Bail-Ins, Optimal Regulation, and Crisis Resolution

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Abstract

We develop a tractable dynamic contracting framework to study bank bail-in regimes. In the presence of a repeated monitoring problem, the optimal bank capital structure combines standard debt, which induces liquidation and provides strong incentives, and bail-in debt, which restores solvency but provides weaker incentives. Optimal policy increases use of bail-in debt when there are fire sales. The social optimum can be implemented using either contracts or a resolution authority. Our framework illuminates important policy questions including the optimal composition of loss-absorbing capital, the trade-offs between ex ante and ex post bail-in implementations, and the relationship between bail-ins and bailouts.

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1 Introduction

In the aftermath of the 2008 financial crisis, the question of orderly bank resolution has received significant attention on both sides of the Atlantic. In many advanced economies, governments employed bailouts to stem financial turbulence in late 2008 and early 2009.[1] Bailouts were arguably very effective at stabilizing financial markets, but have been criticized for leading to moral hazard and perverse redistribution.[2] As a result, the US (Title II of the Dodd-Frank Act) and the EU (Bank Recovery and Resolution Directive) have introduced "bail-ins," which allow resolution authorities to impose haircuts on (long-term) debt holders. The goals of bail-in regimes include ensuring that "creditors and shareholders will bear the losses of the financial company" and that "[n]o taxpayer funds shall be used to prevent the liquidation of any financial company under [Title II]" (Dodd-Frank Act Sections 204, 214). Nevertheless, important concerns emerge with the introduction of bail-ins. If bank solvency can be improved by introducing state contingencies into debt contracts, then what prevents banks from efficiently doing so using private contracts?[3] Moreover, why are bail-ins preferable from a regulatory perspective to other liability instruments—such as (outside) equity—or to bailouts as a recapitalization tool? Studying these issues requires a framework in which debt is part of an optimal liability structure.

The main contribution of this paper is to provide a simple and tractable dynamic contracting model to study bail-in regimes. In particular, the optimal contract of our model can be implemented with a combination of standard and bail-in debt. We leverage this framework to ask whether and how a planner should design bail-in regimes.

Our three period model centers on a repeated incentive problem in the tradition of Innes (1990). Banks raise funds from investors ex ante to finance lending. They must exert monitoring effort

[1] Two examples in the US are the Troubled Asset Relief Program (TARP), which authorized the government to buy toxic bank assets, and the Temporary Liquidity Guarantee Program (TLGP), which provided guarantees of bank debt.

[2] The Dodd-Frank Wall Street Reform and Consumer Act (Dodd-Frank Act) lists "protect[ing] the American taxpayer by ending bailouts" as one of its main objectives, and lists "minimiz[ing] moral hazard" (Section 204) as one of the purposes of bail-ins.

[3] For example, banks could use contingent convertible (CoCo) securities that have gained traction in Europe, which are an internal recapitalization instrument with a trigger event (for example, the bank's capital ratio falling below some threshold) for either a principal write-down or a conversion into equity.

at both the initial date (0) and the middle date (1) in order to ensure the quality of their loans at the onset of the lending relationship and in its continuation. Because monitoring effort is not contractible at either stage, the optimal contract must be written to incentivize effort. Our model remains tractable because we show that the date 1 Innes (1990)-style incentive constraint can be represented as constraint on pledgeable income (Holmstrom and Tirole 1997), that is as a minimum required agency rent in continuation. Banks write optimal liability contracts in a complete markets setting. They can choose to pledge liabilities that will exceed assets in some states. When liabilities exceed assets at date 1, the bank is liquidated. Liquidation implies the bank need not be paid its continuation agency rent, but it is costly because it reduces ex post recovery to both banks and investors.

Our main result is that the privately optimal bank contract can be implemented with a combination of two debt instruments: standard debt and bail-in debt. Standard debt has a face value that does not depend on the bank's date 1 return, and leads to insolvency and liquidation when bank returns are low.[4] This provides strong incentives to the bank for initial monitoring effort because liquidation eliminates the minimum continuation agency rent needed to incentivize effort at date 1. While standard debt ensures the bank receives no payoff in bad states, it requires costly liquidation that reduces investor repayment. Bail-in debt, on the other hand, avoids the resource costs of liquidation. It provides weaker incentives, however, because it transfers all cash flows to investors except for the minimum agency rent. Both instruments retain the upside for the bank, which encourages effort. Other instruments such as outside equity transfer cash flows from the bank to investors when returns are high, and so discourage effort. The bank finds it optimal not to use such instruments.

In practice, banks made little to no use of bail-in debt prior to its introduction in the post-crisis regulatory regime. Our model can generate this outcome as a corner solution of no bail-in debt. We show that higher expected recovery values from liquidation increase the use of standard debt and, if incentive effects are not too strong, reduce or eliminate the use of bail-in debt. Our model can

[4]Our model does not differentiate between standard short-term debt and (uninsured) deposits, and standard debt could be interpreted as a deposit. It could also be interpreted as a repurchase agreement, where insolvency arises when the value of collateral falls sufficiently far that it no longer covers the debt.

thus rationalize scant use of bail-in debt prior to the crisis if banks perceived recovery values to be relatively high, for example due to an expectation that bailouts would raise equilibrium liquidation prices during crises.

Although we frame our model around banks, the core optimal contracting framework can also apply to non-financial corporates. We provide an interpretation of bail-ins in our model as a Chapter 11 bankruptcy reorganization process, with liquidations corresponding to Chapter 7.

The second part of our contribution is to leverage our framework to study optimal policy. Individual banks fail to internalize that more liquidations can reduce the liquidation price due to fire sales. We study the problem of a social planner who uses a complete set of Pigouvian wedges to influence the bank's choice of contract structure, internalizing the fire sale externality. In principle, the social planner can incentivize banks to write any feasible contract, for example incorporating outside equity. However, we show that the social planner finds it optimal for banks to use a combination of standard and bail-in debt. The planner does not require banks to introduce other liability instruments such as outside equity into their capital structures. Intuitively, as with the private bank, the social planner also perceives bail-in debt to be better than outside equity for incentive provision. Relative to the private optimum, however, the social planner increases the use of bail-in debt and reduces the use of standard debt to mitigate the fire sale. We show that the social optimum can also be implemented by requiring minimum issuance of bail-in debt or by instituting a resolution authority that imposes write downs ex post. The structure of this resolution authority closely resembles the architecture of existing resolution regimes such as Title II.

Our model allows us to study the optimal composition of total loss-absorbing capital (TLAC). In practice, regulation includes both minimum bail-in debt requirements and minimum equity capital requirements. First, we extend the model to feature aggregate uncertainty over the magnitude of the fire sale (crisis severity). We show that optimal policy involves a dual trigger: the extent to which bail-in debt is written down increases with crisis severity. The planner implements this by requiring minimum issuance of bail-in debt and then imposing crisis-specific write-downs. Second, we extend the model to incorporate a continuous—rather than binary—effort choice. The optimal contract still combines standard and bail-in debt, but more total debt reduces effort in a continuous sense. Since lower effort raises the probability of liquidation, the planner has a new motive to regulate the total debt level—rather than only its composition—via a minimum equity requirement.

Our model has implications for the desirability of implementing bail-ins via an ex post resolution authority relative to ex ante contractual write-downs. The two approaches are equivalent in our baseline model, but the implementation requires perfect contractibility. When crisis severity is not perfectly contractible, a utilitarian resolution authority is always tempted to resolve as many banks as possible. Absent an incentive mechanism to control the resolution authority, the planner can do no better than to use an ex ante approach without a dual trigger. This leads to too few liqudiations in normal times and too many during severe crises. To manage the resolution authority's time consistency problem while respecting non-contractibility, we propose a plausible incentive mechanism that imposes larger punishments (rewards) for resolving more (fewer) banks. Our mechanism can be implemented under the structure of Title II: The resolution authority can resolve top tier holding companies without cost, but must pair resolution of operating subsidiaries with costly partial bailouts.

The post-crisis regime has emphasized replacing bailouts with bail-ins. We leverage our framework to study this important question. We introduce taxpayer-financed bailouts and ask whether a planner with commitment would find it desirable to commit to resolve some banks with bailouts. We show that the socially optimal contract is exactly the same as before, and that bail-ins fully replace bailouts. Intuitively, bail-ins and bailouts can achieve the same state contingencies in bank debt contracts, so that bailouts are at best Pareto inefficient resource transfers from taxpayers to banks. Our model thus substantiates a core principle of post-crisis regulatory reform, namely that the costs of bank resolution should be borne by bank investors and not by taxpayers.

Finally, we discuss our model in the context of too-big-to-fail institutions and demand-based theories of standard debt. We argue that partial liquidations through a good bank/bad bank approach can be preferable to an all-or-nothing resolution approach for large banks. We compare our model to demand-based (safety premia) theories of debt, and discuss how a combination of the two theories can provide a more complete view of bank capital structure.

Related Literature. We relate to a growing literature on bail-ins.[5] Keister and Mitkov (2021) show that banks may not write down their (deposit) creditors if they anticipate government bailouts, motivating mandatory bail-ins. Chari and Kehoe (2016) use a costly state verification model to show that standard debt is the only renegotiation-proof contract, implying that bail-ins serve only to reduce the level of standard debt. Pandolfi (2021) studies a related Holmstrom and Tirole (1997) incentive problem but takes standard debt contracts as given. The paper argues that bail-in resolution can lead to a credit market collapse by weakening incentives, thus motivating liquidations or partial bailouts. Mendicino et al. (2018) study the optimal TLAC composition for protecting insured deposits under private benefit taking and risk shifting, taking contracts as given. Walther and White (2020) show that precautionary bail-ins can signal adverse information and cause a bank run, resulting in an overly weak bail-in regime and motivating bail-in rules based on public information. Colliard and Gromb (2018) study how bail-ins and bailouts affect the negotation process of distressed bank restructuring. Bolton and Oehmke (2019) study the trade-offs between single- and multi-point-of-entry resolution of global banks. Dewatripont and Tirole (2018) study how bail-ins can complement liquidity regulation. Berger et al. (2020) provide a quantitative analysis of bailouts versus bail-ins. Our main contribution is to develop a tractable dynamic contracting framework based on an incentive problem, in which the privately optimal contract can be implemented with a combination of standard and bail-in debt. We leverage this framework to study the optimal design of bail-in regimes.

A vast literature studies theories of debt.[6] Our paper is closely related to the dynamic optimal contracting literature that studies repeated unobservable effort.[7] DeMarzo and Sannikov (2006) and

[5]There are also related literatures on contingent debt insturments (Flannery 2002, Raviv 2004, Sundaresan and Wang 2015, Pennacchi and Tchistyi 2019, with Flannery 2014 providing a broader overview) and optimal derivatives protection (Biais et al. 2016, Biais et al. 2019).

[6]Apart from incentive problems, theories of debt include costly state verification (Townsend 1979), liquidity provision (Diamond and Dybvig 1983), and asymmetric information (Myers and Majluf 1984, Nachman and Noe 1994). We also connect in particular to the related literature that emphasizes the monitoring role of banks (Diamond 1984, Holmstrom and Tirole 1997).

[7]Relatedly, debt contracts that become *more* expensive to service (higher interest rate or coupon payment) have

DeMarzo and Fishman (2007) implement optimal contracts using combinations of long-term debt, credit lines, and equity. Credit lines provide firms with financial flexibility following bad returns, but are costly to revolve. This costly financial flexibility serves a similar role to our bail-in debt. Long-term debt requires coupon payments, forcing default and liquidation once the credit line is exhausted. The liquidation threat parallels the role of our standard debt. Our paper contributes to this literature by incorporating its ingredients (repeated unobservable effort) and insights into a simple framework that we use to rationalize the coexistence of standard and bail-in debt, which are especially important in the bank regulatory context. We use our framework to study normative policy implications for the design of bail-in regimes. A number of papers separately emphasize the cash flow transfer (Jensen and Meckling 1976, Innes 1990, Dewatripont and Tirole 1994, Hébert 2018) and liquidation threat (Calomiris and Kahn 1991, Diamond and Rajan 2001) values of debt.[8] Our paper is also related to Bolton and Scharfstein (1996), who study strategic default with cash flow diversion. They emphasize the value of easy-to-renegotiate debt for preventing non-strategic default and hard-to-renegotiate debt for preventing strategic default.

