \gamma_i(s)}{\partial \varepsilon} = \frac{\partial \gamma_i(s)}{\partial L_i^A(s)} \left[1 + \frac{\partial [L_{ii}(s) + \int_j L_{ji}(s) dj]}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \varepsilon} \right],$$

where $L_{ii}(s)$ and $L_{ij}(s)$ depend on $\gamma_i(s)$ due to the collateral constraint. Rearranging from here, we obtain the equilibrium country i price response

$$\frac{\partial \gamma_i(s)}{\partial \varepsilon} = \frac{1}{1 - \frac{\partial \gamma_i(s)}{\partial L_i^A(s)} \frac{\partial L_i^A(s)}{\partial \gamma_i(s)}} \frac{\partial \gamma_i(s)}{\partial L_i^A(s)}.$$

This characterization is useful, since externalities in this proof arise from changes in total liquidations.

Now, consider the Lagrangian of the country i planner. The Lagrangian of the planner can be written as

$$\mathcal{L}_i^{SP} = \mathcal{L}_i + \omega_i^T V_i^T(\mathfrak{I}_i) + \lambda_i^T \left[G_i + \mathfrak{I}_i - \int_s \left[T_{i,ii}^1(s) + \int_j T_{i,ji}^1(s) + \int_j T_{i,ij}^1(s) \right] f(s) ds \right]$$

where $\mathcal{L}_i$ internalizes the liquidation response and liquidation price relationships.

We first characterize the regulatory policies (Proposition 20), and then characterize the bailout policies (Proposition 19).

Regulatory Policies. Consider first the domestic allocations of domestic banks. For foreign allocations and consumption of the bank, the planner and bank derivatives coincide, and no wedges are applied, that is $\tau_{i,ij}^D = \tau_{i,ij}^I = 0$ for all $j \neq i$.

For domestic investment, the planner's derivative is

$$\frac{\partial \mathcal{L}_i^{SP}}{\partial I_{ii}} = \frac{\partial \mathcal{L}_i}{\partial I_{ii}} + \int_s \underbrace{\frac{\partial \mathcal{L}_i}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \varepsilon}}_{\equiv \lambda_i^0 \Omega_{i,i}^B(s) f(s)} \frac{\partial L_{ii}(s)}{\partial I_{ii}} ds$$

so that the domestic tax on domestic investment scale is given by

$$\tau_{i,ii}^I = -E \left[\Omega_{i,i}^B(s) \frac{\partial L_{ii}(s)}{\partial I_{ii}} \right],$$

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which is simply the expected spillover effect. Next, we can apply the same argument to taxes on domestic state-contingent securities D_{ii}. We have

$$\frac{\partial \mathcal{L}_i^{SP}}{\partial D_{ii}} = \frac{\partial \mathcal{L}_i}{\partial D_{ii}} + \int_s \frac{\partial \mathcal{L}_i}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \varepsilon} \frac{\partial L_{ii}(s)}{\partial D_{ii}}$$

so that the required tax rate is

$$\tau_{i,ii}^D = E\left[\Omega_{i,i}^B(s) \frac{\partial L_{ii}(s)}{\partial A_{ii}^1(s)}\right].$$

Finally, considering domestic allocations of foreign banks, we only have the price spillover effect. This implies that there is a liquidation ban whenever there is an adverse price spillover, $-\Omega_{i,i}^B(s) > 0$.

Note that we can formally characterize the spillover effect $\Omega_{i,j}^B(s)$ by evaluating

$$\frac{\partial \mathcal{L}_i}{\partial \gamma_j(s)} = \lambda_i^1(s)\left[L_{ij}(s) + (\gamma_j(s) - 1)\frac{\partial L_{ij}(s)}{\partial \gamma_j(s)}\right] f(s)$$

so that we have

$$\Omega_{i,j}^B(s) = \frac{\frac{\partial \mathcal{L}_i}{\partial \gamma_j(s)} \frac{\partial \gamma_j(s)}{\partial \varepsilon}}{\lambda_i^0 f(s)} = \frac{\lambda_i^1(s)}{\lambda_i^0}\left[L_{ij}(s) + (\gamma_j(s) - 1)\frac{\partial L_{ij}(s)}{\partial \gamma_j(s)}\right] \frac{\partial \gamma_j(s)}{\partial \varepsilon}.$$

Bailout Policies.   We next characterize the optimal bailout policies. Consider first the bailout rule for domestic activities of domestic banks, where we have

$$\frac{\partial \mathcal{L}_i^{SP}}{\partial T_{i,ii}^1(s)} = \underbrace{\frac{\partial \mathcal{L}_i}{\partial A_{ii}^1(s)}}_{\equiv \lambda_i^0 B_{ii}^1(s) f(s)} + \frac{\partial \mathcal{L}_i}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \varepsilon} \frac{\partial L_{ii}(s)}{\partial A_{ii}^1(s)} - \lambda_i^T f(s).$$

Now, the FOC for tax collection tells us that $\lambda_i^T = -\omega_i^T \frac{\partial V_i^T}{\partial \mathcal{T}_i}$. Noting that $\frac{\partial V_i^T}{\partial \mathcal{T}_i} < 0$, we rearrange and obtain the bailout rule

$$\frac{\omega_i^T}{\lambda_i^0}\left|\frac{\partial V_i^T}{\partial \mathcal{T}_i}\right| \geq B_{ii}^1(s) + \Omega_{i,i}^B(s)\frac{\partial L_{ii}(s)}{\partial A_{ii}^1(s)}.$$

The remaining two equations follow simply by noting that the spillover term does not appear in the FOC for bailouts of foreign activities of domestic banks, while the bank benefit term does not appear in the FOC for bailouts of domestic activities of foreign banks.

B.6.3 Proof of Propositions 21 and 22

The Lagrangian of the global planner is given by

$$\mathcal{L}_i^G = \int_i \left[\mathcal{L}_i + \omega_i^T V_i^T (\mathfrak{T}_i) + \lambda_i^T \left[G_i + \mathfrak{T}_i - \int_s \left[T_{i,ii}^1(s) + \int_j T_{i,ji}^1(s) + \int_j T_{i,ij}^1(s) \right] f(s) ds \right] + \int_i \left[\lambda^0 T_i + \lambda^T G_i \right] di$$

where the last terms reflect the set of lump sum transfers. The FOC for G_i implies $\lambda^T = \lambda_i^T$ while the FOC for T_i implies $\lambda^0 = \lambda_i^0$. From here, the regulation and bailout rules follow by the same steps as in the non-cooperative equilibrium, except that now the full set of spillovers appear, and the benefits to banks of bailouts are always accounted for.

Next, the relationship $\lambda^T = \lambda_i^T$ gives the tax sharing rule. Bailout irrelevance arises by setting $G_i = \int_s \left[T_{i,ii}^1(s) + \int_j T_{i,ji}^1(s) dj + \int_j T_{i,ij}^1(s) dj \right] dj - \mathfrak{T}_i$, for the desired bailout rule.

B.6.4 Proof of Propositions 16

The country planner Lagrangian is the same as under regulation, except that there is now also tax revenue collected from foreign banks.

The tax revenue collected by country j from country i banks is given by

$$T_{j,ij}^* = \tau_{j,ij} I_{ij} + \tau_{j,ij}^D D_{ij}$$

so that differentiating in total liquidations in state s, we have

$$\frac{\partial T_{j,ij}^*}{\partial \varepsilon} = \frac{\partial}{\partial \gamma_j(s)} \left[\frac{\lambda_i^1(s)}{\lambda_i^0} (\gamma_j(s) - 1) \left[\frac{\partial L_{ij}(s)}{\partial I_{ij}} I_{ij} - \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} D_{ij} \right] \right] \frac{\partial \gamma_j(s)}{\partial \varepsilon} f(s)$$

from here, we note that $L_{ij}(s)$ is homogeneous of degree 1 in $(I_{ij}, A_{ij}^1(s))$, given γ_j, so that we can write

$$\frac{\partial T_{j,ij}^*}{\partial \gamma_j(s)} = \frac{\partial}{\partial \gamma_j(s)} \left[\frac{\lambda_i^1(s)}{\lambda_i^0} (\gamma_j(s) - 1) \left[L_{ij}(s) - \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} T_{ij}^1(s) \right] \right] \frac{\partial \gamma_j(s)}{\partial \varepsilon} f(s)$$

$$= \Omega_{j,i}^B(s) f(s) + \Delta_{ij}^T(s) T_{ij}^1(s) f(s)$$

where we have defined $\Delta_{ij}^T(s) = -\frac{\partial}{\partial \gamma_j(s)} \left[\frac{\lambda_i^1(s)}{\lambda_i^0} \left(\gamma_j(s) - 1 \right) \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} T_{ij}^1(s) \right] \frac{\partial \gamma_j(s)}{\partial \varepsilon}$.

From here, results on regulation follow by the usual steps. Moreover, results on bailouts also follow the usual steps, noting the bailout has indirect effects on tax rates through the liquidation price, but does not have direct effects due to the linear nature of $L_{ij}(s)$.

C General Framework: Non-Cooperative

This Appendix presents the results of the non-cooperative problem in the general framework of Section 6.

C.1 Non-Cooperative Setup

The setup of the non-cooperative problem is analogous to the setup in the baseline model. Country planner i maximizes the welfare of domestic agents using a complete set of wedges $\tau_{i,i} = \{\tau_{i,ij}(m)\}_{j,m}$ on the actions of domestic agents, and wedges $\tau_{i,ji} = \{\tau_{i,ji}(m)\}_m$ on domestic actions of foreign agents. The total tax burden faced by country i agents from the domestic planner (excluding remissions) is therefore $T_{i,i} = \tau_{i,ii} a_{i,ii} + \int_j \tau_{i,ij} a_{i,ij} dj$, while the total tax burden from foreign planner j is given by $T_{j,ij} = \tau_{j,ij} a_{i,ij}$. These taxes appear in the wealth of the multinational agent.

As in the baseline model, under *quantity regulation* wedges are revenue-neutral, while under *Pigouvian taxation* wedges generate revenues from foreign banks.

Implementability. As in the baseline model, the approach to implementability is standard for domestic agents. Moreover, an implementability result analogous to Lemma 2 holds in the general environment, allowing us to apply the standard approach for domestic actions of foreign agents.

Lemma 23. *The domestic actions of foreign agents can be chosen by the domestic planner, with implementing wedges*

$$\tau_{i,ji}(m) = -\tau_{j,ji}(m) + \frac{1}{\lambda_j^0} \left[\omega_j \frac{\partial U_j}{\partial u_j} \frac{\partial u_{ji}}{\partial a_{ji}(m)} + \omega_j \frac{\partial U_j}{\partial u_j^A} \frac{\partial u_{ji}^A}{\partial a_{ji}(m)} + \Lambda_j \frac{\partial \Gamma_j}{\partial \phi_j} \frac{\partial \phi_{ji}}{\partial a_{ji}(m)} + \Lambda_j \frac{\partial \Gamma_j}{\partial \phi_j^A} \frac{\partial \phi_{ji}^A}{\partial a_{ji}(m)} \right]$$

where $\tau_{j,ji}$, λ_j^0, Λ_j, $\frac{\partial U_j}{\partial u_j}$, $\frac{\partial U_j}{\partial u_j^A}$, $\frac{\partial \Gamma_j}{\partial \phi_j}$, and $\frac{\partial \Gamma_j}{\partial \phi_j^A}$ are constants from the perspective of country planner i.