A large literature studies macroprudential regulation in the presence of pecuniary externalities (Bianchi and Mendoza 2010, Bianchi and Mendoza 2018, Caballero and Krishnamurthy 2001, Dávila and Korinek 2018, Farhi et al. 2009, Lorenzoni 2008), aggregate demand externalities (Farhi and Werning 2016, Korinek and Simsek 2016, Schmitt-Grohé and Uribe 2016), and fiscal externalities (Chari and Kehoe 2016, Farhi and Tirole 2012), which motivate ex ante interventions such as leverage requirements. Our model rationalizes interventions that increase bail-in debt relative to other loss-absorbing instruments such as equity, thus emphasizing the importance of the composition rather than the overall level of debt.

been emphasized to promote liquidation in settings with repeated cash flow diversion (Biais et al. 2007) and screening (Manso et al. 2010).

[8] In similar spirits, Philippon and Wang (2022) studies use of bailout tournaments to provide equity-like incentives for lower risk taking while Zentefis (2021) studies disciplining effects of bailouts accompanied with managerial equity stake diluations.

2 Model

The three-period economy, $t = 0, 1, 2$, has a unit continuum of banks, investors, firms, and arbitrageurs. Banks are run by their owners (inside equity). Banks invest in a firm of variable scale $Y_0 = A_0 + I_0 > 0$ by using their own funds, $A_0 > 0$, and by signing contracts with investors to raise $I_0 \geq 0$. Investors are deep-pocketed at date 0 and can finance any investment scale. Firms are penniless and have an outside option of zero. We allocate the entire value of the bank-firm lending relationship to the bank. Arbitrageurs will buy projects (firms) that are liquidated prior to maturity.

Banks and investors are risk-neutral and do not discount the future. We denote bank consumption by (c_0, c_1, c_2), so that bank expected utility is given by $\mathbb{E}_0[c_0 + c_1 + c_2]$. We denote the payments to investors by (x_1, x_2). x_t is the actual amount received by investors, and is distinct from the face value of liabilities (that is, promised repayment). Investor expected utility from the bank contract is $\mathbb{E}_0[-I_0 + x_1 + x_2]$. Contracts are subject to limited liability constraints for banks, given by

$$c_0, c_1, c_2 \geq 0. \tag{1}$$

Limited liability is not required for investors, with $x_t < 0$ denoting investors making a payment to the bank. However, the optimal contract of the model will result in non-negative investor payoffs.

The economy features idiosyncratic uncertainty, but no aggregate uncertainty.[9]

We tailor our model to study the trade-off between standard debt and bail-in debt. Our baseline model will have no role for instruments such as outside equity, or for other trade-offs that affect the use of debt such as tax benefits. We consider such extensions in the appendix.

2.1 Bank Projects

Banks extend financing to firms, thereby establishing a lending and monitoring relationship with those firms. When first extending funds to firms, banks monitor their borrowers, ensuring that the projects undertaken are of good quality. In doing so, banks develop specialized knowledge of that

[9] See Appendix B.2 for aggregate uncertainty.

firm, and are uniquely able to monitor and collect from the firm in continuation. This relationship is the foundation of banking in our model. Because we allocate all value of the lending relationship to the bank, we omit firms going forward and refer to the relationship as bank projects.

Our model proceeds similarly to a multi-period version of Innes (1990). At each of dates 1 and 2, the bank experiences a stochastic quality shock R_t, which adjusts the project scale to $Y_t = R_t Y_{t-1}$. This means the final project scale is $Y_2 = R_2 R_1 Y_0$. The project pays off one unit of the consumption good per unit of final scale if held to maturity at date 2, but yields no dividend at date 1. The shocks R_t are independent and idiosyncratic, with densities $f_{t,e_{t-1}}$ that have support over $[\underline{R}, \overline{R}]$. Both states R_t are contractible. However, the distribution of R_t depends on the bank's non-contractible monitoring effort at the prior date, $e_{t-1} \in \{H, L\}$, where H is high monitoring effort and L is low monitoring effort. We think of e_0 as an initial monitoring/screening of borrowers and e_1 as continued due diligence and collections. For notational convenience, we normalize $\mathbb{E}[R_2|e_1 = H] = 1$.

Define the likelihood ratio $\Lambda_t(R_t) \equiv \frac{f_{t,L}(R_t)}{f_{t,H}(R_t)}$. We assume that $f_{t,e}$ satisfies the *monotone likelihood ratio property* (MLRP), that is $\Lambda_t' < 0$. MLRP is a standard assumption in generating debt contracts, and implies that high (low) returns are a signal that the bank exerted high (low) monitoring effort. However, if a bank exerts low monitoring effort at date $t-1$, it receives a private benefit $B_{t-1} Y_{t-1}$, with $B_0, B_1 > 0$. This private benefit is enjoyed by the bank even if the project is terminated at a later date. We assume throughout the paper that the bank finds it optimal to write a contract that induces high monitoring effort at both dates, that is $e_0 = e_1 = H$.[10]

Because monitoring effort is non-contractible, the bank sequentially chooses effort to maximize its utility after contracts have been signed. Effort e_1 is chosen after R_1 has been observed,

[10] Given our normalization on the date 2 return, jointly sufficient conditions for banks to find it optimal to write a contract inducing high effort are

$$1 < \mathbb{E}[R_1|e_0 = H]$$

$$\mathbb{E}[R_1|e_0 = L] + B_0 < 1$$

$$\mathbb{E}[R_0|e_0 = H](B_1 + \mathbb{E}[R_2|e_1 = L]) < 1$$

in which case the project is NPV positive under repeated high effort, but NPV negative if low effort is ever exerted (note that these conditions jointly imply that $e_0 = e_1 = L$ is not optimal, since $\mathbb{E}[R_0|e_0 = L](B_1 + \mathbb{E}[R_2|e_1 = L]) + B_0 < \mathbb{E}[R_0|e_0 = L]\frac{1}{\mathbb{E}[R_1|e_0 = H]} + B_0 < 1$). If optimal contracts induced low effort, incentive compatibility would either not bind or would lead the bank to adopt a capital structure that rewarded the bank for low returns and punished it for high returns.

assuming the bank is not liquidated at date 1. Given the consumption profile c_t under the contract signed, the bank exerts high monitoring effort at date 1 if $\mathbb{E}[c_1(R_1)+c_2(R_1,R_2)|e_1=H] \geq \mathbb{E}[c_1(R_1)+c_2(R_1,R_2)|e_1=L]+B_1R_1Y_0$ (we include c_1 without loss). We rearrange this incentive compatibility constraint to obtain the representation

$$\mathbb{E}\left[(c_1(R_1)+c_2(R_1,R_2))(1-\Lambda_2(R_2))\middle| e_1=H\right] \geq B_1R_1Y_0, \tag{2}$$

Higher payoffs $c_2(R_1,R_2)$ in states where the likelihood ratio $\Lambda_2(R_2)$ is low relax incentive compatibility because these states signal that monitoring effort was high (note that c_1 drops out).

Because equation (2) must hold for any R_1 under which the bank continues operating at date 2, banks' effort choice at date 0 can be undertaken assuming that high effort will be undertaken at date 1. It is helpful therefore to define

$$c(R_1) \equiv \mathbb{E}\left[c_1(R_1)+c_2(R_1,R_2)\middle| e_1=H\right],$$

which is expected bank consumption when state R_1 is realized under an incentive compatible contract, with $c(R_1)=0$ if the bank is liquidated at date 1. Analogous to the derivation of equation (2), incentive compatibility at date 0 is given by,

$$\mathbb{E}\left[c(R_1)(1-\Lambda_1(R_1))\middle| e_0=H\right] \geq B_0Y_0, \tag{3}$$

so that higher expected payoffs $c(R_1)$ relax date 0 incentive compatibility when the likelihood ratio $\Lambda_1(R_1)$ is low, with the same logic as for date 1 incentive compatibility.

Banks can liquidate their project prematurely at date 1, in which case the project yields $\gamma Y_1 < Y_1$ units of the consumption good at date 1 and nothing at date 2, with the proceeds accruing entirely to investors.[11] Liquidations will be ex post inefficient (money burning) in a sense formalized

[11]We think of the liquidation discount as arising from selling projects to second-best users (Section 2.6), who have not developed the knowledge of the firm lending relationship that the bank has. In this sense, we also assume the banker is not severable from the bank.

below. Because the project pays out at date 2, there is no resource cost associated with insolvency (liabilities exceed assets) at date 2.

2.2 Investors

In order to raise investment $I_0 \geq 0$, banks pledge state-contingent repayment to investors at date 0. The banks' state-contingent repayment contract is a triple (α_1, x_1, x_2). $\alpha_1(R_1) \in \{0,1\}$ specifies whether the bank will be liquidated at date 1, with $\alpha_1 = 1$ denoting liquidation. $x_1(R_1)$ is actual repayment to investors at date 1 and $x_2(R_1, R_2)$ actual repayment at date 2, in each case with negative values denoting a payment from investors to the bank. We first set up the problem in the repayment space, and then in Section 2.5 map into promised liabilities.

Given a repayment contract (α_1, x_1, x_2), the expected payoff to investors at date 1 under an incentive compatible contract is given by

$$x(R_1) = \alpha_1(R_1)\gamma R_1 Y_0 + (1 - \alpha_1(R_1))\mathbb{E}\left[x_1(R_1) + x_2(R_1, R_2)\middle| e_0 = H\right]. \tag{4}$$

When the bank is not liquidated, we have $c_1(R_1) + c_2(R_1, R_2) = R_1 R_2 Y_0 - x_1(R_1) - x_2(R_2)$. This means that expected bank consumption at date 1 is

$$c(R_1) = (1 - \alpha_1(R_1))(R_1 Y_0 - x(R_1)) \tag{5}$$

given that we normalized $\mathbb{E}[R_2|e_1 = H] = 1$.

The voluntary investor participation constraint states that investors must at least break even in expectation on the contract they signed. It is given by

$$Y_0 - A_0 \leq \mathbb{E}\left[x(R_1)\right| e_0 = H]. \tag{6}$$

where $I_0 = Y_0 - A_0$ is the amount financed by investors.

Finally, we assume repayment monotonicity at both dates: $x(R_1)$ must be monotone in R_1 (that is, expected investor repayment is monotone in R_1) and $x_2(R_1,R_2)$ must be monotone in R_2 (for given R_1). Formally, these assumptions are

$$R_1 \geq R_1' \Rightarrow x(R_1) \geq x(R') \tag{7}$$

$$R_2 \geq R_2' \Rightarrow x_2(R_1,R_2') \geq x_2(R_1,R_2'). \tag{8}$$

Monotonicity is a common assumption in many settings of optimal contracts or security design, although in Appendix B.3 we characterize optimal contracts without monotonicity and argue our insights for optimal policy still hold.[12] It generates the flat face value of liabilities in high-return states. Note that monotonicity does not preclude a bank from issuing an individual instrument whose payoff profile is non-monotone, but rather states that the overall structure summed across instruments must be monotone.

2.3 Bank Optimal Contracting

The bank signs a contract with investors, which specifies initial funds provided I_0 in exchange for a promised repayment scheme (α_1, x_t). The contract must be feasible, which we now define.

Definition 1 (Feasible Contracts)**.** A bank contract $\mathscr{C} = (\alpha_1, x_t, I_0, c_t, Y_0)$ is *feasible* if it satisfies: (1) limited liability; (2) incentive compatibility at date 1 when the bank continues; (3) incentive compatibility at date 0; (4) determination of x; (5) determination of c; (6) investor participation; (7) date 1 montonicity; (8) date 2 monotonicity; and the final budget constraint under continuation, $c_1(R_1) + c_2(R_1,R_2) = R_1R_2Y_0 - x_1(R_1) - x_2(R_2)$.

Banks choose a feasible contract $\mathscr{C}$ to maximize their own expected utility,

$$\mathbb{E}\left[c(R_1)|e_0 = H\right]. \tag{9}$$

[12]For example, one justification offered is that banks would be incentivized to pad their returns, for example by secretly borrowing from a third party (Nachman and Noe 1990, Nachman and Noe 1994).

2.4 Pledgeable Income and Continuation Agency Rent

Much of the tractability of our model comes arises because we show that the date 1 Innes (1990) incentive compatibility constraint can be re-represented as Holmstrom and Tirole (1997) style pledgeability constraint on income. This implies a minimum continuation agency rent for the bank, and is implemented by a continuation debt contract. We formalize this notion in the following lemma.

Lemma 2. *When $\alpha_1(R_1) = 0$ and the bank is not liquidated, the optimal contract features debt in the continuation contract. That is, there is a threshold return $R_2^u(R_1)$ such that*

$$x_1(R_1) + x_2(R_1, R_2) = \begin{cases} R_2 R_1 Y_0, & R_2 \leq R_2^u(R_1) \\ R_2^u(R_1) R_1 Y_0, & R_2 > R_2^u(R_1) \end{cases}$$

Incentive compatibility at date 1 (2) can be represented as a constraint on pledgeable income,

$$c(R_1) \geq b R_1 Y_0 \tag{10}$$

where $b \equiv \int_{\overline{R}_2^u}^{\overline{R}} [R_2 - \overline{R}_2^u] f_H(R_2) dR_2$ and where $\int_{\overline{R}_2^u}^{\overline{R}} [R_2 - \overline{R}_2^u](1 - \Lambda_2(R_2)) f_H(R_2) dR_2) = B_1$.[13]

Lemma 2 allows us to re-represent the problem of incentive compatibility at date 1 as a required agency rent b to the bank in continuation as a fraction of its expected final project value, $Y_2 = \mathbb{E}[R_2 | e_1 = H] R_1 Y_0 = R_1 Y_0$. It equivalently tells us that $(1-b) R_1 Y_1$ is the maximum value of the project that is pledgeable to investors. It also provides a contract that respects liability monotonicity at date 2 (equation 8) and the final budget constraint.

For the remainder of the paper, we exploit Lemma 2 to define contracts in terms of date 1 pledged expected values (x, c). This means that x can be interpreted as the market value of external

[13] In equilibrium, date 1 incentive compatibility does not always bind, in which case there can be multiple monotone continuation contracts that achieve the same bank and investor expected payoffs and maintain incentive compatibility at both dates. For consistency, we use a debt contract when (2) does not bind. (2) does not bind when (10) does not bind, that is bank consumption exceeds its minimum agency rent.

liabilities at date 1, with Lemma 2 characterizing the final debt level that achieves that market value.

Finally, we assume that $\gamma \leq 1-b$, which means that pledgeable income is always weakly higher by continuing the project than by liquidating it. As a result, liquidations destroy value ex post for both the bank and investors.

2.5 Liabilities, Market Value, and Face Value

We now map the liquidation rule α_1 and market value x into a more natural setting of promised liabilities. A promised repayment is $L(R_1)$. If $L(R_1) \leq (1-b)R_1Y_0$, then the bank is solvent and hence $x(R_1) = L(R_1)$, that is market value equals promised repayment. If instead $L(R_1) > (1-b)R_1Y_0$, then promised repayment exceeds pledgeable income. In this case, the bank will be liquidated, $\alpha_1(R_1) = 0$, and actual repayment will be $x(R_1) = \gamma R_1 Y_0$. For the rest of the paper, we represent contracts in terms of promised liabilities L.

We refer to L as the face value of liabilities. Face value is the same as promised market value if we implement the optimal contract with short-term state-contingent liabilities that can be rolled over at price 1 if the bank is solvent. The two notions exactly coincide under more general maturity structures if we assume R_2 has a degenerate distribution at 1 if $e_1 = H$ (Holmstrom and Tirole 1997).

Liquidations and Ex Post Renegotiation. Our model has ruled out renegotiation at date 1 when liabilities exceed pledgeable income. Banks can implement the outcome of any feasible renegotiation using state-contingent contracts, meaning renegotiation cannot increase ex ante welfare. However, renegotiation creates a time consistency problem: since $\gamma < 1-b$, liquidations are *ex post* Pareto inefficient, motivating banks and investors to renegotiate ex post. Thus, if a bank ex ante finds it optimal to pledge a liability structure that results in liquidations, it would also prefer to rule out (or make as difficult as possible) renegotiation. For example, if debt is an optimal contract because it liquidates the bank, a bank might implement it using runnable demand deposits dispersed over many creditors in order to make renegotiation difficult. This is in keeping with parts of the banking literature that emphasize the value of demand deposits as a threat to liquidation (Calomiris and Kahn 1991, Diamond and Rajan 2001). This idea is also consistent with the design of the Title II process, which focuses debt write-downs on long-term debt and not on short-term debt or deposits, due to a concern that "the threat of a restructuring may cause clients to flee and short-term creditors to withdraw their capital" (French et al. 2010).[14]

2.6 Arbitrageurs and Liquidation Prices

We introduce a simple fire sale externality into the model to motivate studying optimal policy. Banks are small and take γ as given. A representative arbitrageur purchases bank projects at date 1 and converts them into the consumption good using a technology $\mathscr{F}(\Omega)Y_0$, where Ω is the fraction of total bank projects purchased relative to initial scale. Arbitrageur surplus at date 1 from purchasing projects is $\mathscr{F}(\Omega)Y_0 - \gamma\Omega Y_0$, yielding an equilibrium liquidation price

$$\gamma = \gamma(\Omega) = \frac{\partial \mathscr{F}}{\partial \Omega}, \qquad \Omega = \int_{R_1} \alpha(R_1) R_1 f_{1,H}(R_1) dR_1. \tag{11}$$

If $\frac{\partial \mathscr{F}}{\partial \Omega} = \gamma$ is a constant which does not depend on Ω, then there is no fire sale. By contrast if $\frac{\partial \gamma}{\partial \Omega} = \frac{\partial^2 \mathscr{F}}{\partial \Omega^2} < 0$, there is a *fire sale*: more liquidations reduce the liquidation value. To ease exposition, we assume constant elasticity of the liquidation price, $\frac{\Omega}{\gamma}\frac{\partial \gamma}{\partial \Omega} = -\sigma$, with $\sigma < 1$.[15] $\sigma = 0$ is the case of a constant liquidation discount.

Arbitrageurs have initial wealth $\overline{A} - A_0$, but cannot borrow against future income. Their total date 0 welfare is $u(\overline{A} - A_0) + (\mathscr{F}(\Omega)Y_0 - \gamma\Omega)Y_0$, with $u'(\overline{A}) > 1$ so that the borrowing constraint binds. The intertemporal borrowing constraint gives rise to a distributive externality (Dávila and Korinek 2018) that makes fire sales Pareto inefficient (see Appendix B.1). The inefficient

[14]Moreover, Title II resolution includes a "clean holding company" requirement, which bars the top tier holding company (the target of resolution) from issuing any short-term debt to external investors (12 CFR §252.64).

[15]Formally, we set $\mathscr{F} = \overline{\gamma}(\sigma)\frac{\Omega^{1-\sigma}-1}{1-\sigma}$. We restrict $\sigma < 1$ so that total funds raised from liquidations, $\frac{\partial \mathscr{F}}{\partial \Omega}\Omega = \overline{\gamma}(\sigma)\Omega^{1-\sigma}$, increases in total liquidations and goes to zero as liquidations go to zero. We make the constant $\overline{\gamma}(\sigma) = \overline{\gamma}\underline{\Omega}^{\sigma}$ for some sufficiently small $\underline{\Omega}$, which ensures that $\gamma(\sigma, \underline{\Omega}) = \overline{\gamma}$ for any σ and that $\frac{\partial \gamma}{\partial \sigma} < 0$ for all $\Omega \geq \underline{\Omega}$. We generally focus on cases where either $\sigma = 0$ or $\Omega \geq \underline{\Omega}$ in equilibrium.

distributive externality arises because the borrowing constraint creates means arbitrageurs have a higher marginal value of wealth at date 0 than at date 1.[16]

3 Privately Optimal Contracts

In this section, we show that the privately optimal contract written by banks can be implemented by a combination of two debt instruments. The first, which we call *standard debt*, has a fixed face value that does not depend on R_1, and liquidates the bank in low-return states. The second, which we call *bail-in debt*, has a face value that can be written down based on R_1, and restores bank solvency when total debt exceeds the pledgeable income.

We begin by characterizing the privately optimal bank contract in terms of two thresholds in the date 1 return, R_ℓ and R_u. We then associate these two thresholds with the two debt instruments. These thresholds summarize the privately optimal liability structure of the bank.

Proposition 3. *A privately optimal bank contract has a liability structure*

$$L(R_1) = \begin{cases} (1-b)R_\ell Y_0, & R_1 \leq R_\ell \\ (1-b)R_1 Y_0, & R_\ell \leq R_1 \leq R_u \\ (1-b)R_u Y_0, & R_u \leq R_1 \end{cases}$$

where $0 \leq R_\ell \leq R_u \leq \overline{R}$. The bank is liquidated if and only if $R_1 \leq R_\ell$. These thresholds, when interior and not equal,[17] *are given by*

$$\underbrace{\mu b\left(\Lambda_1(R_\ell) - 1\right)}_{\text{Incentive Provision}} = \underbrace{b + \lambda\left(1 - b - \gamma\right)}_{\text{Liquidation Costs}} \tag{12}$$

[16]This externality is similar to the case where there are multiple date 1 aggregate states, and incomplete markets prevent arbitrageurs from equating the marginal value of wealth across date 1 states.

[17]For the remainder of the paper, we assume that the thresholds are interior and not equal, except when explicitly stated otherwise. Generally speaking, R_ℓ will be interior when the likelihood ratio $\Lambda(\underline{R})$ is sufficiently large, that is when $\underline{R}$ is a sufficiently good signal of low effort. R_u will be interior when $\Lambda(\overline{R})$ is sufficiently small and $\mu > \lambda - 1$, that is when $\overline{R}$ is a sufficiently good signal of high effort. Section 3.3 studies the possibility that $R_\ell = R_u$.

$$0 = \mathbb{E}\left[\underbrace{\lambda - 1}_{\text{Investor Repayment}} \; - \; \underbrace{\mu\left(1 - \Lambda_1(R_1)\right)}_{\text{Incentive Provision}} \;\middle|\; R_1 \geq R_u, e = H \right] \tag{13}$$

where $\mu > 0$ is the Lagrange multiplier on date 0 incentive compatibility (3) and $\lambda > 1$ is the Lagrange multiplier on investor participation (6).

All proofs are contained in Appendix A.[18] An optimal bank contract is defined by three regions, illustrated in Figure 1. In the first region, $R_1 \leq R_\ell$, the face value of liabilities exceeds pledgeable income and the bank is liquidated following a low date-1 return. In the second region, $R_\ell \leq R_1 \leq R_u$, all pledgeable income of the bank is transferred to investors, leaving the bank with only the continuation agency rent to induce high effort at date 1. In the third region, $R_1 \geq R_u$, all additional income generated by higher returns accrues to the bank and investors receive the same amount $(1-b)R_u Y_0$ regardless of the return realization.

Equation (12) describes the marginal trade-off the bank faces in choosing the liquidation threshold R_ℓ. On the one hand, liquidating the bank results in a total resource loss $b + \lambda(1 - b - \gamma)$ to the bank and investors. On the other hand, pledging to liquidate the bank provides higher-powered monitoring incentives at date 0, reflected in the term $\mu b\left(\Lambda_1(R_\ell) - 1\right)$, by depriving the bank of its continuation agency rent $bR_\ell Y_0$ necessary to ensure date 1 incentive compatibility. The optimal choice of R_ℓ trades off these two effects. In particular, the liquidation threshold features $\Lambda_1(R_\ell) > 1$, that is at R_ℓ the likelihood ratio is greater than 1 and is more in line with low effort having been exerted.

Equation (13) summarizes the marginal trade-off in choice of R_u. On the one hand, the binding investor participation constraint implies that transfering pledgeable income to investors is valuable because it allows the bank to increase project scale ($\lambda - 1 > 0$). On the other hand, increasing the total debt level reduces bank consumption in high-return states, where the likelihood ratio $\Lambda_1(R_1)$ is

[18] In the proof of this proposition, see Appendix A.2.1 for a comment on non-uniqueness of total promised repayment $L(R_1)$ below R_ℓ. Non-uniqueness arises in this region because any face value of liabilities above $(1-b)R_1 Y_0$ results in bank liquidation. We have chosen the face value of liabilities that correspond to standard debt, which seems most natural in the context of banks and bail-ins. Moreover, uniqueness is restored if there is an $\varepsilon \to 0$ premium for standard debt, for example due to tax benefits of debt. The face value of liabilities is unique above R_ℓ.

low and the signal of high effort is stronger. This weakens bank monitoring incentives and tightens the date 0 incentive compatibility constraint (3). The optimal choice of R_u equalizes these two effects on the margin.

We now associated the privately optimal contract with two liability instruments: standard debt and bail-in debt.