The intuition behind these implementability conditions is analogous to the baseline model: the planner first unwinds the wedge placed by the foreign planner, and then sets the residual wedge equal to the benefit to foreign agents of conducting that activity.

C.2 Non-Cooperative Quantity Regulation

We now characterize the non-cooperative equilibrium under quantity regulation, where wedges are revenue neutral. We obtain the following characterization of the non-cooperative equilibrium.

Proposition 24. *Under non-cooperative quantity regulation, the equilibrium has the following features.*

1. *The domestic wedges on domestic activities of domestic agents are*

$$\tau_{i,ii}(m) = -\Omega_{i,i}(m)$$

   while the domestic wedges on foreign activities of domestic agents are 0.

2. *The domestic wedges on foreign banks generate an allocation rule*

$$\Omega_{i,i}(m)a_{ji}(m) = 0$$

   so that foreign activities are allowed only up to the point they increase domestic welfare.

Proposition 24 reflects logic closely related to the baseline model. On the one hand, regulatory policies applied to domestic agents account for spillovers to domestic agents, but not to foreign agents. On the other hand, regulatory policies applied to foreign agents' domestic activities do not account for benefits to foreign agents of domestic activities. Foreign agents are allowed to conduct domestic activities only to the extent the *domestic* benefits of those activities outweigh *domestic* costs.

This characterization leads to a generic inefficiency result in the presence of cross-border activities. We say that there are *cross border activities* if $\exists M' \subset M$ and $I, J \subset [0,1]$ such that $a_{ii}(m), a_{ji}(m) > 0 \; \forall m \in M', i \in I, j \in J$.

Proposition 25. *Suppose that a globally efficient allocation features cross border activities over a triple (M', I, J). The non-cooperative equilibrium under quantity regulation generates this globally efficient allocation* only if *the globally efficient allocation features $\Omega_{i,i}(m) = \int_{i'} \Omega_{i',i}(m) di' = 0$ $\forall m \in M', i \in I$.*

Proposition 25 provides a strong and generic result that quantity regulation does not generate an efficient allocation when there are regulated cross-border activities. In particular, cross-border activities must generate no *net* domestic externality to avoid the problem of unequal treatment, and cross-border activities must generate no *net* foreign externalities to avoid the problem of uninternalized foreign spillovers. Notice that efficient under Proposition 25 requires no regulation of cross-border activities in the globally efficient policy.

C.3 Non-Cooperative Pigouvian Taxation

Finally, we characterize non-cooperative Pigouvian taxation and its optimality.

Proposition 26. *Suppose Assumption 9 holds. The equilibrium under non-cooperative Pigouvian taxation has the following features.*

1. *The domestic wedges on domestic activities of domestic agents are*

$$\tau_{i,ii}(m) = -\Omega_{i,i}(m) - \int_j \Omega_{j,i}(m) dj$$

   while domestic wedges on foreign activities of domestic agents are 0.

2. *The domestic wedges on domestic activities of foreign agents are*

$$\tau_{i,ji}(m) = \tau_{i,ii}(m) - \frac{\partial \tau_{i,ji}}{\partial a_{ji}(m)} a_{ji}$$

As in the baseline model, the derivatives of foreign tax revenue in domestic liquidation prices yield the foreign spillovers, so that planners account for these effects in designing policy. However, revenue collection generates a monopolistic distortion. The generalized problem therefore reflects

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the same logic as Proposition 4, with the only difference being the nature of the spillovers and of the monopolistic distortion. As in the baseline model, when this monopolist distortion is zero, non-cooperative Pigouvian taxation results in a globally efficient allocation.

Proposition 27. *Suppose Assumption 9 holds, and suppose that $u_{ij}, \phi_{ij}, u_{ij}^A, \phi_{ij}^A$ are linear in a_{ij} (given a_j^A) for all i and $j \neq i$. Then the non-cooperative equilibrium under taxation is globally efficient, and there is no scope for cooperation.*

Proof. Observe that when $u_{ij}, \phi_{ij}, u_{ij}^A, \phi_{ij}^A$ are linear in a_{ij} (given a_j^A) for all i and $j \neq i$, then the non-cooperative tax rates align with the cooperative ones, resulting in an efficient allocation. ∎

Non-cooperative taxation is globally efficient if Assumption 9 holds, and if the monopolistic distortions are zero. The assumption of linearity on $u_{ij}, \phi_{ij}, u_{ij}^A, \phi_{ij}^A$ ensures that (partial equilibrium) elasticities of foreign activities with respect to tax rates are infinite, so that monopolistic distortions are zero. This reflects the same notion of sufficient substitutability as in the baseline model, and generalizes Proposition 5 to a broader class of problems.

As in the baseline model, Proposition 27 provides a limiting case of exactly efficiency. Comparing Propositions 26 and 8 reveals that even without exact efficiency, there are three appealing properties of Pigouvian taxation. The first is that the need for cooperation is restricted to foreign activities of multinational agents. The second is that cooperation is needed only to correct bilateral monopolist problems. The third is that the information needed to determine the magnitude of these problems is a set of partial equilibrium elasticities. This provides a potential method to evaluate the need for cooperation in practice.

C.4 Proofs

C.4.1 Proof of Lemma 23

Taking the Lagrangian of bank i

$$\mathcal{L}_i = \omega_i U_i \left(u_i(a_i), u_i^A(a_i, a^A)) \right) + \Lambda_i \Gamma_i \left(A_i - T_i, \phi_i(a_i), \phi_i^A(a_i, a^A) \right)$$

and taking the first order condition in $a_{ij}(m)$, we obtain

$$0 = \omega_i \frac{\partial U_i}{\partial u_i} \frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i}{\partial u_i^A} \frac{\partial u_{ij}^A}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial W_i} \left(-\tau_{i,ij}(m) - \tau_{j,ij}(m) \right) + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i} \frac{\partial \phi_{ij}}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A} \frac{\partial \phi_{ij}^A}{\partial a_{ij}(m)}.$$

Defining $\lambda_i^0 = \Lambda_i \frac{\partial \Gamma_i}{\partial W_i}$ and rearranging, we obtain

$$\tau_{j,ij}(m) = -\tau_{i,ij}(m) + \frac{1}{\lambda_i^0} \left[\omega_i \frac{\partial U_i}{\partial u_i} \frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i}{\partial u_i^A} \frac{\partial u_{ij}^A}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i} \frac{\partial \phi_{ij}}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A} \frac{\partial \phi_{ij}^A}{\partial a_{ij}(m)} \right]$$

giving the relevant equation. From here, notice that the allocations a_{ij} and aggregates a_j^A appear to first order only in the tax rate equations in country j. As a result, considering any candidate equilibrium, the first order conditions for optimality for allocations by country i banks outside of country j are not affected (to first order) by policies in country j, and so continue to hold independent of a_{ij} and a_j^A. As a result, any allocation a_{ij} can be implemented with the above tax rates. The implementability result follows.

C.4.2 Proof of Proposition 24

Substituting in the equilibrium tax revenue, the optimization problem of the country i social planner is

$$\max_{a_i, \{a_{ji}\}} \omega_i U_i \left(u_i(a_i), u_i^A(a_i, a^A)) \right) \quad \text{s.t.} \quad \Gamma_i \left(A_i - \int_j \tau_{j,ij} a_{ij} dj, \phi_i(a_i), \phi_i^A(a_i, a^A) \right) \geq 0$$

and subject to the implementability conditions of Lemma 23. Note that the wedges rates $\tau_{i,ji}$ do not appear except in the implementability conditions, meaning that they are set to clear implementability but do not contribute to welfare. As a result, the Lagrange multipliers on implementability are 0, and the Lagrangian of planner i is given by

$$\mathcal{L}_i^{SP} = \omega_i U_i \left(u_i(a_i), u_i^A(a_i, a^A)) \right) + \Lambda_i \Gamma_i \left(A_i - \int_j \tau_{j,ij} a_{ij} dj, \phi_i(a_i), \phi_i^A(a_i, a^A) \right).$$

First of all, note that the social planner does not internalize impacts on foreign aggregates. As a result, $\frac{d\mathcal{L}_i^{SP}}{da_{ij}(m)} = \frac{\partial \mathcal{L}_i^{SP}}{\partial a_{ij}(m)}$. Social and private preferences align, and therefore we have $\tau_{i,ij}(m) = 0$.

Next, consider a domestic policy $a_{ii}(m)$. Here, we have $\frac{d\mathcal{L}_i^{SP}}{da_{ii}(m)} = \frac{\partial\mathcal{L}_i^{SP}}{\partial a_{ii}(m)} + \frac{\partial\mathcal{L}_i^{SP}}{\partial a_i^A(m)}$. To align preferences, the domestic planner therefore sets

$$\tau_{i,ii}(m) = -\frac{1}{\lambda_i^0}\frac{\partial\mathcal{L}_i^{SP}}{\partial a_i^A(m)} = -\Omega_{i,i}(m)$$

where the final equality follows as in the proof of Proposition 8.

Finally, consider $a_{ji}(m)$. Here, we have $\frac{d\mathcal{L}_i^{SP}}{da_{ji}(m)} = \frac{\partial\mathcal{L}_i^{SP}}{\partial a_i^A(m)} = \lambda_i^0\Omega_{i,i}(m)$, giving the allocation rule.

C.4.3 Proof of Proposition 25

Given a globally efficient allocation with cross border activites over (M', I, J), suppose that the non-cooperative equilibrium under quantity regulation generates this allocation. From Proposition 24, $a_{ji}(m) > 0$ implies that $\Omega_{i,i}(m) = 0$ over (M, I, J). Using Propositions 8 and 24, $\tau_{i,ii}(m) = \tau_{ii}(m)$ and $\Omega_{i,i}(m) = 0$ implies that $\int_{i'}\Omega_{i',i}(m)di' = 0$ over (M, I, J), completing the proof.