Corollary 4. *The privately optimal contract can be implemented with a combination of standard debt with face value $(1-b)R_\ell Y_0$, which cannot be written down contingent on the idiosyncratic state R_1, and bail-in debt with face value $(1-b)(R_u - R_\ell)Y_0$, which can be written down contingent on the idiosyncratic state.*

Corollary 4 provides a natural implementation of the optimal contract in this setting. In the region $R_1 \leq R_\ell$, standard debt face value exceeds pledgeable income and results in liquidation. In the region $R_\ell \leq R_1 \leq R_u$, bail-in debt is written down to $(1-b)(R_1 - R_\ell)Y_0$, so that the bank is just held to its minimum agency rent. In the region $R_1 \geq R_u$, bail-in debt is not written down and investors receive the full face value of both contracts.

For the remainder of the paper, we associate standard and bail-in debt with the thresholds R_ℓ and R_u, respectively, rather than writing out their associated (face value) liabilities.

Alternate Contract Implementations. The implementation of Proposition 3 that most closely resembles bail-in regimes in practice is short-term standard debt and long-term bail-in debt.[19] This implementation is not unique. First, the maturity structure is not unique: it could be implemented (in principle) with entirely short-term debt, or with some long-term standard debt (and sufficient short-term debt to force liquidation).[20] Second, the implementation could involve different instruments

[19] Bail-in debt can also be interpreted as a *contingent convertible* (CoCo) debt instrument (see Avdjiev et al. 2017 and Flannery 2014 for more background). Bail-in debt in our model is a principal write-down CoCo debt security that applies at the point of non-viability.

[20] Note that due to the liquidation discount, R_ℓ can be implemented with $\gamma R_\ell < (1-b)R_\ell$ of short-term standard debt and $(1-b-\gamma)R_\ell$ of long-term standard debt, since short-term debt can force full liquidation at date 1 due to the discount when $R_1 < R_\ell$.

entirely, for example: (i) standard debt and outside equity, with a managerial compensation scheme to pay the bank $c(R_1)$;[21] (ii) partial-bail-in debt, which can only be written down to $(1-b)R_\ell Y_0$.

3.1 The Role of Agency Problems and Costly Liquidation

Our model features three key ingredients: an initial incentive problem ($B_0 > 0$), a continuation agency rent ($b > 0$), and costly liquidations ($\gamma > 0$). Absent all three ingredients, the optimal contract in our model can be implemented without combining standard and bail-in debt.

Proposition 5. *The privately optimal contract can be implemented with a single liability instrument if $B_0 = 0$, $b = 0$, or $\gamma = 1$. In particular,*

(a) If $B_0 = 0$, then the privately optimal contract can be implemented with bail-in debt.

(b) If $b = 0$, then the privately optimal contract can be implemented with long-term debt.[22]

(c) If $\gamma = 1$, then the privately optimal contract can be implemented with standard debt.

When $B_0 = 0$, there is no required agency rent at date 0 (incentive compatibility is maintained with any contract with monotone bank payoff), but there is a required agency rent at date 1. Therefore, the bank can ensure incentive compatibility at both dates by using a debt contract set according to Lemma 2. However, R_1 still requires the contract to adjust the level of debt in continuation to maintain date 1 incentive compatibility. As a result, a bail-in debt contract suffices. As a result, a date 0 incentive problem, that is $B_0 > 0$, is necessary in our model to generate an optimal contract that combines standard and bail-in debt.

The second and third cases of Proposition 5 show that $B_0 > 0$ alone is not sufficient to generate a privately optimal contract that combines standard and bail-in debt. When $B_0 > 0$, the privately optimal contract employs some debt instrument for ex ante incentive reasons. In the second case with $b = 0$, all income is pledgeable to investors, and the bank can guarantee zero consumption,

[21] Under this implementation, outside equity would have the same payoff profile bail-in debt did in Corollary 4.
[22] Note that it could also be implemented with bail-in debt.

$c(R_1) = 0$, without having to liquidate the project prior to maturity. This case is analogous to Innes (1990), and means that the bank finds it optimal to only use long-term debt to avoid costly liquidations. In contrast in the third case, with $\gamma = 1$ but $b > 0$, there is a limit to pledgeable income, but no bankruptcy costs from liquidation. Banks can repay any amount $x(R_1) \leq R_1 Y_0$ by liquidating bank projects, and the pledgeability constraint ceases to be relevant. Banks use only standard debt.

In all cases of Proposition 5, the key property of debt is the cash flow transfer from the bank to investors in low-return states and fixed repayment in high-return states (e.g., Innes 1990, Hébert 2018). In absence of ex ante incentive problems, cash flow transfer is achieved with bail-in debt. In the absence of continuation agency rents, cash flow transfer is achieved with long-term debt. In the absence of bankruptcy costs, cash flow transfer is achieved with standard debt. However, if there are ex ante incentive problems, continuation agency rents, and bankruptcy costs, then bail-in debt cannot enact a full cash flow transfer, while standard debt enacts a full cash flow transfer at a resource cost. A role emerges for both forms of debt in the optimal contract.

3.2 Bail-in Debt or (Outside) Equity?

Proposition 3 and Corollary 4 highlight why bail-in debt can be a valuable loss-absorbing instrument for banks, relative to equity. Bail-in debt combines the incentive properties of standard debt with the loss-absorbing properties of equity. It generates a maximal cash flow transfer below R_u and a flat investor payoff above R_u, similar to standard debt, but does so without liquidating the bank (as standard debt does). By contrast, equity transfers the upside of the bank to investors. Transferring more of the upside of the bank to investors worsens incentives due to MLRP, since higher returns signal that the bank likely exerted high effort. Bail-in debt recapitalizes the bank in the same manner as equity on the downside, but generates better incentives on the upside. This leads banks to prefer bail-in debt to equity as a loss-absorbing instrument even in the private optimum without any intervention by a social planner.[23]

[23] In Appendix B.6 we add a role for outside equity in the model by incorporating risk aversion and risk shifting. We show that the core trade-off between standard debt and bail-in debt exists as in the baseline model.

3.3 Why Didn't Banks Issue Bail-in Debt before 2008?

Although Proposition 3 states that privately optimal bank contracts combine standard and bail-in debt, bail-in debt is largely a post-crisis innovation that was "introduced" by bail-in regulation. We now show that if liquidation values are sufficiently high, the bank can find it optimal to use only standard debt.

Proposition 6. *Standard debt, R_ℓ, and total debt, R_u, are both increasing in γ.*

Moreover, suppose that $R_\ell f_H(R_\ell) \leq \frac{(\lambda-1)(1-b)}{b+\lambda(1-b-\gamma)}$.[24] *Then, there exists $\overline{\gamma} \in [0, 1-b]$ such that:*

(a) If $\gamma > \overline{\gamma}$, then there is no bail-in debt ($R_\ell = R_u$).

(b) If $\gamma < \overline{\gamma}$, then higher γ decreases bail-in debt $R_u - R_\ell$

Proposition 6 shows that higher liquidation values always lead the bank to issue more standard debt and more total debt. The intuition comes from equation (12): the marginal liquidated bank has $\Lambda_1(R_\ell) > 1$, meaning its return R_ℓ signaled lower effort was exerted. An increase in R_ℓ slackens incentive compatiblity by encouraging higher effort, which allows the bank to issue more total debt R_u. An increase in γ reduces the cost of liquidations, and leads the bank to increase R_ℓ and R_u.

The second part of the proposition then tells us that as long as $R_\ell f_H(R_\ell)$ is not too large, the increase in standard debt increases total debt but crowds out bail-in debt. Intuitively, the upper bound on $R_\ell f_H(R_\ell)$ implies that the marginal incentive effect of an increase in R_ℓ is not so large that it allows a more than one-for-one increase in R_u. If the condition is violated, then both standard and bail-in debt increase γ.

Proposition 6 helps shed light on why banks didn't make use of bail-in debt prior to the 2008 crisis. Small banks perceive a higher γ than that of a social planner that internalizes the fire sale (Section 4). Moreover, expectation of bailouts would increase recovery values both directly by protecting creditors (debt guarantees such as TLGP) and indirectly by stabilizing resale markets

[24]Note that we know that $\lambda \geq \mathbb{E}[R|e_0 = H]$, and so we can bound the RHS away from zero.

(asset purchases such as TARP). Resale market stabilization in particular can help explain why even smaller banks, which may not have expected direct bailouts, would nevertheless not use bail-in debt.

3.4 Nonfinancial Firms and Bankruptcy

Although our model is framed in terms of banks, our optimal contracting framework could also be applied to nonfinancial corporates. This suggests that nonfinancial corporates might also wish to use bail-in debt. One interpretation in this spirit can be provided in the context of bankruptcy. Chapter 7 of the US Bankruptcy Code provides for liquidation, while Chapter 11 provides for reorganization and debt restructuring process. Chapter 11 reorganization requires that creditors in impaired classes should either have voted to accept the plan or be no worse off than in Chapter 7 liquidation (11 U.S.C. §1129).[25] Impaired classes can push for liquidation under Chapter 7, or can accept concessions such as the bail-in haircuts of our model in a reorganization plan. It is well known that different creditors have different incentives in the renegotiation process (e.g., Bolton and Scharfstein 1996). Senior secured creditors often favor liquidation to avoid further impairment, while junior unsecured creditors often prefer reorganization to capitalize on convexity. Moreover, dispersing (secured senior) claims over many creditors can lead to disorderly collateral seizures and hold out problems that inhibit reorganization, whereas concentrating claims can mitigate hold out problems. One interpretation of our model is that standard debt parallels dispersed senior secured claims that promote Chapter 7, and bail-in debt parallels concentrated junior unsecured claims that promote Chapter 11.[26]

One important concern is that Chapter 11 may be imperfectly designed for banks, in part because the automatic stay might disrupt liquidity services from short-term debt (French et al. 2010). Reflecting this, the US Treasury Department has adopted a proposal for a Chapter 14 bankruptcy process, with the aim of creating a bankruptcy process tailored to banks (Scott and Taylor 2012,

[25] This mirrors the no-creditor-worse-off condition of bail-in regimes, see Section 4.2.

[26] Drawing on this analogy, our normative results in Section 4 could also be viewed as rationalizing intervention in the non-financial corporate bankruptcy process, for example requiring greater issuance of easier to resolve junior unsecured debt. Relatedly, see Antill and Clayton (2021) for a related analysis of optimal intervention in the insolvency process for nonfinancials.

US Department of Treasury 2018). Interestingly, our model suggests that difficulties of resolving banks under Chapter 11 results from banks' deliberate capital structure decisions rather than a shortcoming of the Chapter 11 process, and that exemption from the automatic stay may be a desirable means of promoting liquidation. However, our normative results in Section 4 suggest a role for the government to require greater use of bail-in debt even under a Chapter 14 process.

In practice, Chapter 11 reorganization is common for distressed large nonfinancials and conversion to Chapter 7 liquidation is relatively uncommon (Wruck 1990, Bernstein et al. 2019, Antill 2022). In addition, nonfinancials often have lower leverage (lower R_u), while financials often use very short maturity contracts (higher R_ℓ) such as repurchase agreements that are exempt from the automatic stay (Gorton and Metrick 2012).[27] One explanation for why nonfinancials may make themselves fairly resolvable and have lower leverage is that they may have fairly high average liquidation discounts (Proposition 6). In this direction, Antill (2022) shows in a sample excluding acquisitions that switching from reorganization to liquidation in a given bankruptcy reduces expected creditor recovery across all debt claims by 42 cents on the dollar.[28] Financials might expect higher recovery values in part due to expectations of fiscal support during crises (moral hazard), and so undertake higher leverage and lower resolvability.

4 Optimal Policy

In this section, we study optimal policy. We do so in the context of the fire sale externality outlined in Section 2.6, with fire sales being a common motivation for studying government intervention in regulation and bailouts. The planner of our model has a complete set of regulatory wedges, and so can incentivize the bank to adopt any feasible capital structure, for example requiring issuance of outside equity as a loss absorbing instrument. Nevertheless, we show that the social planner finds it

[27] For example, the Flow of Funds (B.103) suggests that the debt-to-equity of nonfinancial corporates has been in the 20-40% range over the past decade, whereas capital requirements for systemically important financial institutions are in the range of 20% (i.e. a debt-equity ratio well above one).

[28] Relatedly, Bernstein et al. (2019) provides evidence that firm liquidation persistently reduces the utilization of the firm's real estate assets.

optimal for banks to write contracts that combine standard and bail-in debt, so that bail-in debt is the socially optimal loss absorbing instrument. Relative to private banks, the planner prefers greater use of bail-in debt to mitigate the fire sale. We use our results to shed further light on the optimal design of bail-in regimes.