C.4.4 Proof of Proposition 26

It is helpful to begin by characterizing the derivative of revenue from foreign agents in the domestic aggregate. Using the implementability conditions of Lemma 23, the revenue collected by planner j from country i agents is

$$T_{j,ij}^* = \tau_{j,ij}a_{ij} = -\tau_{i,ij}a_{ij} + \frac{1}{\lambda_i^0}\left[\omega_i\frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}}a_{ij} + \omega_i\frac{\partial U_i}{\partial u_i^A}\frac{\partial u_{ij}^A}{\partial a_{ij}}a_{ij} + \Lambda_i\frac{\partial\Gamma_i}{\partial\phi_i}\frac{\partial\phi_{ij}}{\partial a_{ij}}a_{ij} + \Lambda_i\frac{\partial\Gamma_i}{\partial\phi_i^A}\frac{\partial\phi_{ij}^A}{\partial a_{ij}}a_{ij}\right].$$

Applying Assumption 9, $\frac{\partial u_{ij}^A}{\partial a_{ij}}a_{ij} = u_{ij}^A$ and $\frac{\partial\phi_{ij}^A}{\partial a_{ij}}a_{ij} = \phi_{ij}^A$, so that we obtain

$$T_{j,ij}^* = -\tau_{i,ij}a_{ij} + \frac{1}{\lambda_i^0}\left[\omega_i\frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}}a_{ij} + \omega_i\frac{\partial U_i}{\partial u_i^A}u_{ij}^A + \Lambda_i\frac{\partial\Gamma_i}{\partial\phi_i}\frac{\partial\phi_{ij}}{\partial a_{ij}}a_{ij} + \Lambda_i\frac{\partial\Gamma_i}{\partial\phi_i^A}\phi_{ij}^A\right].$$

Finally, differentiating in $a_j^A(m)$, we obtain

$$\frac{\partial T_{j,ij}^*}{\partial a_j^A(m)} = \frac{1}{\lambda_i^0}\left[\omega_i\frac{\partial U_i}{\partial u_i^A}\frac{\partial u_{ij}^A}{\partial a_j^A(m)} + \Lambda_i\frac{\partial\Gamma_i}{\partial\phi_i^A}\frac{\partial\phi_{ij}^A}{\partial a_j^A(m)}\right] = \Omega_{i,j}(m)$$

which is the spillover effect. From here, the country i social planner's Lagrangian is given by

$$\mathcal{L}_i^{SP} = \omega_i U_i \left(u_i(a_i), u_i^A(a_i, a^A)) \right) + \Lambda_i \Gamma_i \left(A_i - \int_j \tau_{j,ij} a_{ij} dj + \int_j T_{i,ji}^* dj, \phi_i(a_i), \phi_i^A(a_i, a^A) \right).$$

From here, derivation follows as in the proof of Proposition 24, except for the additional derivative in revenue. For $a_{ij}(m)$, there is no additional revenue derivative, and so $\tau_{i,ij}(m) = 0$ as before. For $a_{ii}(m)$, we have following the steps of Proposition 24

$$\tau_{i,ii}(m) = -\frac{1}{\lambda_i^0} \frac{\partial \mathcal{L}_i^{SP}}{\partial a_i^A(m)} - \frac{1}{\lambda_i^0} \Lambda_i \frac{\partial \Gamma_i}{\partial W_i} \int_j \frac{\partial T_{i,ji}^*}{\partial a_i^A(m)} dj = -\Omega_{i,i}(m) - \int_j \Omega_{j,i}(m) dj$$

giving the first result.

Finally, considering a foreign allocation a_{ji}, we have

$$0 = \frac{d\mathcal{L}_i^{SP}}{da_{ji}(m)} = \frac{\partial \mathcal{L}_i^{SP}}{\partial a_i^A(m)} + \lambda_i^0 \left[\frac{dT_{i,ji}^*}{da_{ji}(m)} + \int_j \frac{\partial T_{i,ji}^*}{\partial a_i^A(m)} dj \right].$$

From here, noting that we have $\frac{dT_{i,ji}^*}{da_{ji}(m)} = \tau_{i,ji}(m) + \frac{\partial \tau_{i,ji}}{\partial a_{ji}(m)} a_{ji}$, we obtain

$$0 = -\tau_{i,ii}(m) + \tau_{i,ji}(m) + \frac{\partial \tau_{i,ji}}{\partial a_{ji}(m)} a_{ji}$$

which rearranges to the result.

D Extensions of the Banking Model

In this Appendix, we present extensions to and discussions of the model, as applied to the banking context. To ease exposition, we express all results in this appendix for interior solutions, except for foreign allocations under non-cooperative regulation.

D.1 Dispersed Bank Ownership

Banks in practice are multinational not only in their activities, but also in ownership: even though a bank is headquartered in one country, part of its equity can be owned by foreigners. This invites

a natural question: do regulatory incentives change when part of the value of banks accrues to non-domestic agents, and if so does it cause inefficiencies?

This model is a straight-forward extension of the baseline model, and an application of the general theory. In particular, suppose that there is a disconnected set of global "equity" investors, who have preferences given by $U_{e,0}(c_{e,0}) + E[U_{e,1}(c_{e,1} + c_{e,2})]$.[81] These global investors imply that the global (probability-normalized) price of equity payoff in state s is given by

$$q(s) = \frac{U'_{e,1}(c_{e,1} + c_{e,2})}{U'_{e,0}(c_{e,0})},$$

that is the stochastic discount factor of global equity arbitrageurs.

Now, suppose that bank i sells "equity" payoff claims $\alpha_i(s)$ to global investors. It receives total revenue from this equity sale of

$$E_i = \int_s q(s)\alpha_i(s)f(s)ds.$$

Equity issuance may be constrained by some general constraint set, as in Section 6, for example incentive constraints. From here, the results are an application of Appendix E.1. Globally efficient regulation does not regulate issuance of equity, since the global price $q(s)$ only generates distributive externalities that net out in equilibrium.[82] As such, the core results of the baseline model carry through, and the efficiency results are the same. This generalizes the results to include common ownership of cross-border banks.

D.2   Local Capital Goods and Protectionism

Although financial stability and fire sales have been highlighted as justifications for post-crisis regulation, cooperative agreements predate the crisis, including the previous Basel accords. In this context, regulators may care about additional considerations such as domestic spillovers. Additionally, regulators may care about controlling local costs of investment, for example wishing to ensure that (less strictly regulated) foreign banks are not at a competitive advantage over domestic banks while also wishing to allow domestic banks to expand in more-regulated foreign markets

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[81]We could alternatively consider a representative equity investor existing within each country.

[82]The key here is not that equity is not regulated, but rather that there is not a welfare-relevant pecuniary externality from the global price.

where lending costs are cheaper. This form of trade-off underlies prior literature on international cooperation, such as Dell'Ariccia and Marquez (2006).[83]

We consider such a motivation by extending the model to include a common domestic investment price. In particular, we augment the model with local capital goods, which are used to produce domestic projects. For simplicity, we rule out all spillovers besides capital good prices in this sections. As a result, there are no fire sales and no extended stakeholder spillovers.

Banks can produce projects using both initial endowment, and with units of a local capital good. Bank i purchases a vector k_i of local capital goods, with k_{ij} being the capital good of country j, at prices p_j. When using k_i of the capital good, it costs an additional $\Phi_{ii}(I_{ii}, k_{ii}) + \int_j \Phi_{ij}(I_{ij}, k_{ij}) dj$ to produce the vector I_i of projects. The date 0 budget constraint of bank i is

$$p_i k_{ii} + \int_j p_j k_{ij} dj + \Phi_{ii}(I_{ii}, k_{ii}) + \int_j \Phi_{ij}(I_{ij}, k_{ij}) dj \leq A_i + D_i.$$

The optimization problem of banks is otherwise unchanged, except that k_i is now a choice variable of banks.

In each country, there is a representative capital producing firm. The capital producing firm produces the capital good out of the consumption good with an increasing and weakly convex cost function $\mathcal{K}_i(K_i)$, and so has an optimization problem

$$\max_{K_i} p_i K_i - \mathcal{K}_i(K_i).$$

The resulting equilibrium capital good price in country i is

$$p_i = \frac{\partial \mathcal{K}_i(K_i)}{\partial K_i}, \quad K_i = k_{ii} + \int_j k_{ji} dj. \tag{40}$$

The local capital producing firm cannot be controlled by country planners, so that equation (40) is an implementability condition of the model. Note that $\frac{\partial p_i}{\partial K_i} \geq 0$.[84]

Finally, the social planner places a welfare weight ω_i^K on the capital producing firm, so that

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[83]In this Appendix, our main contribution relative to their paper is to allow for common agency, to study the impacts of Pigouvian taxation, and to relate this mechanism to fire sales.

[84]In order to ensure that firm profits are bounded above, we will assume that $\frac{\partial p_i}{\partial K_i} = 0$ above some point K^*, which amounts to assuming that $\mathcal{K}_i(K_i)$ becomes linear on the margin above K^*.

the social welfare function is

$$V_i^P = \int_s c_i(s)f(s)ds + \omega_i^K \left[p_i(K_i)K_i - \mathcal{K}_i(K_i) \right].$$

From here, note that the model is in the form of Section 6 when we interpret profits of the capital producing firm as a utility spillover to the domestic representative bank.

From here, we see that there are spillovers to both domestic and foreign agents from changes in capital purchases, given by

$$\Omega_{i,i}^K = -\frac{\partial p_i}{\partial K_i}k_{ii} + \frac{\omega_i^K}{\lambda_i^0}\frac{\partial p_i}{\partial K_i}K_i$$

$$\Omega_{j,i}^K = -\frac{\partial p_i}{\partial K_i}k_{ji}$$

The spillover from the capital price increase is the additional resource cost to the bank of purchasing their existing level of the capital good. This is closely related to the direct price spillover under fire sales.

Let us suppose that we are in an environment where the domestic planner wishes to subsidize domestic banks by keeping capital cheap. We represent this by the limiting case $\omega_i^K = 0$. In this case, there is a *negative* spillover from increases in the capital price to both domestic and foreign banks, which make capital more expensive.

The globally efficient policy subsidizes capital by limiting capital purchases of all banks. By contrast, non-cooperative quantity regulation is protectionist and bans foreign banks from purchasing the domestic capital. In effect, it shields domestic banks from foreign competition.

Nevertheless, the "pecuniary externality" here falls within the class of problems under Assumption 9. As a result, assuming no monopoly power, the non-cooperative equilibrium under Pigouvian taxation is globally efficient.

Relationship to the Pre-Crisis World. In addition to understanding the Basel accords, this result also helps contextualize the historical aversion to capital control measures or other barriers to capital flows. In a purely non-cooperative environment, countries are tempted to engage in inefficient protectionism to shield domestic banks from foreign competition. Protectionism is inefficient because all countries do so, and so countries benefit from agreements against protectionist policies.

For example, agreements might allow expansion via branches, rather than subsidiaries, in addition to lifting other barriers to capital flows. Our results suggest that although quantity-based measures lead to inefficient protectionist policies, priced-based measures (taxes) do not. This provides another advantage of tax-based policies in the international context.

Differences from Fire Sales. Although the general characterizations in this extension are closely related in a general sense to the characterizations of the main paper under fire sales, there are two important differences.

The first important difference is the form of restrictions on foreign banks. Under fire sales, non-cooperative policies were meant to restrict premature liquidations. This corresponded most naturally to either ring fencing type policies, or to restrictions on capital outflows. By contrast, with local capital prices, non-cooperative policies are meant to restrict investment in the first place, and so more closely resemble either greater regulation on domestic activities of foreign banks, or bans on capital inflows. The motivation under the former is to enhance domestic financial stability, while the motivation under the latter is more protectionist in nature.