4.1 Social Optimum

The social planner possesses a complete set of Pigouvian taxes (wedges) τ on the contract terms $\mathscr{C}$ of banks. Wedges are fully state-contingent.[29] Wedge revenues are remitted lump-sum to banks at date 0, so the project scale of banks can be written as $Y_0 = A_0 + I_0 + T^* - \tau\mathscr{C}$, where $T^* = \tau\mathscr{C}^*$ is equilibrium revenue collected and remitted. We have adopted inner product notation to simplify exposition, where for example $\tau^L L = \mathbb{E}[\tau(R_1)L(R_1)|e_0 = H]$. Possessing a complete set of Pigouvian wedges, the planner can incentivize banks to write any feasible contract (satisfying Definition 1) through appropriate choice of wedges τ. We solve directly for the optimal contract $\mathscr{C}$ chosen by the social planner, rather than its decentralization τ.

The social welfare function of the planner is bank welfare (9), that is a welfare weight of 0 is assigned to arbitrageurs. In Appendix B.1, we show that the qualitative properties of the social optimum are the same even with positive welfare weights on arbitrageurs.[30] The planner's problem is to choose a feasible contract $\mathscr{C}$ to maximize social welfare, internalizing the equilibrium pricing relationship (11).

In principle, the planner can choose a contract of any feasible form, even if that contractual form differs from that chosen privately by banks. For example, the planner might prefer banks to choose a contract featuring outside equity. The following result characterizes the socially optimal contract.

[29] Note that the planner's problem also satisfies Lemma 2, and so we directly use the same reduction of the contract as we did for private banks.

[30] In particular, fire sale spillover term in equation (14) is still positive but is lower in magnitude due to arbitrageur surplus from liquidations. To achieve Pareto efficiency, the social optimum combines a reduction in standard debt with a lump sum transfer from banks to arbitrageurs at date 0 to compensate them for losses on purchases.

Proposition 7. *A socially optimal bank contract has a liability structure*

$$L(R_1) = \begin{cases} (1-b)R_\ell Y_0, & R_1 \le R_\ell^* \\ (1-b)R_1 Y_0, & R_\ell^* \le R_1 \le R_u^* \\ (1-b)R_u^* Y_0, & R_u^* \le R_1 \end{cases}$$

The threshhold R_ℓ^* is given by

$$\mu b\left(\Lambda_1(R_\ell^*) - 1\right) = b + \lambda\left((1-b) - \gamma\right) + \underbrace{\lambda\sigma\gamma}_{\text{Fire Sale Spillover.} \ge 0} \tag{14}$$

while the threshold R_u^* is given by equation (13).

Therefore, the socially optimal contract can be implemented with a combination of standard and bail-in debt. Under an analogous condition to Proposition 6, the social optimum features less standard debt ($R_\ell^* < R_\ell$) and more bail-in debt ($R_u^* - R_\ell^* > R_u - R_\ell$) than the private optimum.

Even though the social planner has the ability to write any feasible contract $\mathscr{C}$, such as requiring some issuance of outside equity, Proposition 7 shows that the social planner finds it optimal to write a contract of the same structural form as private banks chose. That is to say, the socially optimal contract, like the privately optimal contract, can be implemented with a combination of standard and bail-in debt. The planner thus agrees with the bank that the optimal capital structure should make use of these two debt instruments, and not other instruments such as outside equity. We denote R_ℓ^* and R_u^* to be the planner's choices.

Even though the planner uses the same debt instruments as the bank, the fire sale spillover results in an additional *social* cost of liquidation in the planner's optimality condition for R_ℓ^*: the project liquidations of one bank increase the resource loss to all other banks that liquidate projects at the same depressed prices. This liquidation cost term represents the only difference between the private and social optimality conditions in equations (12) and (14), respectively. By contrast, there is no additional wedge in the determination of R_u^*, since a greater total debt level arising from more bail-in debt does not change total liquidations. Relative to private banks, the planner on the margin prefers lower issuance of standard debt but the same issuance of total debt, that is an increase in bail-in debt.

The constant elasticity σ implies that the social planner perceives an effective liquidation value of $(1-\sigma)\gamma$, rather than just γ. That is, the planner's solution is as-if the liquidation price was lower than it truly is. Proposition 6 tells us that the planner's solution has less standard debt and less total debt than the bank's private optimum. Proposition 6 also tells us that the planner uses more bail-in debt if incentive effects are not too strong.

4.2 Ex Post (Bail-in) Resolution as Optimal Policy

In our model, bail-in debt involves pre-specified (ex ante) contractual write-downs. Bail-ins are also implemented in practice via a resolution authority that imposes write downs ex post. Both forms of authority are used in practice, with the US emphasizing ex-post resolution and the EU being more acommodating of contractual recapitalization.[31] In this section, we show that the social optimum can also be implemented using an ex-post resolution authority. In Section 4.4, we discuss conditions under which this duality fails.

We define the resolution authority as follows. The resolution authority has discretion at date 1 to impose write downs on liability contracts that are designated "bail-inable," but is prohibited from imposing write downs on contracts that are designated "non-bail-inable." The objective of the ex-post bail-in authority is, at the level of the individual bank (i.e. taking equilibrium prices as given), to maximize total recovery value to creditors, subject to write downs being Pareto efficient. In the implementation that follows, it will be necessary to allow for debt seniority.

Corollary 8. *The social optimum can be implemented using macroprudential policy and a resolution*

[31] In the US, banks are required to maintain a certain level of *total loss-absorbing capital* (TLAC), principally long-term debt and equity, to safeguard the bank against poor returns. Debt used to satisfy TLAC requirements must be plain-vanilla, implying a fixed face value, while debt with contractual contingencies cannot generally be used to satisfy TLAC requirements. In particular, "eligible external LTD [is] prohibited from including contractual triggers for conversion into or exchange for equity." 82 FR 8266. See Avdjiev et al. (2017) for background on the European case.

authority. The social planner imposes an ex ante requirement for the bank to issue R_ℓ^* in non-bail-inable senior debt and $R_u^* - R_\ell^*$ in bail-inable junior debt, where R_ℓ^* and R_u^* are given as in Proposition 7. The resolution authority implements the write downs of Proposition 7 ex post.

Corollary 8 provides an implementation of the social optimum using a resolution authority. If $R_1 < R_\ell^*$, the resolution authority lacks capability of imposing sufficient write downs, and so cannot intervene. If $R_\ell^* \leq R_1 \leq R_u^*$, the resolution authority writes down bail-inable debt to $R_1 - R_\ell^*$. This maximizes creditor recovery and is Pareto efficient, since bail-inable debt is junior. If $R_1 > R_u^*$, the resolution authority's objective is achieved without write downs. Thus the same outcome is achieved. Finally, note the planner must impose restrictions on non-bail-inable debt ex ante, for the same reason as under the contractual implementation.[32]

The implementation of optimal policy in Corollary 8 is consistent with the design of bail-in regimes in practice, for example Title II. Bail-in regimes subordinate bail-in(able) (long-term) debt to standard (short-term) debt, that is standard debt enjoys absolute priority in bankruptcy, liquidation, and resolution.[33] Moreover, the objective of Pareto efficiency is consistent with the No Creditor Worse Off principle of bank resolution (BRRD Article 73).

4.3 Composition of TLAC and Need for Bail-in Regulation

In the environment of Proposition 7, the planner disagrees with the bank on the margin only over the composition but not the total level of debt. The planner also agrees with the bank that bail-in debt is preferable to outside equity. Therefore, the planner could implement the optimum by placing a tax on standard debt, and allowing the bank to freely choose between outside equity and bail-in debt. The bank then chooses to use bail-in debt rather than outside equity, achieving the social optimum.

[32] In our model, bank fundamentals R_1 are common knowledge, so there is no informational time consistency problem as in Walther and White (2020).

[33] In practice, short-term debt priority has three implementations. The first is contractual: bail-in debt is junior to short-term debt. The second is organizational: short-term debt is issued at the operating subsidiary, whereas long-term debt is issued at the top-tier holding company. The third is legal: national bankruptcy law confers priority to short-term debt in the case of banks. The US induces seniority through organizational form, and we could implement the US approach under Corollary 8 by assuming that bail-inable debt is held at a resolvable holding company, whereas non-bail-inable is held at a non-resolvable operating subsidiary.

The social planner thus needs to specify minimum total loss-absorbing capacity (TLAC) but not its composition (where TLAC instruments protect a bank from liquidation, for example bail-in debt and equity). In this subsection, we develop two extensions that highlight the need for the planner to specify minimum bail-in debt and minimum (inside) equity separately. Each extension represents the minimal required departure from the baseline model.

Need for Minimum Bail-in Debt. We extend the model to allow for aggregate uncertainty over the magnitude of the fire sale.[34] Formally, we have a stochastic liquidation price elasticity $\sigma \in \Sigma$ (a finite set), with aggregate probabilities $\pi(\sigma)$. The model is otherwise unchanged.[35] We obtain the following result.

Proposition 9. *The socially optimal contract takes an aggregate-state-contingent form of that in Proposition 7, where $R_\ell^*(\sigma)$ decreases in σ and R_u^* is constant in σ.*

Proposition 9 states that uncertainty over the magnitude of the fire sale, σ, leads to a fixed total stock of debt but a state-contingent debt composition. Intuitively, the planner prefers fewer liquidations in more severe crises (higher σ) and more liquidations in less severe crises. The contract implementation involves two instruments: standard debt, and bail-in debt with a *dual trigger*.[36] Standard debt is associated with $R_\ell^*(\sigma_{\max})$, where $\sigma_{\max} = \max\Sigma$. The dual trigger on bail-in debt specifies the portion $R_u^* - R_\ell^*(\sigma)$ of bail-in debt that can be written down. The portion $R_\ell^* - R_\ell^*(\sigma_{\max})$ cannot be written down, and so mimics standard debt ex post.

With aggregate uncertainty, a tax on standard debt only gives the planner control over the minimum threshold $R_\ell^*(\sigma_{\max})$, but not separately thresholds in states $\sigma > \sigma_{\max}$. The fire sale implies, however, that the planner prefers on the margin less standard debt in every state $\sigma > 0$. This means the planner also has to regulate the dual trigger, i.e., how much of bail-in debt can be resolved based

[34] For simplicity, we abstract away from aggregate return risk. Appendix B.2 studies the case where aggregate uncertainty affects the return distribution.

[35] Note that monotonicity is now σ-adapted.

[36] This terminology alludes to that of a dual price trigger for a CoCo security (e.g., McDonald 2013), where write downs or conversions to equity are based on both an idiosyncratic measure (e.g., individual bank stock price) and an aggregate measure (e.g., aggregate stock market performance).

on the severity of the crisis. Proposition 9 provides a role for explicit regulation of bail-in debt contracts, and helps explain post-crisis requirements for minimum holdings of (long-term) bail-in debt in the top tier bank holding company (82 FR 8266).

Need for Minimum Equity Requirements. Our model gives no concrete role for minimum equity capital requirements – that is, maximum R_u – which are common in practice. This results from the binary effort problem, and can be resolved by making date 0 effort continuous (maintaining binary date 1 effort for simplicity). Formally, let $e_0 \in [0,1]$, with $f(R_1|e_0) = e_0 f_H(R_1) + (1-e_0) f_L(R_1)$ and a private benefit $\frac{1}{2}B_0(1-e_0^2)$ to shirking. Our baseline model is obtained by imposing the restriction $e_0 \in \{0,1\}$, with $e_0 = 1$ being high effort. In the proof of Proposition 10, we show that date 0 incentive compatibility is given by

$$\int c(R_1)\Big(1-\Lambda_1(R_1)\Big) f_H(R_1) dR_1 = e_0 B_0 Y_0,$$

where $\Lambda_1(R_1) = \frac{f_L(R_1)}{f_H(R_1)}$ as in the baseline model. Incentive compatibility here defines the effort level e_0 that is chosen by the bank as a function of its capital structure. As in the baseline model, liquidations in low-return states and cash flow transfers in intermediate states promote higher effort e_0, while cash flow transfers in high return states discourage effort.[37] We obtain the following result.