The second important distinction is in the implications for cooperation. Under fire sales, cooperation was required among countries who invest across borders *and* who share common crisis states. By contrast under local capital goods, cross border investment alone determines the need for cooperation.

D.3   Quantity Restrictions in the Form of Ceilings

In this appendix, we show how the global optimum (Proposition 1) and the non-cooperative optimum under quantity restrictions (Proposition 3) can be achieved using explicit quantity restrictions, rather than revenue-neutral taxes. Moreover, we show this forms an optimal policy under quantity restrictions. In this manner, we will show that duality between quantity restrictions and revenue-neutral taxes holds in our baseline model.

The argument will proceed in two steps. First ("Step 1"), we will argue that provided we can find an implementation of Propositions 1 and Proposition 3 using explicit quantity restrictions, then that implementation is in fact optimal among all possible quantity restrictions. Second ("Step 2"), we will show how quantity restrictions implement these outcomes.

D.3.1 Global Optimum

We can formally define quantity restrictions set by the global planner to be a set of restrictions $\Phi_i(c_i, D_i, I_i, L_i) \leq 0$ on the bank i contract. For example, possible restrictions include: (i) a ceiling on liquidations, $L_{ij}(s) \leq \overline{L}_{ij}(s)$ (e.g. a quantity-based capital control restricting outflows); and, (ii) a ceiling on debt, $D_i \leq \overline{D}_i$ (e.g. a leverage requirement).

Step 1. To start with the first step of the argument, suppose that we can in fact find set of quantity restrictions Φ_i that implements the cooperative outcomes $(c_i^*, D_i^*, I_i^*, L_i^*)$ of Proposition 1, where we have used the asterisk notation to denote the optimal quantities.[85] It then in fact follows that we have found a global optimum under all possible quantity restrictions. The reason is that Proposition 1 is already the solution to the global constrained efficient planning problem in which the global planner directly chooses the contracts (c_i, D_i, I_i, L_i) of all banks, subject to the same constraints faced by banks. The only difference is in how the global planner implements this Pareto efficient allocation. In Proposition 1, it is implemented via revenue-neutral Pigouvian wedges. Here, we are looking to implement it via quantity restrictions.

Step 2. Now, we argue how the Pareto efficient allocation (global optimum) of Proposition 1 can be implemented using quantity restrictions. In particular, consider a ceiling on liquidations $L_{ij}(s) \leq \overline{L}_{ij}(s) = L_{ij}^*(s)$, that is the ceiling $\overline{L}_{ij}(s)$ is set equal to the globally optimal liquidation rule $L_{ij}^*(s)$. To complete the argument, we need to show that there are non-negative Lagrange multipliers such that the allocation $(c_i^*, D_i^*, I_i^*, L_i^*)$ satisfies the Lagrangian optimality conditions.

Taking the bank Lagrangian in Appendix A.1 and incorporating the quantity ceiling restrictions,

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[85] As in Proposition 3, cooperation may also involve date 0 lump-sum transfers $\mathcal{T}_i$ to guarantee Pareto efficiency.

we have,

$$\mathcal{L}_i = \omega_i \int_s c_i(s)f(s)ds + \lambda_i^0 \left[A_i + D_i - \Phi_{ii}(I_{ii}) - \int_j \Phi_{ij}(I_{ij})dj \right]$$

$$+ \int_s \lambda_i^1(s) \left[\gamma_i(s)L_{ii}(s) + (1 + r_{ii})(R_i(s)I_{ii} - L_{ii}(s)) \right.$$

$$+ \int_j \left[\gamma_j(s)L_{ij}(s) + (1 + r_{ij})(R_j(s)I_{ij} - L_{ij}(s)) \right] dj - c_i(s) - D_i \right] f(s)ds$$

$$+ \int_s \Lambda_i^1(s) \left[-D_i + \gamma_i(s)L_{ii}(s) + \int_j \gamma_j(s)L_{ij}(s)dj + (1 - h_i(s))\mathcal{C}_{ii}(s) + \int_j (1 - h_j(s))\mathcal{C}_{ij}(s)dj \right] f(s)ds$$

$$+ \int_s \left[\underline{\xi}_{ii}(s)L_{ij}(s) + \overline{\xi}_{ii}(s)(R_iI_{ii} - L_{ii}(s)) + \int_j \left(\underline{\xi}_{ii}(s)L_{ij}(s) + \overline{\xi}_{ij}(s)(R_jI_{ij} - L_{ij}(s)) \right) \right] f(s)ds$$

$$+ \int_s \left[\kappa_{ii}(s)(L_{ii}^*(s) - L_{ii}(s)) + \int_j \kappa_{ij}(s)(L_{ij}^*(s) - L_{ij}(s))dj \right] f(s)ds$$

where $\kappa_{ij}(s)$ is the Lagrange multiplier on the regulatory constraint $L_{ij}(s) \leq L_{ij}^*(s)$. We now construct the multipliers $\lambda_i^0, \Lambda_i^1(s), \Lambda_i^1(s), \underline{\xi}_{ij}(s), \overline{\xi}_{ij}(s), \kappa_{ij}(s)$ so that the allocation $(c_i^*, D_i^*, I_i^*, L_i^*)$ satisfies the optimality conditions. In particular, suppose that we set the Lagrange mulitpliers $\lambda_i^0 = \lambda_i^{0*}, \lambda_i^1(s) = \lambda_i^{1*}(s), \Lambda_i^1(s) = \Lambda_i^{1*}(s)$ to coincide with the social planner's Lagrange multiplier values in the constrained efficient planning problem of Proposition 1, and further set $\underline{\xi}_{ij}(s) = \overline{\xi}_{ij}(s) = 0$. It follows as in the proof of Proposition 1 that the optimality conditions now coincide for c_i, I_i, D_i, leaving only optimality of liquidations L_i. Here, we have the optimality condition for $L_{ij}(s)$ given by

$$0 = \lambda_i^1(s)(\gamma_j(s) - (1 + r_{ij})) + \Lambda_i^1(s)(\gamma_j(s) - (1 - h_j(s))\gamma_j(s)) - \kappa_{ij}(s).$$

Therefore, construct the multiplier $\kappa_{ij}(s) = \lambda_i^{0*}\tau_{ij}(s)$ using the optimal wedge $\tau_{ij}(s)$ given in Proposition 1. Because the wedge $\tau_{ij}(s)$ is non-negative, the multiplier $\kappa_{ij}(s)$ is also non-negative. Under this multiplier, this optimality condition is the same condition as in Proposition 1 and so holds under the optimal allocation $(c_i^*, D_i^*, I_i^*, L_i^*)$. As a result, we have found non-negative Lagrange multipliers such that the optimality conditions are satisfied under the quantity ceilings $L_{ij}(s) \leq L_{ij}^*(s)$. This shows how the globally optimal allocation of Proposition 1 can be implemented via use of explicit quantity restrictions, rather than revenue-neutral wedges.

Summary. In sum, we have shown how the global planner can implement the global optimum of Proposition 1 using explicit quantity restrictions in the form of ceilings on liquidations, and moreover that this constitutes optimal policy under quantity restrictions. This illustrates more concretely the connection between revenue-neutral Pigouvian wedges and explicit quantity restrictions, showing that duality between them holds in the global planning problem of the baseline model.

In the next subsection, we show that the same applies in the non-cooperative optimum.

D.3.2 Non-Cooperative Optimum.

We will now proceed to show that non-cooperative ceilings on liquidations $L_{i,ji}(s) \leq \overline{L}_{i,ji}(s)$ form an optimal policy, and implement the equilibrium of Proposition 3. The argument will follow in two steps. First, we will consider country planner i, whose banks face quantity restrictions of the form $L_{j,ij}(s) \leq \overline{L}_{j,ij}(s)$ imposed upon them by foreign country planners, where the ceilings are set in according with the equilibrium outcomes under Proposition 3. In this setting, we will show that the domestic planner can do no better than to implement the allocations under Proposition 3. Second, we will argue that quantity restrictions in the form of liquidation ceilings can be used to implement the outcomes of Proposition 3.

Step 1. Consider the allocation achieved under Proposition 3, denoted by $(c_i^*, D_i^*, I_i^*, L_i^*)$. Suppose that foreign planners have set quantity restrictions $L_{j,ij}(s) \leq \overline{L}_{j,ij}(s) \equiv L_{j,ij}^*(s)$, that is they have imposed ceilings on liquidations by bank i, with the ceiling set equal to the allocation under Proposition 3. Note that in any state s in which there was a positive foreign spillover in country j, this implies $\overline{L}_{j,ij}(s) = L_{j,ij}^*(s) = 0$, that is the quantity restriction explicitly bans foreign liquidations (rather than implicitly banning it via a large wedge). Now to proceed with the first step of the argument, suppose we study the following relaxed problem: planner i can directly choose allocations (c_i, D_i, L_i, I_i) of domestic banks and (I_{ji}, L_{ji}) of foreign banks, regardless of whether there is a set of quantity restrictions that implements this allocation, subject to the limitations $L_{j,ij}(s) \leq \overline{L}_{j,ij}(s)$ on domestic banks imposed by foreign planners. If the solution to the relaxed problem can be achieved via quantity restrictions, then it is clearly optimal for planner i. Thus, we will first show that the solution to the relaxed problem is the solution of Proposition 3, and then we will show that it can be implemented by ceilings on liquidations.

First, consider the solution to the relaxed problem. Recall that Lemma 2 provides an imple-mentability result whereby planner i directly chooses allocations, and then backs out implementing wedges. In other words, Lemma 2 and Proposition 3 already study a relaxed problem, but with one key difference. This key difference is that in the environment of Lemma 2 and Proposition 3, bank i faces wedges on liquidations imposed by foreign planners, $\tau^L_{j,ij}(s)$, rather than ceilings $\overline{L}_{j,ij}(s)$. Nevertheless, employing the same strategy as for the global optimum, suppose we now conjecture that planner i faces Lagrange multipliers $\lambda_i^0 = \lambda_i^{0*}$, $\lambda_i^1(s) = \lambda_i^{1*}(s)$, $\Lambda_i^1(s) = \Lambda_i^{1*}(s)$ that are the same as the Lagrange multipliers of the planning problem of Proposition 3, and moreover set $\underline{\xi}_{ij}(s) = \overline{\xi}_{ij}(s) = 0$. Suppose finally that we set the Lagrange multiplier $\kappa_{ij}(s)$ on the foreign planner regulatory constraint $L_{j,ij}(s) \le L^*_{j,ij}(s)$ to be $\kappa_{ij}(s) = \lambda_i^{0*}\tau^{L*}_{j,ij}(s)$, where $\tau^{L*}_{j,ij}(s)$ is the optimal liqui-dation wedge set by planner j in Proposition 3. Because $\tau^{L*}_{j,ij}(s) \ge 0$, this multiplier is non-negative, and so we have constructed non-negative Lagrange multipliers such that $(c_i^*, D_i^*, I_i^*, \{L_{ji}^*, I_{ji}^*\})$ is a solution of the relaxed problem of planner i who faces the quantity restrictions $L_{j,ij}(s) \le L^*_{j,ij}(s)$ imposed by foreign planners. Thus, the optimal allocation of Proposition 3 is an optimum of the relaxed problem here.

Step 2. Second, we have to show that the solution of the relaxed problem here can be implemented via quantity restrictions. In particular, define ceilings $L_{i,ii}(s) \le \overline{L}_{i,ii}(s) = L^*_{i,ii}(s)$ for domestic banks and $L_{i,ji}(s) \le \overline{L}_{i,ji}(s) = L^*_{i,ji}(s)$ for foreign banks (recall that the foreign planner has imposed the ceilings on foreign liquidations by domestic banks). From here, construction of non-negative Lagrange multipliers of the bank i problem proceeds exactly as before, with $\kappa_{ii}(s) = \lambda_i^{0*}\tau^{L*}_{i,ii}(s)$ and $\kappa_{ij}(s) = \lambda_i^{0*}\tau^{L*}_{j,ij}(s)$, where the wedges are the optimal wedges in Proposition 3, verifying that the bank optimality conditions hold.

D.4 Commitment and Time Consistency

The baseline model of Sections 2-4 assumes that banks and planners operate under commitment when choosing ex-post liquidation policies, L, or wedges on ex-post liquidation policies, τ^L. In this appendix, we study the role of planner commitment over wedges in the baseline model, and show that absent commitment a time consistency problem arises owing to the revenue collection motive. In particular, revenue from taxes on investment is collected at date 0, but this revenue is affected

by taxes on liquidations, which are set at date 1. This leads to a time consistency problem as the date 1 planner does not internalize the effect on date 0 revenue. The effect of this time consistency problem is that non-cooperative planners using Pigouvian taxation partially neglect the value of date 0 investment for use as collateral at date 1 when setting date 1 taxes, that is they fail to fully account for the collateral externality that arises from the domestic fire sale. By extension, in the limiting case where there is a full haircut and the domestic asset cannot be used as collateral, Pigouvian efficiency is restored.

Importantly, this appendix also highlights a key difference between the baseline model and the model of Section 5. In Section 5, all regulatory decisions are taken at date 0, and hence no commitment problem exists.

Concretely, suppose that wedges on liquidations are set at date 1 without commitment. Notice that this implies the global planner also lacks commitment, and so we will study whether the date 1 cooperative and non-cooperative solutions coincide. The net debt position of bank i at date 1, accounting for tax burdens and remissions, is given by $D_i + \tau_i^L(s)L_i(s) - \Pi_i^*(s)$, where $\Pi_i^*(s)$ is revenue remissions to bank i. Hence, the consolidated dates 1 and 2 budget constraint of bank i in state s is

$$c_i(s) \leq \mathcal{R}_{ii}(s) + \int_j \mathcal{R}_{ij}(s)dj - \left(D_i + \tau_i^L(s)L_i(s) - \Pi_i^*\right),$$

where $\mathcal{R}_{ij}(s) = \gamma_j(s)L_{ij}(s) + (1+r_{ij})(R_j(s)I_{ij} - L_{ij}(s))$ is the total return on initial investment I_{ij}, as in the baseline model. Similarly, the collateral constraint of bank i in state s is

$$D_i + \tau_i^L(s)L_i(s) - \Pi_i^* \leq h_i(s)\gamma_i(s)L_{ii}(s) + \int_j h_j(s)\gamma_j(s)L_{ij}(s)dj$$

$$+ (1 - h_i(s))\gamma_i(s)R_i(s)I_{ii} + \int_j (1 - h_j(s))\gamma_j(s)R_j(s)I_{ij}dj$$

which is the same as that in the baseline model, except for the addition of the tax burden and revenue remissions. Importantly, notice that the investment portfolio I_i that appears in the second line is taken as given by bank i and by all planners, since it was determined at date 0. However, liquidations $L_{(s)}i$ are a choice variable at date 1. Thus, the problem of bank i is to choose liquidations $L_i(s)$ in order to maximize its consumption value $c_i(s)$, subject to its collateral constraint and taking as given its investment portfolio I_i and inherited debt D_i.

Global Optimum. Consider the global spillover that arises from liquidations in country i. The spillover of an increase in liquidations $L_i^A(s)$ onto the welfare of country i' banks in wealth equivalent is given by

$$
\underbrace{\frac{1}{\lambda_{i'}^1(s) + \Lambda_{i'}^1(s)}}_{\text{Wealth Equivalent}} \underbrace{\frac{\partial \gamma_i(s)}{\partial L_i^A(s)}}_{\text{Price Impact}} \left[\underbrace{\lambda_{i'}^1(s) L_{i'i}(s)}_{\text{Distributive Externality}} + \Lambda_{i'}^1(s) \left(\overbrace{h_i(s) L_{i'i}(s)}^{\text{Date 1 Liquidations}} + \overbrace{(1 - h_i(s)) R_i(s) I_{i'i}}^{\text{Date 0 Investment}} \right) \right]
$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{Collateral Externality}}$$

(41)

Notice that once again, this spillover is of the same general form as the spillovers $\Omega_{i'i}^L(s)$ in Proposition 1, and combines distributive and collateral externalities. Importantly, there are two components of the collateral externality. The first relates to how date 1 liquidations appear in the collateral constraint. The collateral constraint is relaxed by liquidations proportional to the haircut $h_i(s)$, which is the *excess* date 1 funds that can be raised by liquidating country i assets rather than by using them as collateral. This excess value is relative to the baseline where all date 0 investments are used as collateral. The value of this baseline is the second term of the collateral externality. This second term will be the key source of inefficiency in this model lacking commitment.

Non-Cooperative Optimum. In the problem of bank i accounting for the wedges imposed by country planners, denote $\lambda_i^1(s)$ to be the Lagrange multiplier on the consolidated budget constraint and $\Lambda_i^1(s)$ the Lagrange multiplier on the collateral constraint. Following the same steps as the proof of Lemma 2, we obtain the implementability condition for the domestic liquidations of foreign banks as

$$
\tau_{i,ji}^L(s) = -\tau_{j,ji}^L(s) + \frac{\lambda_j^1(s)}{\lambda_j^1(s) + \Lambda_j^1(s)} \left[\gamma_i(s) - (1 + r_{ji}) \right] + \frac{\Lambda_j^1(s)}{\lambda_j^1(s) + \Lambda_j^1(s)} h_i(s) \gamma_i(s).
$$

First, let us consider quantity regulation. Under quantity regulation, Π_i^* is taken as given by country planner i, and hence by the same steps as the proof of Proposition 3 we obtain a ban on liquidations of the domestic asset by foreign banks whenever there is an adverse domestic spillover, and further than the wedge set on domestic liquidations by domestic banks only accounts for domestic spillovers. Thus, under quantity regulation, lack of commitment does not affect the qualitative insights of the baseline model with commitment.

By contrast, let us consider Pigouvian taxation. Under non-cooperative Pigouvian taxation, $\Pi_i^* = \int_j \tau_{i,ji}^L L_{ji}(s) dj$ is the total revenue collected from liquidation taxes on foreign banks. Substituting in the implementing tax rate and using $\tau_{j,ji}^L(s) = 0$, we obtain that the tax revenue collected from bank i' is

$$\tau_{i,i'i}^L(s) L_{i'i}(s) = \frac{1}{\lambda_{i'}^1(s) + \Lambda_{i'}^1(s)} \left[\lambda_{i'}^1(s) \left[\gamma_i(s) - (1 + r_{i'i}) \right] + \Lambda_{i'}^1(s) h_i(s) \gamma_i(s) \right].$$

Finally, following the proof of Proposition 4, we have the derivative of tax revenue from bank i' in aggregate liquidations $L_i^A(s)$ given by

$$\underbrace{\frac{\partial \tau_{i,i'i}^L(s) L_{i'i}(s)}{\partial L_i^A(s)}}_{} = \underbrace{\frac{1}{\lambda_{i'}^1(s) + \Lambda_{i'}^1(s)}}_{\text{Wealth Equivalent}} \underbrace{\frac{\partial \gamma_i(s)}{\partial L_i^A(s)}}_{\text{Price Impact}} \left[\underbrace{\lambda_{i'}^1(s) L_{i'i}(s)}_{\text{Distributive Externality}} + \Lambda_{i'}^1(s) \left(\overbrace{h_i(s) L_{i'i}(s)}^{\text{Date 1 Liquidations}} \right) \right] \quad (42)$$

In the baseline model, the proof of Proposition 4 relied on showing that the tax revenue derivative for revenues collected from bank i' coincided with the global spillover effect onto bank i', resulting in efficiency. Here, we have conducted the same exercise without commitment, looking to compare the global spillover effect (equation 41) with the tax revenue derivative (equation 42). In this model without commitment, we see that these two expressions differ from each other by the collateral externality term denoted "Date 0 Investment." Assuming investment $I_{i'i}$ is positive, then this term is zero only provided that $h_i(s) = 1$, that is there is a full haircut and debt cannot be rolled over at all. Otherwise it is generally positive, and the tax revenue derivative is generally not equal to the global spillover.

To understand why this spillover is correctly internalized in the model with commitment but not in the model without commitment, consider the problem at date 0. At date 0, banks choose investment scale, anticipating the outcome of the date 1 equilibrium. As a result, the implementability condition on date 0 investment is

$$\tau_{i,ji}^I = -\tau_{j,ji}^I - \frac{\partial \Phi_{ji}}{\partial I_{ji}} + E\left[\frac{\lambda_j^1}{\lambda_j^0} (1 + r_{ji}) R_i \right] + \frac{1}{\lambda_j^0} E\left[\Lambda_j^1 (1 - h_i) \gamma_i R_i \right].$$

As a result, revenue collected from bank i' *at date 0* is given by

$$\tau^I_{i,i'i}I_{i'i} = -\frac{\partial \Phi_{i'i}}{\partial I_{i'i}}I_{i'i} + E\left[\frac{\lambda^1_{i'}}{\lambda^0_{i'}}(1+r_{i'i})R_iI_{i'i}\right] + \frac{1}{\lambda^0_{i'}}E\left[\Lambda^1_{i'}(1-h_i)\gamma_iR_iI_{i'i}\right]$$

and hence, the revenue derivative at date 0 in date 1 aggregate liquidations $L^A_i(s)$ is

$$\frac{\partial \tau^I_{i,i'i}I_{i'i}}{\partial L^A_i(s)} = \frac{1}{\lambda^0_{i'}}\frac{\partial \gamma_i(s)}{\partial L^A_i(s)}\Lambda^1_{i'}(s)(1-h_i(s))R_i(s)I_{i'i}f(s).$$

In the baseline model with commitment, this was the source of the term missing in equation (42) in the model without commitment. Economically, this term reflects the baseline collateral value of date 0 investment. As the date 1 fire sale worsens, this baseline value falls, and so the tax rate on investment must fall, reducing tax revenue.