Proposition 10. *With continuous effort, the privately and socially optimal contracts can be implemented by combinations of standard and bail-in debt. Decentralizing the social optimum requires a positive tax on total debt.*

In the baseline model with binary effort choice, R_u discouraged high effort and tightened incentive compatibility. With continuous effort, higher R_u lowers e_0 in a continuous sense. Lower e_0 increases the probability that the bank finds itself with $R_1 \le R_\ell$, leading to larger fire sales. Small

[37] Analogous to the baseline model, we assume that a commitment to higher effort would be desirable. As in the baseline model, this implies a positive Lagrange multiplier on incentive compatiblity, $\mu > 0$. As in the baseline model, violation of this condition would either lead to a corner solution in effort or lead to contracts that punished the bank for high returns.

banks do not internalize this effect and so over-pledge total debt and under-provide effort relative to the social optimum. This gives the planner a role to restrict total debt issuance, that is to implement a form of minimum (inside) equity capital requirement.

4.4 Ex Ante vs Ex Post: Commitment vs Flexibility

Our baseline model features an equivalence between ex ante contractual write-downs and ex post bail-ins via a resolution authority. Both of these institutional setups are used in practice, making it important to understand when one method is preferable to the other. We extend the model to capture a particularly relevant practical dimension: As a crisis unfolds, it is difficult to assess and, in particular, verify its severity in real-time. A trade-off emerges between ex ante contractual write-downs and ex post resolution: A resolution authority with flexibility to resolve more banks in severe crises is also tempted to resolve more banks in normal times to prevent costly liquidations. We first show that absent a mechanism to control the resolution authority, the planner can do no better than an ex ante contractual approach. However, ex post resolution can improve on ex ante contractual write-downs if the resolution authority is designed with an appropriate incentive mechanism. The optimal incentive mechanism has similarities to the structure of Title II.

Consider the aggregate risk environment of Section 4.3 and suppose that $\sigma \in \Sigma$ is non-contractible.[38] An ex ante contract can only specify R_ℓ and R_u that are invariant to σ, resulting in too few liquidations when σ is small ("normal times") and too many when σ is large ("crisis times").[39] Suppose then that we delegate to a utilitarian resolution authority the ability to choose R_ℓ ex post at date 1 from a pre-specified set $\mathcal{R}_\ell$. Formally, the resolution authority chooses R_ℓ to maximize $U(\sigma, R_\ell) = (\gamma(\sigma, R_\ell) - 1)\Omega(R_\ell)$, that is to minimize the total costs of liquidations. It follows immediately that U is decreasing in R_ℓ, since more liquidations both increase the number of liquidated banks, Ω, and reduce the fire sale price, γ. Therefore, the resolution authority selects

[38] If crisis severity also has a verifiable component, we can think of our analysis as being about deviations around that verifiable component.

[39] It is interesting to note that the ex ante optimal contract still involves a combination of standard and bail-in debt, since these instruments are still optimal incentive devices. The cost of standard debt is the average cost across aggregate states, that is $\mathbb{E}[\gamma]$ for the bank and $\mathbb{E}[(1-\sigma)\gamma]$ for the social planner.

$R_\ell(\sigma) = \inf \mathcal{R}_\ell$ for all σ. Intuitively, the utilitarian resolution authority always wants to prevent as many costly liquidations as possible, and so always chooses to resolve as many banks as it can. Absent further incentives, an ex post resolution authority cannot improve on an ex ante implementation under commitment. This suggests that an ex ante approach has an advantage over the ex post approach.[40]

We now propose a Pigouvian method of implementing the efficient rule $R^*_\ell(\sigma)$ of Proposition 9 using an ex post resolution authority. In particular, the key friction we must overcome is the incentive outlined above for too many resolutions. We allow the planner to assign a penalty (or transfer) T to the resolution authority based on its chosen R_ℓ, so that the resolution authority achieves total utility $U(\sigma, R_\ell) - T$. We provide plausible practical implementations of T after presenting our result. A mechanism in our model operates as follows: After the resolution authority observes σ, it can report that it observed any $\hat{\sigma} \in \Sigma$. It is then assigned a pair $(R_\ell(\hat{\sigma}), T(\hat{\sigma}))$ based on this report. The mechanism must be incentive compatible, i.e., a resolution authority must find it optimal to report $\hat{\sigma} = \sigma$.[41] Formally, this requires $U(\sigma, R_\ell(\sigma)) - T(\sigma) \geq U(\sigma, R_\ell(\hat{\sigma})) - T(\hat{\sigma})$ for all $\sigma, \hat{\sigma} \in \Sigma$. We now show that there is always an incentive compatible mechanism that implements the efficient allocation of Proposition 9, and afterwards discuss the form of the mechanism.

Proposition 11. *There is an incentive compatible mechanism that implements the allocation $R^*_\ell(\sigma)$ of Proposition 9, where $T(\sigma)$ increases in σ.*

Proposition 11 shows that an incentive compatible mechanism for implementing the efficient liquidation rule with a resolution authority does in fact exist. To make the mechanism concrete, we discuss it in terms of a stylized example: let $\Sigma = \{0, \overline{\sigma}\}$, that is there is a normal state without fire sales and a crisis state with fire sales. Define $\Delta\Omega = \Omega(R^*_\ell(0)) - \Omega(R^*_\ell(\overline{\sigma}))$ as the difference

[40] It is helpful to draw parallels to the commitment versus flexibility literature in the contexts of fiscal and monetary policy. Athey et al. (2005) and Amador et al. (2006) show that threshold rules are optimal in environments with money burning, which mirrors our simple ex ante approach. Several papers (Beshears et al. 2022, Clayton and Schaab 2022, Halac and Yared 2022) study mechanisms with transfers or penalties in a similar method to our Pigouvian approach.

[41] As usual, we exploit the revelation principle and focus on direct mechanisms where the resolution authority truthfully reports its type.

in liquidations that occur in the normal and crisis states. The proof of Proposition 11 shows that the smallest possible punishments that maintain incentive compatibility are $T(\underline{\sigma}) = 0$ and $T(\overline{\sigma}) = (1-\overline{\gamma})\Delta\Omega > 0$, where $\overline{\gamma}$ is the liquidation price absent fire sales. Intuitively, the resolution authority can always choose $R_\ell^*(\underline{\sigma})$ without incurring a penalty, but must incur a penalty $T(\overline{\sigma})$ to resolve more banks. The level of the penalty in a crisis is set to discourage the resolution authority in *normal times* from behaving as if there was a crisis. Intuitively, this happens because during a crisis, deep fire sales increase the incentive for the resolution authority to resolve more banks. Thus the binding temptation is for the resolution authority to resolve too many banks in normal times.[42] The size of the penalty is equal to the marginal cost of liquidation, $1-\overline{\gamma}$, times the number of liquidations prevented by resolving more banks, $\Delta\Omega$.

Proposition 11 implies that a resolution authority approach can be preferable to an ex ante contractual approach provided that an appropriate incentive mechanism is established to control its behavior. The incentive mechanism allows the resolution authority to respond to non-contractible information about crisis severity while curbing its incentives to over-respond to less severe crises.

An important practical question is how to implement this mechanism. Although mechanisms such as monetary incentives and threats of firing are possible, we suggest one novel and plausible implementation that exploits the structure of the resolution framework. The implementation involves pairing larger-scale bail-ins with costly bailouts.[43] In particular, Title II of the Dodd-Frank Act provides for resolution of the top tier bank holding company while operating subsidiaries continue operations unimpeded. One implementation of the mechanism of Proposition 11 is to divide loss-absorbing capital between the top tier holding company and operating subsidiaries. The ex post resolution authority is given full flexibility to resolve the top tier holding company, but resolution of operating subsidiaries requires a partial bailout of the operating subsidiaries. This mechanism defines a contractible event associated with larger bail-ins – the resolution of an operating subsidiary

[42] This is a typical direction of binding constraints result: the high type (here low σ) must be deterred from imitating the low type (here high σ).

[43] Our proposal of using bailouts as an incentive mechanism is related to Philippon and Wang (2022), but takes a different form and context. They propose awarding higher bailouts to better performing banks to incentivize lower ex ante risk taking, whereas our focus is on bailouts as an incentive mechanism for the resolution authority's ex post resolution choices.

– and assigns a punishment based on that contractible event – need to employ costly bailouts.[44]

4.5 Bail-ins versus Bailouts

One of the stated goals of bail-in regimes is to replace bailouts. A strength of our framework is that we can leverage it to examine important policy questions such as this one. In this subsection, we allow the planner to engage in bailouts and show that under commitment the planner prefers to commit to bail-ins rather than bailouts. This result provides a stylized benchmark that sheds light on the policy goal of replacing bailouts with bail-ins. In practice, commitment is a strong assumption, and we discuss time consistency at the end of this subsection and in Appendix B.9.

We take as our starting point the model of Section 4.1 and allow the planner to bail out insolvent banks under commitment. The smallest bailout that recapitalizes an insolvent bank with return R_1 is $T(R_1) = L(R_1) - (1-b)R_1Y_0$, which allows the bank to repay its liabilities and maintain its minimum agency rent. Bailout funds are raised from taxpayers at date 1. Taxpayers have utility $u_0(c_0^T) + u_1(c_1^T)$, where $c_0^T = A_0^T + T_0 - B_0$ and $c_1^T = A_0^T + B_0 - \mathbb{E}[T(R_1)]$. B_0 is taxpayer savings. T_0 is a "bailout fund," i.e., a transfer from banks to households at date 0 to compensate for ex post bailouts. A bailout fund is necessary to achieve a Pareto improvement if bailouts are used in equilibrium, but is not needed if bailouts do not occur.

We study the social planning problem of choosing a feasible contract $\mathscr{C}$ and bailout rules (T_0, T_1) to maximize social welfare, with a Pareto weight ω^T placed on taxpayers. The problem is otherwise the same as in Section 4. We obtain the following result.

Proposition 12. *In the model with committed bailouts, the socially optimal contract of Proposition 7, with no bailouts ($T_0 = T_1 = 0$), is Pareto efficient.*

Proposition 12 shows that a planner achieves Pareto efficiency in the model with committed bailouts by exclusively relying on bail-ins to recapitalize insolvent banks. To understand why,

[44]One simple formalization of this mechanism is to assume that a bailout of size T_1 at date 1 has a political or deadweight cost $\kappa > 0$ per unit. This means the total deadweight cost of bailouts is κT_1. As a result, implementing the minimal punishments above would mean setting $T_1(0) = 0$, that is no bailouts, and setting $T_1(\overline{\sigma}) = \frac{(1-\overline{\gamma})\Delta\Omega}{\kappa}$.

suppose the planner bailed out the marginal insolvent bank, $R_1 \uparrow R_\ell^*$. There are two effects of a bailout. First, the insolvency threshold is lowered, that is R_ℓ^* effectively falls. But under the socially optimal contract of Proposition 7, the marginal cost of liquidation costs is perfectly balanced against the marginal benefit of incentive provision, meaning a reduction in R_ℓ^* has zero net welfare effect. Second, a bailout also transfers resources from taxpayers to banks. The marginal social benefit of the resource transfer is the value of relaxing the participation constraint, λ. The marginal social cost of the resource transfer is the burden to taxpayers, $\omega^T u_1'(c_1^T)$. Thus selecting the Pareto weight $\omega^T = \frac{\lambda}{u_1'(c_1^T)}$, from taxpayer optimization over savings B_0 we have $\lambda = \omega^T u_1'(c_1^T) = \omega^T u_0'(c_0^T)$. Thus the net welfare gain from the resource transfer is also zero, and there is no welfare gain from bailing out the marginal insolvent bank. Thus we have found a Pareto weight ω^T such that the contract of Proposition 7, with no bailouts, maximizes the planner's welfare criterion, and so we have found a Pareto efficient allocation without bailouts.[45]

It is well known that when debt contracts are non-contingent by assumption, bailouts can be Pareto efficient because they insert contingencies into otherwise non-contingent contracts (Bianchi 2016, Jeanne and Korinek 2020). Our model endogenously generates not only non-contingent standard debt, but also contingent bail-in debt. Thus the planner can achieve the same state-contingencies with bail-in debt as it could with bailouts. As a result under full commitment, the planner can achieve an efficient outcome by using more bail-in debt, rather than by bailing out standard debt.

In Appendix B.9, we study bailouts in the absence of commitment under two special cases of collective moral hazard and finite fiscal capacity. We show the two cases have different implications for whether bailouts occur, but that in both cases the possibility of bailouts either reduces or does not alter welfare. With collective moral hazard, fixed costs of bailouts imply all banks get bailed out if there are sufficiently many bank failures. The planner restricts standard debt so that no bailouts occur. With finite fiscal capacity, higher standard debt exhausts fiscal capacity faster. Thus

[45] Bailouts can be optimal in our model for insurance reasons, as in Dewatripont and Tirole (2018), Farhi and Tirole (2021), and Keister and Mitkov (2021). In our model, this can happen in reduced form by assuming taxpayers are borrowing constraint, $B \geq 0$, so that $u_0'(c_0^T) \geq u_0'(c_1^T)$.

counterintuitively, the planner *increases* use of standard debt to exhaust fiscal capacity and achieve the efficient liquidation rule, which results in some bailouts.