In the model with commitment, taxes on investment and liquidation are set simultaneously, and hence planner i internalizes how a decrease in the tax on liquidations leads to a worse fire sale and a forces (via implementability) a lower tax on investment. By contrast, in the model without commitment, the tax on liquidation is set after the tax on investment. Although changes in the liquidation tax at date 1 affects revenue collected from the date 0 investment tax, by the time the date 1 tax is being set the date 0 tax revenue has already been collected. This leads to the time consistency problem and the deviation from efficiency.

D.5 Real Economy or Arbitrageur Spillovers and Quantity Regulation

In the baseline model, the lack of any benefit from foreign banking led to the strong result of Proposition 3 of a ban on liquidations by foreign banks. In this appendix, we study the impact of benefits from foreign banking on the quantity regulation game, adopting an extension of the simple form provided in Examples 1 and 2 in Section 6. In this setting, we show that benefits from foreign banking still lead to under-regulation of domestic banks, but can lead to either under- or over-regulation of foreign banks. In particular, as in the baseline model, over-regulation arises when at the margin the domestic fire sale spillover is greater than the domestic benefit from foreign banking, as was the case by assumption in the baseline model. On the other hand, there is under-regulation (on the margin) when at the global optimum, the marginal domestic economic benefit from foreign

banks outweighs the marginal fire sale spillover to domestic banks, but does not outweigh the total fire sale spillover to all banks (domestic and foreign).

In particular, suppose there is a domestic spillover $u_i^A(I_i^A, L_i^A)$ from total bank activities in country i. This could be a real economy spillover from credit extension (Example 1), or alternatively could capture surplus of domestic arbitrageurs (Example 2).

We will focus here on the optimal liquidation rule. Define $\Omega_{ii}^A(s) = \frac{1}{\lambda_i^0} \frac{\partial u_i^A}{\partial L_i^A(s)}$ to be the marginal spillover associated with the real economy/arbitrageur surplus. It follows from the same steps as the proof of Proposition 1 that the globally optimal wedge on liquidations is

$$\tau_{ji}^L(s) = -\Omega_{ii}(s) - \int_{i'} \Omega_{i'i}(s) di' - \Omega_{ii}^A(s).$$

On the other hand, following the steps of the proof of Proposition 3, we obtain that the non-cooperative optimum generates wedges on domestic liquidations of domestic banks of

$$\tau_{ii}^L(s) = -\Omega_{ii}(s) - \Omega_{ii}^A(s),$$

while for foreign banks it generates an allocation rule

$$L_{ji}(s)\left[-\Omega_{ii}(s) - \Omega_{ii}^A(s)\right] = 0.$$

First considering domestic regulation of domestic banks, we have the same under-regulation of domestic banks as in the baseline model, as foreign spillovers are neglected.

Now, let us consider regulation of foreign banks. First, the non-cooperative rule tells us that either it is the case that $\Omega_{ii}(s) + \Omega_{ii}^A(s) = 0$ or otherwise $L_{ji}(s) = 0$. This captures the basic logic of the baseline model: liquidations by foreign banks are allowed only up to the point that they do not contribute adversely to domestic spillovers. In the baseline model with $\Omega_{ii}^A(s) = 0$, this meant it had to be the case that $L_{ji}(s) = 0$ whenever $\Omega_{ii}(s) \neq 0$, resulting in the ban on liquidations. However with additional spillovers (for example, surplus to domestic arbitrageurs) it can arise that $\Omega_{ii}(s) + \Omega_{ii}^A(s) = 0$ and $L_{ji}(s) > 0$, so that the domestic planner allows some liquidation of domestic assets by foreign banks. However, note that $\Omega_{ii}(s) + \Omega_{ii}^A(s) = 0$ does not generally imply foreign

banks are unregulated.[86]

The key question is whether the domestic planner is more or less stringent than the global optimum with regards to foreign banks. Consider the marginal incentives. Recall that $\tau_{ji}^L(s)$ is always (by implementability) equal to the marginal benefit to foreign banks of liquidating assets in country i. Therefore, we have from the tax rate

$$\underbrace{\tau_{ji}^L(s) + \int_{i'} \Omega_{i'i}(s)di'}_{\text{Total MB to Foreign Banks}} = \underbrace{-\Omega_{ii}(s) - \Omega_{ii}^A(s)}_{\text{MC to Domestic Banks}} .$$

Suppose first that at the global optimum, $-\Omega_{ii}(s) > \Omega_{ii}^A(s)$. In this case, the net domestic cost of fire sales $-\Omega_{ii}(s)$ exceeds the net domestic benefit of other spillovers $\Omega_{ii}^A(s)$, and hence country planner i on the margin prefers fewer liquidations. However, the positive marginal cost to domestic banks means that there must be an equal and positive marginal benefit to foreign banks, which is neglected by the domestic planner when designing regulation. This leads to over-regulation of foreign banks. The baseline model is the limiting case where $\Omega_{ii}^A(s) = 0$, and hence the marginal cost to domestic banks must be non-negative at the optimum, resulting in the ban.

Conversely if $-\Omega_{ii}(s) < \Omega_{ii}^A(s)$, then the domestic benefit outweighs the domestic cost, and hence country planner i prefers more liquidations on the margin. In contrast to the previous case, the fact that the domestic marginal benefit is positive means that the foreign marginal cost is also positive, due to the foreign fire sale spillover. In this case, the domestic planner designing regulation neglects the net cost to foreign banks, and under-regulates foreign banks relative to the optimum. The logic of this case is economically similar therefore to under-regulation of domestic banks, which is also driven by the domestic planner considering positive domestic benefits but ignoring foreign costs.

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[86]A simple example of this is the knife-edge case where the optimum features $\Omega_{ii}(s) + \Omega_{ii}^A(s) = 0$ and $L_{ji}(s) = 0$, that is when equilibrium liquidations by domestic banks put the planner exactly on their first order condition. In this case, implementability implies that $\tau_{i,ji}^L(s) = \frac{\lambda_j^1(s)}{\lambda_j^0}(\gamma_i(s) - (1 + r_{ji})) + \frac{1}{\lambda_j^0}\Lambda_j^1(s)h_i(s)\gamma_i(s)$ and so the wedge on foreign bank liquidations is positive provided that r_{ji} is sufficiently large. Notably, in this case we nevertheless have $\tau_{i,ii}^L(s) = 0$, that is domestic banks are unregulated. Economically, this difference reflects unequal treatment. The domestic planner values domestic banks, and since in equilibrium the domestic spillover is zero then the optimal tax rate is zero. However, suppose that there is a foreign bank which, at $L_{ji}(s) = 0$, places positive value on liquidations, and so $\tau_{i,ji}^L(s) > 0$. If the planner instead allowed foreign banks to liquidate assets, they would set $L_{ji}(s) > 0$ and push the marginal cost of liquidations to be positive, rather than zero. The domestic planner internalizes the social cost this would generate, but not the marginal benefit to foreign banks. Hence, the domestic planner imposes a positive wedge on foreign banks.

Taken together, this extension reflects the same underlying forces as were present in the baseline model, with the same policy implications for under-regulation of domestic banks. However, it caveats and qualifies the implications for over-regulation of foreign banks.

E   Extensions of the General Model

In this Appendix, we provide extensions of the general model presented in Section 6.

E.1   World Prices

We now extend the model to incorporate world prices, for example allowing for state contingent securities prices at date 0 to be endogenous. We show that provided that global prices only enter constraints through the wealth level, the problem is unaffected. This result is in line with Korinek (2017) and follows similarly.

Let $x_i = \{x_i(n)\}_{n \in N}$ be a vector of global goods held by country i, so that market clearing implies $\int_i x_i(n) di = 0$. Global goods trade at prices q, so that the wealth level of country i multinational agents is

$$W_i = A_i - T_i - \sum_n q(n) x_i(n).$$

Global goods enter into $u_{ii}, u_{ii}^A, \phi_{ii}, \phi_{ii}^A$, but prices do not enter except through the wealth level. Note that because global goods enter into domestic functions, they do not influence Assumption 9. From here, we obtain the following result.

Proposition 28. *The optimal cooperative wedges are of the same form as Proposition 8, with no wedges on x_i. Pigouvian taxation is efficient under the same conditions as Proposition 27.*

Proposition 28 may apply, for example, to a global market for liabilities at date 0.

E.1.1   Proof of Proposition 28

The global planning problem has a Lagrangian

$$\mathcal{L}^G = \int_i \left[\omega_i U_i \left(u_i(a_i, x_i), u_i^A(a_i, x_i, a_i^A) \right) + \Lambda_i \Gamma_i \left(A_i + \mathcal{T}_i, \phi_i(a_i, x_i), \phi_i^A(a_i, x_i, a_i^A) \right) - \lambda^0 \mathcal{T}_i - \lambda^0 Q x_i \right] di$$

where we have suggestively denoted $Q(n)$ to be the Lagrange multiplier on the global goods market clearing for good n. Differentiating in $x_i(n)$, we obtain

$$0 = \frac{\partial \mathcal{L}_i}{\partial x_i(n)} - \lambda^0 Q(n)$$

so that world prices $q(n) = Q(n)$ form an equilibrium (recall that $\lambda_i^0 = \lambda^0$). Globally efficient policy is as in Proposition 8, with no wedges placed on x_i.

E.2   Local Constraints on Allocations

We extend Section 6 to incorporate local constraints on allocations. Note that such constraints are already available through Γ_i for domestic allocations, but that such constraints are not available in countries $j \neq i$. The extension captures, for example, the constraints $0 \leq L_{ij}(s) \leq R_j(s)I_{ij}$ and $I_{ij} \geq 0$ imposed in the main paper.

Suppose that in country j, there is a vector of linear constraints $\chi_{ij}(a_j^A)a_{ij} \leq b_{ij}$ on allocations, where $\chi_{ij}(a_j^A)$ potentially depends on aggregates in country j and where $b_{ij} \geq 0$.[87] We impose linearity in the spirit of the required conditions for optimality of Pigouvian taxation in Proposition 27. We obtain the following revised implementability result for foreign allocations, which mirrors Lemma 23

Lemma 29. *Any domestic allocation of foreign agents satisfying constraints $\chi_{ij}(a_j^A)a_{ij} \leq b_{ij}$ is optimally implemented with the wedges in Lemma 23.*

Lemma 29 implies that implementability constraints are the same as in Section 6. The only difference is that now the constraint set on local allocations is a constraint of the local planner. Note that this implies that the local planner directly internalizes spillovers of domestic aggregates onto the constraint set $\chi_{ij}(a_j^A)a_{ij} \leq b_{ij}$, so that such spillovers are not an issue.

From here, all results proceed as in Section 6.[88] Intuitively, the only adjustment we need to make is that $\chi_{ij}(a_j^A)a_{ij} \leq b_{ij}$ is now a constraint set of planner j. Without loss of generality,

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[87]We impose $b_{ij} \geq 0$ to ensure that non-participation ($a_{ij} = 0$) is always feasible.
[88]Notice that it is expositionally convenient to define the decentralization of the global optimum in an analogous manner to the corner solutions associated in Lemma 29, where taxes are set to make banks indifferent at the corner solution with Lagrange multipliers of zero on local constraints.

scale the Lagrange multiplier v_{ij} by λ_i^0, and define the "local constraint spillover" of a change in aggregates by

$$\Omega_{j,ij}^{LC}(m) = -v_{ij}\frac{\partial \chi_{ij}}{\partial a_j^A(m)}a_{ij}$$

so that we can define the total domestic local constraint set spillover as

$$\Omega_j^{LC}(m) = -\int_i \Omega_{j,ij}^{LC}(m)di$$

From here, it follows that the efficiency results of Section 6 apply, treating the total domestic spillover as $\Omega_{j,j}(m) + \Omega_j^{LC}(m)$.[89]

Note that if the local constraints were non-linear, this would not generally hold, as we would not be able to recover the complementary slackness condition precisely in the above proof. As a result, the domestic planner may have an incentive to manipulate the tax rates that implement corner solutions in order to increase revenue. This would amount to another form of "monopolistic" revenue distortion in the model.

E.2.1   Proof of Lemma 29

For expositional ease, we suppress the notation $\chi_{ij}(a_j^A)$ and simply write χ_{ij}. Let $v_{ij} \geq 0$ be the Lagrange multipliers on the local feasibility constraints $b_{ij} - \chi_{ij}a_{ij} \geq 0$. The first order condition for an action m is

$$0 = \omega_i\frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i\frac{\partial U_i}{\partial u_i^A}\frac{\partial u_i^A}{\partial a_{ij}(m)} + \Lambda_i\frac{\partial \Gamma_i}{\partial W_i}\left(-\tau_{i,ij}(m) - \tau_{j,ij}(m)\right) + \Lambda_i\frac{\partial \Gamma_i}{\partial \phi_i}\frac{\partial \phi_{ij}}{\partial a_{ij}(m)} + \Lambda_i\frac{\partial \Gamma_i}{\partial \phi_i^A}\frac{\partial \phi_i^A}{\partial a_{ij}(m)}$$

$$- v_{ij}\chi_{ij}(m)$$

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[89] To see that $\tau_{i,ij}(m) = 0$ constitutes an equilibrium policy for $j \neq i$, suppose that $\tau_{j,ij}(m)$ is set to clear the first-order condition. Then, the first order condition of country planner i for $a_{ij}(m)$ is satisfied with equality, and so we must have $v_{ij} = 0$, so that there is no value to country planner i of relaxing the local constraints in country j at the equilibrium. As a result, the preferences of country planner i align with country i agents over actions in country j, and we have $\tau_{i,ij}(m) = 0$.

So that rearranging, we obtain

$$\lambda_i^0 \tau_{j,ij}(m) + \nu_{ij}\chi_{ij}(m) = -\lambda_i^0 \tau_{i,ij}(m)$$

$$+ \omega_i \frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i}{\partial u_i^A}\frac{\partial u_i^A}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i}\frac{\partial \phi_{ij}}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A}\frac{\partial \phi_i^A}{\partial a_{ij}(m)}.$$

Notice that the right-hand side is constant for a given allocation, and is the same formula as in Lemma 23. Denote it to be $\lambda_i^0 \tau_{j,ij}^*(m)$, so that we have $\tau_{j,ij}(m) = \tau_{j,ij}^*(m)$ if $\nu_{ij} = 0$. Given corner solutions, there may be multiple vectors of tax rates that implement this allocation. We can express the problem of country planner j therefore maximizing tax revenue collected while implementing the same allocation, that is

$$\max_{\nu_{ij}, \tau_{j,ij}} \tau_{j,ij}a_{ij}^* \quad \text{s.t.} \quad \lambda_i^0 \tau_{j,ij}(m) + \nu_{ij}\chi_{ij}(m) = \lambda_i^0 \tau_{j,ij}^*(m), \quad \nu_{ij}\left(b_{ij} - \chi_{ij}a_{ij}^*\right) = 0$$

where the second constraint is complementary slackness. Substituting in for $\tau_{j,ij}$ and substituting in the complementary slackness condition, we obtain

$$\max_{\nu_{ij} \geq 0} \tau_{j,ij}^* a_{ij}^* - \frac{1}{\lambda_i^0}\nu_{ij}b_{ij}$$

Because $b_{ij} \geq 0$, revenue collection is maximized at $\nu_{ij} = 0$, so that we have $\tau_{j,ij} = \tau_{j,ij}^*$. As a result, the implementability conditions of Lemma 23 hold.

E.3 Heterogeneous Agents

We extend the model of Section 6 by allowing for heterogeneous agents within a country. Suppose that in each country, there are $\mathcal{K} = \{1,...,K\}$ agents, who differ in their utility functions and constraint sets, whom we index i_k. Some agents may not be able to conduct cross-border activities, in which case foreign actions would not appear in their utility function or constraint sets. Agents of type i_k have relative mass μ_{i_k} and are assigned a social welfare weight ω_{i_k}.

It is easy to see that we can treat the problem as if there were a single representative agent in country i. In particular, define $a_i = \{a_{i_k}\}_{k \in K}$, $U_i = \sum_k \mu_{i_k}\omega_{i_k}U_{i_k}$, and $\Gamma_i = (\Gamma_{i_1},...,\Gamma_{i_K})$. The

problem is as-if we have a single representative agent who solves

$$\max_{a_i} U_i \quad \text{s.t.} \quad \Gamma_i \geq 0,$$

since this decision problem is fully separable in a_{i_k} and yields the optimality conditions of each agent type. The only difference relative to Section 6 is that there are K different measures of wealth, W_{i_k}. Domestic lump sum transfers imply that $\lambda_{i_k}^0 = \lambda_i^0$ is independent of k, and the characterization of optimal policy follows as in Section 6.

E.4 Nonlinear Aggregates

In Section 6, we assumed that aggregates are linear, that is $a_i^A(m) = a_{ii}(m) + \int_j a_{ij}(m)dj$. The welfare-relevant aggregates may not necessarily be linear. We can represent this by

$$Z_i\left(z_{ii}(a_{ii},a_i^A) + \int_j z_{ji}(a_{ji},a_i^A)dj, a_i^A\right) = 0$$

for some functions z and Z. The key change in the model is that we now have spillover effects that depend on the identity of the country investing, as in the bailouts model. The optimality of non-cooperative Pigouvian taxation follows from the same steps and logic as the baseline model, simply incorporating the change in aggregates that arises through this nonlinear relationship. This clarifies once again that the homogeneity property of Assumption 9 applies to allocations, not to aggregates.

The possibility for non-linear aggregation helps to generalize the results to settings where regulation is set at an initial date, but the economy is not regulated thereafter (Section 5).

E.5 General Government Actions

We extend the model to feature more general government actions, for example bailouts as in Appendix B.4. In particular, country planner i can take actions $g_{i,jk}(m) \geq 0$ (for either $i = j$ or $i = k$), which affect country j agents in the same way as action m in country k. As such, we can

define the total domestic action of agent i as

$$\bar{a}_{ii}(m) = a_{ii}(m) + g_{i,ii}(m)$$

and the total foreign action of agent i as

$$\bar{a}_{ij}(m) = a_{ij}(m) + g_{i,ij}(m) + g_{j,ij}(m).$$

This classification allows for a rich set of both agent and government actions. For example, a domestic action m that can only be taken by the government, such as government debt issuance or a bailout, could feature a feasibility constraint $a_{ii}(m) = 0$. From here, the domestic aggregates are given by

$$a_i^A(m) = \bar{a}_{ii}(m) + \int_j \bar{a}_{ji}(m)dj.$$

The flow utility of the country i representative agent is now given by

$$\max_{a_i} U_i \left(u_i(a_i, g_i, \bar{a}_i), u_i^A(a_i, g_i, \bar{a}_i, a_i^A)) \right) \quad \text{s.t.} \quad \Gamma_i \left(W_i, \phi_i(a_i, g_i, \bar{a}_i), \phi_i^A(a_i, g_i, \bar{a}_i, a^A) \right) \geq 0,$$

where we have $u_i(a_i, g_i, \bar{a}_i) = u_{ii}(a_{ii}, u_{ii}^g(g_{i,ii}) + \int_j u_{ij}^g(g_{j,ji})dj, \bar{a}_{ii}) + \int_j u_{ij}(a_{ij}, g_{i,ij}, \bar{a}_{ij})dj$ and so on. It simplifies exposition to include in Γ_i any government feasibility constraints, for example government budget constraints. Observe that such constraints would be assigned Lagrange multipliers of 0 by the representative agent, but not by the social planner.

From here, we begin by characterizing the globally efficient allocation. Observe first that the optimal wedges for private actions are still given by the equations in Proposition 8.

Proposition 30. *The globally efficient allocation can be decentralized by the wedges of Proposition 8. The globally efficient government actions $g_{i,jk}$ (for either $i = j$ or $i = k$) are given by*

$$-\frac{\partial \mathcal{L}_i}{\partial g_{i,jk}(m)} \geq \underbrace{\frac{\partial \mathcal{L}_j}{\partial \bar{a}_{jk}(m)}}_{\text{Country } j \text{ Benefit}} + \underbrace{\frac{\partial \mathcal{L}_k}{\partial a_k^A(m)}}_{\text{Country } k \text{ Spillover}} + \underbrace{\int_{i'} \frac{\partial \mathcal{L}_{i'}}{\partial a_k^A(m)}di'}_{\text{Foreign Spillovers}} \tag{43}$$
$$\underbrace{\phantom{-\frac{\partial \mathcal{L}_i}{\partial g_{i,jk}(m)}}}_{\text{Country } i \text{ Cost}}$$

where $\frac{\partial \mathcal{L}_i}{\partial g_{i,jk}(m)} = \omega_i \frac{\partial U_i}{dg_{jk}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial g_{jk}(m)}$ and so on.

Proof. The proof of the decentralizing wedges follows as in the proof of Proposition 8. The government action rules follow directly from the derivatives of the global Lagrangian. ∎

The globally efficient allocation of government actions is a generalization of the optimal bailout rule of Proposition 14, with analogous intuition. Note that for $j \neq i$, we have an action smoothing result: $\frac{\partial \mathcal{L}_i}{\partial g_{i,ji}(m)} = \frac{\partial \mathcal{L}_j}{\partial g_{j,ji}(m)}$, that is the marginal cost of providing the action is smoothed across countries. For example, this corresponds to bailout sharing.

From here, the non-cooperative results on quantity regulation follow as in the baseline model and bailouts section. Taking either $i = j$ or $i = k$, the neglected terms are always the terms that affect other countries, namely the foreign spillovers and either the spillover ($i = j$) or the benefit ($i = k$). For domestic actions, there are neglected foreign spillovers, while for domestic actions on foreign agents there is unequal treatment when the cost of providing the action is held fixed.

On the other hand, suppose that choices of foreign government actions $g_{i,ij}$ and $g_{i,ji}$ are delegated to agents, but can be taxed.[90] Once this is imposed and governments use Pigouvian taxation, these foreign government actions are no different from regular actions from a technical perspective,[91] and the efficiency of Pigouvian taxation is restored.