4.6 Discussion and Additional Extensions

We provide additional discussion of how our model can speak to too-big-to-fail institutions, as well as its connection to demand-based (safety premia) theories of standard debt.

Too-Big-To-Fail. Our model assumes all banks are small, but in practice bail-in regulation is often tailored to "too-big-to-fail" banks. Failure of a single large bank could generate a large enough fire sale that the planner would prefer to avoid liquidation. In our model, this would mean $R_\ell \leq \underline{R}$.[46] Such regulation is privately costly because it reduces effort incentives, forcing lower debt and smaller project scale. However, this cost arises because we have modeled liquidation as an all-or-nothing decision. We now argue that partial liquidation of large banks can be desirable to mitigate externalities while providing incentives. We connect this idea to proposals for a good bank/bad bank approach to resolution.

We model $N \geq 1$ ex ante identical large banks (in place of small banks). A large bank is an investment family ("holding company") consisting of $\frac{1}{N}$ managers ("subsidiaries"), each of whom can undertake a project of the form in Section 2. Each holding company divides inside equity, $\frac{1}{N}A_0$, equally among its subsidiaries, and then coordinates external capital structure decisions for the family (i.e., the contract of each subsidiary).[47] We treat x as subsidiary cash flows pledged to external investors and c as cash flows pledged to the family.[48] Each manager independently

[46]This is a more extreme version of current regulations that impose surcharges in capital requirements for systemically important financial institutions.

[47]For simplicity, our model abstracts away from the possibility that contracts are interconnected (that is, $\mathscr{C}$ is only adapted to the individual subsidiary's R_1) – for example a threat to liquidate other subsidiaries as well if one subsidiary performs badly. This is consistent with the idea that the manager must act to maximize payoff to her subsidiary's equity holders, and not equity holders of other subsidiaries.

[48]In our simple framework, there are two equivalent methods this could be achieved. One is that the subsidiary directly holds external bail-in debt. The other is that the holding company holds bail-in debt in the subsidiary, and then issues the same instrument externally. Thus a loss at the subsidiary is indirectly passed on to external investors. We treat these two methods as equivalent in the sense that both generate the same division of final payoffs x_t and c_t among outside investors and the family.

operates her subsidiary to maximize the payoff of her project to her family. Incentive compatibility, monotonicity, and limited liability are specified at the subsidiary level. For simplicity, suppose there are no fire sales. It is easy to see in this case that every large bank chooses the same privately optimal contract as Proposition 3.[49] The privately optimal contract therefore results in a partial liquidation of each large bank, where subsidiaries with $R_1 \leq R_\ell$ are liquidated.

This simple extension highlights how our model can be used to speak to too-big-to-fail institutions. Because effort choice is at the subsidiary level and the holding company coordinates capital structure decisions, partial liquidations targeting poorly performing subsidiaries replace all-or-nothing liquidation rules. The importance of subsidiary-level (partial) liquidations highlights the desirability of not only a holding company / operating subsidiary company structure, which allows individual units of the group to fail in isolation, but also a novel rationale for clean holding company requirements: liquidations occur at the subsidiary level, so standard debt is held at the subsidiary level. This also rationalizes a good bank/bad bank approach to resolution. Under this implementation, the best performing assets or subsidiaries (with $R_1 \geq R_\ell$) of the large bank would be separated into the "good" bank and reorganized. The worst performing assets or subisidiaries (with $R_1 < R_\ell$) would be placed into the "bad" bank, and liquidated. The size of the bad bank increases in R_ℓ, resulting in larger partial liquidations.

Relation to Demand-Based Explanations. Our model provides an incentive-based explanation for standard debt. Another important explanation is demand-based: investors assign a special preference to safe debt, which makes safe debt cheaper to issue (Bolton and Oehmke 2019, Walther and White 2020). Demand-based explanations are by no means mutually exclusive with ours, but it is important to highlight some of the differences. We then discuss how a combination of our theory and a demand-based one can provide a more complete perspective on bank capital structure choices.

Our model uses a single contracting friction, repeated unobservable effort, to rationalize the joint existence of standard and bail-in debt. A pure demand-based (safety premium) story has

[49] If there were fire sales, each large bank would internalize only a fraction $\frac{1}{N}$ of the fire sale cost, generating a role for the planner to intervene and impose the contract of Proposition 7.

two important points to reconcile. First, a safety premium means that costly liquidations are also inefficient ex ante. Banks thus have strong incentives to write hedging contracts to protect standard debt and prevent liquidation. Yet, the costly insolvencies we see in practice are a basis for introducing bail-in regimes. Both Bolton and Oehmke (2019) and Walther and White (2020) reconcile this in part by assuming that banks are unable to write such hedging contracts (incomplete markets).[50] Our model predicts that banks do not hedge standard debt, that is banks use noncontingent contracts even though markets are complete. Our model thus provides a unified explanation for why we observe costly liquidations in practice. Second, a pure safety premium story implies Modigliani-Miller holds for the residual capital structure of the bank, giving no clear role for bail-in debt per se. Both Bolton and Oehmke (2019) and Walther and White (2020) incorporate one period of unobservable effort to give a clear role for bail-in debt over equity. Our model unifies the explanation for the desirability of both standard debt and bail-in debt over equity from repeated unobservable effort. Our model also highlights that total debt discourages effort in a continuous sense, providing a unified perspective on the role of minimum equity capital requirements in conjunction with standard debt and minimum bail-in debt.

Incorporating a special preference for safe debt into our model can offer a more complete perspective on bank capital structure. For example, our model provides no role for deposit insurance, which impairs the incentive benefits of standard debt. A safety premium could motivate deposit insurance (Dewatripont and Tirole 2018, Farhi and Tirole 2021). Appendix B.8 provides one simple extension in which a planner protects a class of insured deposits to preserve their safety premium.[51] If liquidations are not too costly, the social planner allows banks to issue both insured standard debt (e.g., retail deposits) and uninsured standard debt (e.g., wholesale funding). Costly liquidations happen whenever total standard debt exceeds pledgeable income. This rationalizes capital structures that combine insured and uninsured deposits. It also motivates the common FDIC practice of

[50] Bolton and Oehmke (2019) also offers an insightful observation that resolution authorities themselves may prevent hedging across countries "by ring-fencing assets" (p. 2390).

[51] There is little meaningful difference between public and private insurance in this extension, so we could also view this as an explanation of secured standard debt, possibly also protected by hedges, and unsecured standard debt, not protected by hedges.

resolving small insolvent banks by using either liquidation or merger, both of which could be seen as a possibly costly reallocation of the bank to the next best user. It can also explain the exemption of repurchase agreements from the automatic stay: disorderly collateral seizure both protects secured creditors and promotes liquidation.

Further Extensions. We briefly summarize several additional extensions in the Appendix that have not yet been discussed. In Appendix B.4, we introduce multiple investment projects and show that asset-side regulation is a necessary complement to liability-side regulation because asset composition affects the probability of failure and liquidation. In Appendix B.5, we study the allocation of bail-in securities among heterogeneous investors and show that retail investors, with greater exposures to idiosyncratic banks, and institutional investors that experience greater fire sale spillovers should hold safer (standard debt) claims. In Appendix B.7, we allow standard debt to command a premium over other instruments, including bail-in debt. This helps rationalize the high level of standard debt banks employ in practice.

5 Conclusion

We develop a simple and tractable dynamic contracting model in which the privately optimal bank contract can be implemented with a combination of standard and bail-in debt. Banks privately under-use bail-in debt in the presence of fire sale externalities from bank failures, motivating the government to set up a bail-in resolution regime. Bail-ins are a desirable addition to the regulatory regime because bail-in debt is better suited than (outside) equity to address the incentive problem that motivated banks to issue standard debt in the first place. Our model sheds light on a number of normative issues in bank regulation, including TLAC composition, trade-offs between ex ante and ex post approach, and interaction with bailouts.

There are a few interesting possible directions in which our framework could be extended. First, our model had two periods of effort choice and assumed high effort choice was always exerted.

We conjecture that the key insights of our model would arise in a longer horizon model: liquidations would be costly but strong incentive devices, while maximal cash flow transfers ("bail-ins") would be less costly but weaker incentive devices. It is possible that pledging contracts that induce future shirking (low effort) after low returns could be an optimal form of money burning, similar to liquidation, by reducing continuation agency rents. The policy implications of such a model would be an interesting avenue for future research. Second, our model assumes that the banker is not severable from the bank, that is the banker cannot be fired without also liquidating the bank. Standard debt thus transfers control rights to investors, whose best option is liquidation. If the banker were severable but firings were costly, we conjecture the government might prefer to use costly firings and bail-ins to provide high-powered incentives, rather than liquidations. Another interesting avenue for future research would be to consider socially optimal punishment schemes in this context.

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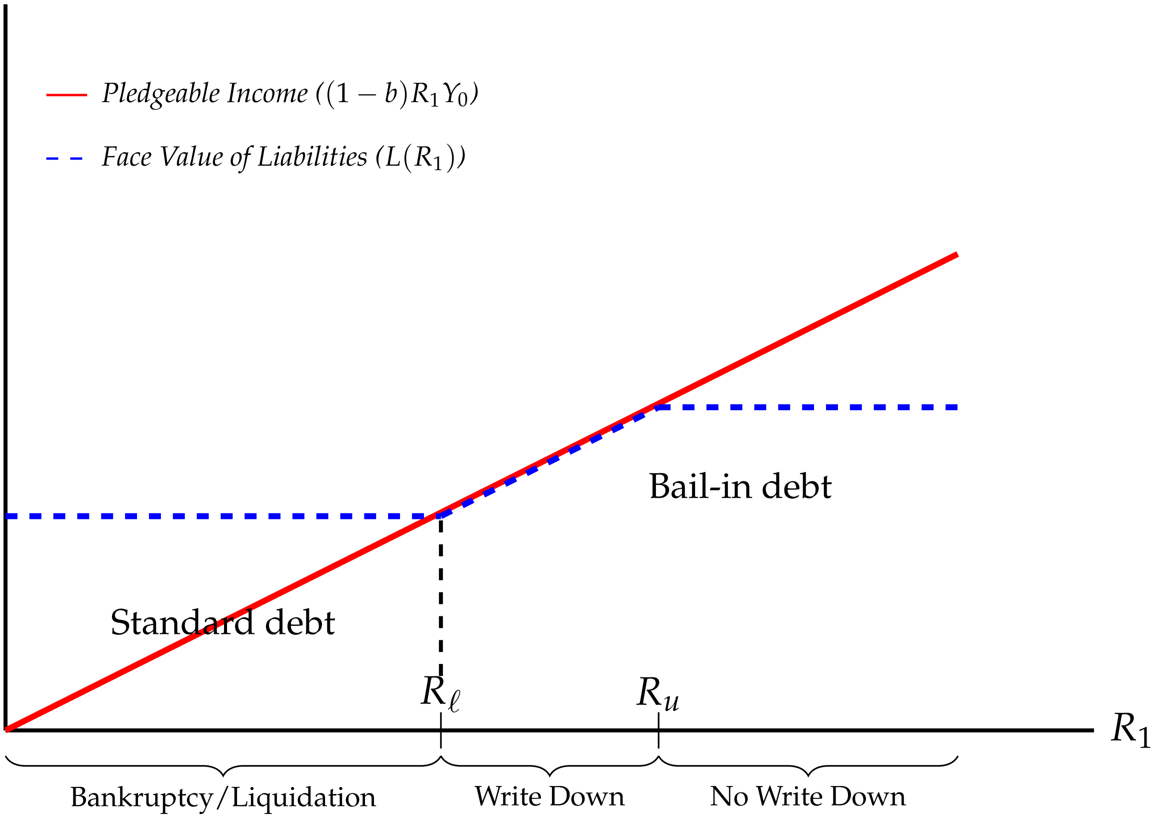

Figure 1: This figure provides an illustration for the privately optimal contract. Up to a threshold R_l, bank liabilities are constant and exceed pledgeable income, leading to liquidations ("standard debt"). Between R_l and R_u, the face value of liabilities is written down to coincide with pledgeable income ("bail-in" or "write down"). Above R_u, the face value of liabilities is constant ("bail-in debt").