E.6 Preference Misalignment

We now suppose that there is a difference in preferences between country planners and multinational agents, that is country planners have a utility function $V_i(v_i(a_i), v_i^A(a_i, a^A))$. For example, preference differences may arise due to paternalism, control by special interest groups, or corruption. For simplicity, we incorporate the welfare weights into the planner utility function.

We define efficient policies with respect to those of country planners. This is a natural efficiency benchmark, as country planners agree to cooperative agreements.[92] Under this definition, globally efficient policy can be characterized as follows.

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[90]Notice that $g_{i,ij}$ is delegated to country i agents and $g_{i,ji}$ to country j agents.

[91]Excepting that there is a non-linear aggregate arising from u_{ii}^g, which is covered above.

[92]See Korinek (2017) for the same argument.

Proposition 31. *The globally efficient wedges are given by*

$$\tau_{ji}(m) = -\Delta_{ji}(m) - \Omega^v_{i,i}(m) - \int_{i'} \Omega^v_{i',i}(m) di' \tag{44}$$

where we have

$$\Delta_{ji}(m) = \frac{1}{\lambda^0_j} \left[\frac{\partial V_j}{\partial v_j} \frac{\partial v_{ji}}{\partial a_{ji}(m)} - \frac{\partial U_j}{\partial u_j} \frac{\partial u_{ji}}{\partial a_{ji}(m)} \right]$$

and where $\Omega^v_{i,j}$ are defined analogously to $\Omega_{i,j}$, but with the planner utility functions.

Proof. The proof follows as usual by writing country social welfare as $U_i + (V_i - U_i)$ and comparing the planner and agent first order conditions. ∎

Globally efficient policy accounts for spillovers onto the welfare of country planners in a standard way. However, it also must correct for the difference in preferences, yielding the first term $\Delta_{ji}(m)$.

From here, characterization of optimal quantity regulation follows as in Section 6, except with the spillovers defined above. Regulation of domestic agents accounts for both the preference difference and spillovers to country planner welfare, but does not account for spillovers to foreign planners. Regulation of foreign agents allows them to conduct activities only to the point that it increases domestic planner welfare. The result is uninternalized spillovers and unequal treatment.

The result for Pigouvian taxation is more subtle. Considering tax revenue collections with no monopolist distortion, we have the tax revenue collection $\tau_{ji}(m)a_{ji}(m)$. Note first that differentiating in $a_{ji}(m)$, we obtain the total revenue impact (assuming no monopoly rents)

$$\tau_{ji}(m) + \int_{i'} \frac{\partial \tau_{i'i}}{\partial a^A_i(m)} a_{i'i}(m) = \tau_{ji}(m) + \int_{i'} \Omega_{i',i}(m) di'$$

where we note that $\tau_{i,ji}(m)$ is now the benefit to the foreign agent net of the wedge placed by the foreign planner, which unwinds the preference difference. This results in the difference $\Delta_{ji}(m)$ being correctly accounted for. However, the spillovers defined above are the spillovers to the agent, not the planner. This implies setting correct policy requires $\Omega^v_{j,i} = \Omega_{j,i}$ when $j \neq i$. The simplest way for this requirement to hold is if spillovers onto foreign agents are limited to constraint set spillovers, for example the fire sales of the baseline model.

Finally, it should be noted that these results imply that country planners can achieve the cooperative outcome using Pigouvian taxation. However, this section does not address whether the cooperative outcome is superior to the non-cooperative outcome. This latter claim requires a normative stand on whether the preferences of the planner or the agent are the normatively legitimate preferences, which depends on the source of preference difference. Although interesting for future work, such analysis is beyond the scope of this paper.

E.7   A Finite Country Game

We now consider a game with a finite number of countries, and show that the optimality of Pigouvian taxation is obtained up to a set of new external reoptimization effects. Provided that these external reoptimization effects are negligible, for example in the limit with a large number of countries, the results of the paper are obtained.

Suppose that rather than a continuum of countries, we have a finite set $I = \{1, \ldots I\}$ of countries, each of measure $\frac{1}{I}$. To simplify exposition, we assume that there is a single action $M = \{m\}$ and rule out constraint sets. As a result, we write

$$\max_{a_i} U_i \left(u_i(a_i, I), u_i^A(a_i, a^A, I), W_i \right)$$
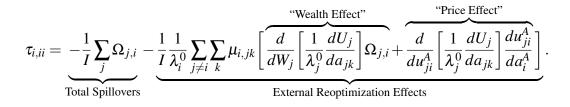
where we have

$$u_i(a_i, I) = \sum_{j \in I} u_{ij}(a_{ij}, I)$$

$$a_i^A = \frac{1}{I} \sum_{j \in I} a_{ij}$$

and so on. We use the functional dependency on I to capture scaling as we take the limit $I \to +\infty$, which will allow for home bias and marginal foreign investment.

The following Proposition characterizes the equilibrium under non-cooperative Pigouvian taxation. For expositional purposes, we focus on the domestic tax rate $\tau_{i,ii}$.

Proposition 32. *Suppose that Assumption 9 holds. In the finite country game, the non-cooperative equilibrium under Pigouvian taxation has the following tax rate on the domestic activity of domestic*

agents

$$
\tau_{i,ii} = \underbrace{-\frac{1}{I}\sum_j \Omega_{j,i}}_{\text{Total Spillovers}} - \underbrace{\frac{1}{I}\frac{1}{\lambda_i^0}\sum_{j\neq i}\sum_k \mu_{i,jk}\left[\overbrace{\frac{d}{dW_j}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\Omega_{j,i}}^{\text{``Wealth Effect''}} + \overbrace{\frac{d}{du_{ji}^A}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\frac{du_{ji}^A}{da_i^A}}^{\text{``Price Effect''}}\right]}_{\text{External Reoptimization Effects}}.
$$

where $\mu_{i,jk}$ is a Lagrange multiplier defined in the proof.

In the finite country game, the intuition behind the internalization of foreign spillovers is the same as the baseline model. However, there is also an additional set of *external reoptimization effects* that arise due to global monopoly power: the *entire contracts* of foreign agents are affected to first order by changes in domestic activities and aggregates, including allocations and aggregates outside the domestic economy.

These external reoptimization effects consist of two effects. The "Wealth Effect" arises because taxes on foreign agents reduce their wealth level, impacting their preferences over their entire contract. The "Price Effect" arises because a change in the domestic aggregate affects the benefit foreign agents get from activities, which in turn affects their entire contract. These additional forces amount to an additional form of monopolist distortion. When these monopolist distortions disappear, efficiency is restored.

In the baseline model, we have taken a continuous limit, where the marginal presence in foreign countries implies that the wealth effects and price effects are negligible.

Notice that if we characterized the tax rate $\tau_{i,ij}$ on foreign activities of agents, it would now account for the fact that agents' contribution to the foreign aggregate spills back to domestic agents. This would result in a form of excessive taxation, because the domestic planner is also taxing this externality. This term would disappear in limit, as the contribution to the foreign aggregate becomes negligible, so that excessive taxation disappears in limit, as in the baseline model.

E.7.1 Proof of Proposition 32

Given this setup, the demand functions of the country i multinational agent are given by the system of equations

$$\tau_{i,ii} = \frac{1}{\lambda_i^0} \frac{dU_i}{da_{ii}}$$

$$\tau_{i,ij} + \tau_{j,ij} = \frac{1}{\lambda_i^0} \frac{dU_i}{da_{ij}}$$

where we have defined $\lambda_i^0 = \frac{\partial U_i}{\partial W_i}$ and $\frac{dU_i}{da_{ij}} = \frac{\partial U_i}{\partial u_i} \frac{\partial u_{ij}}{\partial a_{ij}} + \frac{\partial U_i}{\partial u_i^A} \frac{\partial u_{ij}^A}{\partial a_{ij}}$.

Now, consider the optimization problem of country planner i, which is given by

$$\max_{a,\tau_i} U_i \left(u_i(a_i, I), u_i^A(a_i, a^A, I), A_i + \sum_{j \neq i} \left[\tau_{i,ji} a_{ji} - \tau_{j,ij} a_{ij} \right] \right)$$

subject to the above implementability conditions in all countries, taking as given τ_{-i}. Notice that tax collections do not need to be scaled by $\frac{1}{I}$ since countries have equal measure. We write the Lagrangian as

$$\mathcal{L}_i = U_i \left(u_i(a_i, I), u_i^A(a_i, a^A, I), A_i + \sum_{j \neq i} \left[\tau_{i,ji} a_{ji} - \tau_{j,ij} a_{ij} \right] \right) + \sum_{j,k} \mu_{i,jk} \text{FOC}_{jk}$$

where $\mu_{i,jk}$ is the Lagrange multiplier on the FOC of agent j for its action in country k.

From here, note that we have $\mu_{i,ik} = 0$, given the complete set of controls on domestic agents. Moreover, the FOC for the tax on foreign agents $\tau_{i,ji}$ is given by

$$0 = \lambda_i^0 a_{ji} - \mu_{i,ji} + \sum_k \mu_{i,jk} \frac{d\text{FOC}_{jk}}{dW_j} a_{ji}$$

$$= \lambda_i^0 a_{ji} - \mu_{i,ji} + \sum_k \mu_{i,jk} \frac{d}{dW_j} \left[\frac{1}{\lambda_j^0} \frac{dU_j}{da_{jk}} \right] a_{ji}$$

where the final term reflects wealth effects on foreign multinational agent k.

From here, let us take the FOC in the domestic action a_{ii}. We have

$$0 = \frac{dU_i}{da_{ii}} + \frac{1}{I} \frac{dU_i}{da_i^A} + \frac{1}{I} \sum_{j \neq i} \sum_k \mu_{i,jk} \frac{d\text{FOC}_{jk}}{da_i^A}.$$

Taking the derivatives, substituting in for $\mu_{i,ji}$, and applying Assumption 9, we obtain

$$0 = \frac{dU_i}{da_{ii}} + \frac{1}{I}\frac{dU_i}{da_i^A} + \frac{1}{I}\sum_{j\neq i}\left[\lambda_i^0 + \sum_k \mu_{i,jk}\frac{d}{dW_j}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\right]\frac{1}{\lambda_j^0}\frac{dU_j}{da_i^A} + \frac{1}{I}\sum_{j\neq i}\sum_k \mu_{i,jk}\frac{d}{du_{ji}^A}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\frac{du_{ji}^A}{da_i^A}$$

and finally, substituting in the tax rate,

$$\tau_{i,ii} = \underbrace{-\frac{1}{I}\sum_j \Omega_{j,i}}_{\text{Total Spillovers}} \underbrace{-\frac{1}{I}\frac{1}{\lambda_i^0}\sum_{j\neq i}\sum_k \mu_{i,jk}\left[\frac{d}{dW_j}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\Omega_{j,i} + \frac{d}{du_{ji}^A}\left[\frac{1}{\lambda_j^0}\frac{dU_j}{da_{jk}}\right]\frac{du_{ji}^A}{da_i^A}\right]}_{\text{External Reoptimization Effects}}.$$