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Internet Appendix

A Proofs

A.1 Proof of Lemma 2

Consider the problem of the bank. The bank chooses a contract $\mathscr{C}$ in order to maximize utility,

$$\mathbb{E}\left[c(R_1)|e_0 = H\right]$$

subject to (IC-0),

$$\mathbb{E}\left[c(R_1)(1-\Lambda_1(R_1))\middle|e_0 = H\right] \geq B_0 Y_0,$$

to (IC-1),

$$\mathbb{E}\left[(c_1(R_1)+c_2(R_1,R_2))(1-\Lambda_2(R_2))\middle|e_1 = H\right] \geq B_1 R_1 Y_0,$$

to (P),

$$Y_0 - A_0 \leq \mathbb{E}\left[x(R_1)|\, e_0 = H\right].$$

and to monotonicity,

$$R_1 \geq R_1' \Rightarrow x(R_1) \geq x(R')$$

$$R_2 \geq R_2' \Rightarrow x_2(R_1, R_2') \geq x_2(R_1, R_2').$$

Throughout here, we have adopted defintions,

$$x(R_1) = \alpha_1(R_1)\gamma R_1 Y_0 + (1-\alpha_1(R_1))\mathbb{E}\left[x_1(R_1) + x_2(R_1,R_2)\right].$$

$$c(R_1) = (1-\alpha_1(R_1))(R_1 Y_0 - x(R_1))$$

Consider now that utility, (IC-0), (M1), and (P) depend only on (c,x) and not on the distributions (c_t,x_t). Therefore, we can define a set of *equivalent date* 1 *expected contracts* as contracts $\mathscr{C}(\alpha,c,x)$ as the set of contracts that yield the same (α,c,x) through different payoff splits (c_t,x_t). Take a contract $\mathscr{C}(\alpha,c,x)$ that satisfies (IC-0), (P), and (M1).

We now look to characterize whether a contract $\mathcal{C} \in \mathscr{C}(\alpha,c,x)$ can be made to satisfy (IC-1) and (M2) with a payoff split (c_t,x_t). Formally, we represent this problem as maximizing slack in incentive compatibility,

$$\mathcal{S}(R_1) = \mathbb{E}\left[(c_1(R_1)+c_2(R_1,R_2))(1-\Lambda_2(R_2))\middle| e_1 = H\right] - B_1R_1Y_0$$

subject to the constraints

$$\mathbb{E}[c_1(R_1)+c_2(R_1,R_2)|e_1 = H] = c(R_1)$$

$$R_2 \geq R_2' \Rightarrow x_2(R_1,R_2') \geq x_2(R_1,R_2').$$

$$\mathbb{E}[x_1(R_1)+x_2(R_1,R_2)|e_1 = H] = x(R_1)$$

along with the resource constraint. The contract $\mathcal{C}$ is feasible iff $\mathcal{S}(R_1) \geq 0$ for all R_1. This problem is the dual of a standard Innes (1990) type problem with a binary effort choice, MLRP, and monotonicity, and hence we know the solution is a debt contract,

$$x_1(R_1)+x_2(R_1,R_2) = \begin{cases} R_2R_1Y_0, & R_2 \leq R_2^u(R_1) \\ R_2^u(R_1)R_1Y_0, & R_2 > R_2^u(R_1) \end{cases}$$

Thus, it remains only to check that $\mathcal{S}(R_1) \geq 0$ and the contract is indeed incentive compatible. Given the debt contract, we have

$$\mathcal{S}(R_1) = \int_{R_2^u(R_1)}^{\overline{R}} \left[R_2 - R_2^u(R_1)\right] R_1Y_0(1-\Lambda_2(R_2))f_{2,H}(R_2)dR_2 - B_1R_1Y_0,$$

which is nonnegative if

$$\int_{R_2^u(R_1)}^{\overline{R}} \left[R_2 - R_2^u(R_1) \right] (1-\Lambda_2(R_2)) f_{2,H}(R_2) dR_2 \geq B_1.$$

Thus, define $\overline{R}_2^u$ as the higest value of $R_2^u(R_1)$ that satisfies the above equation, and note that it does not depend on R_1. Thus, IC is satisfied only if $R_2^u(R_1) \leq \overline{R}_2^u$. This equivalently means that the contract can be made date 1 incentive compatibility under a monotone continuation contract only if

$$c(R_1) \geq \int_{\overline{R}_2^u}^{\overline{R}} [R_2 - \overline{R}_2^u] R_1 Y_0 f_{2,H}(R_2) dR_2 = bR_1 Y_0$$

where we have defined $b \equiv \int_{\overline{R}_2^u}^{\overline{R}} [R_2 - \overline{R}_2^u] f_H(R_2) dR_2$. This concludes the proof.

A.2 Proof of Proposition 3

We will represent the problem in the space (α, x, c) and then back out the liability structure L that implements it.

Exploiting Lemma 2, we represent the program as

$$\max \mathbb{E}\left[c(R_1) | e_0 = H\right]$$

subject to

$$\mathbb{E}\left[c(R_1)\left(1-\Lambda_1(R_1)\right) | \, e_0 = H\right] \geq B_0 Y_0$$

$$Y_0 - A = \mathbb{E}[x(R_1) | e_0 = H]$$

$$R_1 \geq R_1' \Rightarrow x(R_1) \geq x(R_1')$$

$$c(R_1) \geq (1-\alpha(R)) bR_1 Y_0$$

where the last equation is the representation of (IC-1) as a minimum agency rent. Note that we can drop limited liability due to the pledgeable income constraint (given we already assign $c(R_1) = 0$ in liquidation).

Given representation in the investor payoff space, we have

$$x(R_1) = \alpha(R_1)\gamma R_1 Y_0 + (1-\alpha(R_1))R_1 Y_0 - c(R_1)$$

Note that because banks are repaid 0 when $\alpha(R_1) = 1$, it is irrelevant whether we multiply $c(R_1)$ by $1-\alpha(R_1)$. Given this characterization, investor voluntary participation can be rewritten as

$$Y_0 - A = \mathbb{E}\left[\alpha\gamma R_1 Y_0 + (1-\alpha)R_1 Y_0 - c|e = H\right].$$

We begin by studying the optimization problem not subject to monotonicity in x, and show that it generates a non-monotone contract. The Lagrangian of this relaxed problem is

$$\begin{aligned}
\mathscr{L} =& \mathbb{E}\left[c|e_0 = H\right] + \mu\left[\mathbb{E}\left[c(1-\Lambda_1)|e_0 = H\right] - BY_0\right] \\
&+ \lambda\left[\mathbb{E}\left[\alpha\gamma R_1 Y_0 + (1-\alpha)R_1 Y_0 - c|e_0 = H\right] + A_0 - Y_0\right] \\
&+ \mathbb{E}\left[\chi\left(c - (1-\alpha)\,bR_1 Y_0\right)|e_0 = H\right] \\
&+ \mathbb{E}\left[\zeta\left((\alpha\gamma R_1 Y_0 + (1-\alpha)R_1 Y_0 - c + \bar{x})\right)|e_0 = H\right]
\end{aligned}$$

where the last line is a limited liability constraint $x \geq -\bar{x}$ that we introduce for purely technical reasons in showing that the optimal contract not subject to monotonicity is in fact non-monotone (anticipating it will take a live-or-die form). Limited liability for investors will play no role in the optimal monotone contract. From here, first order condition for bank consumption as

$$\begin{aligned}
0 &= f_{1,H}(R_1) + \mu(1-\Lambda_1(R_1))f_{1,H}(R_1) - \lambda f_{1,H}(R_1) + \chi(R_1)f_{1,H}(R_1) - \zeta(R_1)f_{1,H}(R_1) \\
&= \left[1 - \lambda + \mu(1-\Lambda_1(R_1))\right]f_{1,H}(R_1) + \chi(R_1) - \zeta(R_1)
\end{aligned}$$

By MLRP, there is a threshold R^* such that $\chi(R_1) > 0$ for $R_1 \leq R^*$ and $\zeta(R_1) > 0$ for $R_1 \geq R^*$. This threshold is given by

$$1-\lambda+\mu\left(1-\Lambda_1(R^*)\right)=0. \tag{A.1}$$

However, this contract violates monotonicity unless $x(R_1)$ is constant for all R_1. Therefore, we have an upper pooling region in the optimal contract, where investor repayment is constant.[52]

It is worth remarking that monotonicity therefore binds in the optimal contract. We relax the assumption of monotonicity in Appendix B.3, and show that the optimal contract takes a live-or-die form (Innes 1990). We interpret the optimal contract as a combination of standard debt, bail-in debt, and an "insurance" contract that pays off to banks in a high-return states. The limited liability $\bar{x}$ prevents writing contracts that deliver arbitrarily large payoffs to the bank at only the possible highest effort level.

We now characterize the optimal contract using the following strategy. First, we conjecture a pooling thresholds R_u with corresponding $x_u \equiv x(R_u)$, so that $x(R_1)=x_u$ for all $R_1 \geq R_u$. The live-or-die result of the contract not subject to monotonicity implies such a pooling threshold exists.[53] We then solve for the optimal contract below R_u, taking as given R_u and x_u, subject to a relaxed monotonicity constraint $x(R_1) \leq x_u \; \forall R_1 \leq R_u$, and verify that the resulting contracting is monotone. In doing so, we characterize the space of implementable contracts (that satisfy monotonicity). Finally, we optimize over the choice of R_u and x_u.

Conjecture pooling thresholds R_u with liabilities x_u. The associated Lagrangian is given by

$$\begin{aligned}
\mathscr{L} =& \mathbb{E}\left[c|e_0=H\right]+\mu\left[\mathbb{E}\left[c(1-\Lambda_1)|e_0=H\right]-B_0Y_0\right] \\
&+\lambda\left[\mathbb{E}\left[\alpha\gamma R_1Y_0+(1-\alpha)R_1Y_0-c(R_1)|e_0=H\right]+A_0-Y_0\right] \\
&+\mathbb{E}\left[\chi\left(c-(1-\alpha)bR_1Y_0\right)|e_0=H\right] \\
&+\mathbb{E}\left[\nu\left(x_u-(\alpha\gamma R_1Y_0+(1-\alpha)R_1Y_0-c)\right)|e_0=H\right]
\end{aligned}$$

[52] If $L(R_1)$ is constant, then the entire contract is pooled. If $R_1^*=\overline{R}$, then the results that follow apply setting $R_u=\overline{R}$ to be the pooling threshold.

[53] Note that this is without loss, since the pooling threshold could be $R_u=\overline{R}$ if $R^*=\overline{R}$.

where the final line is the relaxed monotonicity constraint (note that we have now dropped investor limited liability as a constraint). Taking the derivative in consumption $c(R_1)$ for $R_1 \leq R_u$, we obtain

$$0 = 1 + \mu(1 - \Lambda_1(R_1)) - \lambda + \chi(R_1) + \nu(R_1).$$

Observe that the resulting contract is non-monotone if $R_u > R^*$, by the same logic as above. Therefore, we can discard candidate contracts with $R_u > R^*$. This implies that $1 + \mu(1 - \Lambda_1(R_1)) - \lambda < 0$ among the set of feasible contracts.

Now, consider the derivative in liquidations $\alpha(R_1)$, given by[54]

$$\frac{\partial \mathscr{L}}{\partial \alpha(R_1)} \propto \lambda(\gamma - 1) + \chi(R_1)b + \nu(R_1)(1 - \gamma)$$

When $\alpha(R_1) = 1$, $\nu(R_1) = 1$ is possible at at most a single point, in particular at $\gamma R_1 Y_0 = x_u$. $\alpha(R_1) = 1$ therefore generically implies $\chi(R_1) > 0$ and $\nu(R_1) = 0$. From the FOC for $c(R_1)$, we have (almost everywhere) that when $\alpha(R_1) = 1$

$$\chi(R_1) = \lambda - 1 - \mu(1 - \Lambda_1(R_1))$$

which, combined with the liquidation rule, yields

$$\frac{\partial \mathscr{L}}{\partial \alpha(R_1)} \propto \lambda(\gamma - 1) + (\lambda - 1 - \mu(1 - \Lambda_1(R_1)))\,b.$$

By MLRP, there is a threshold rule $R_1 \leq R_\ell$ for liquidations.

Finally, in the region (if non-empty) between R_ℓ and R_u, by MLRP we have

$$1 + \mu(1 - \Lambda_1(R_1)) - \lambda < 1 + \mu(1 - \Lambda_1(R_u)) - \lambda < 0$$

[54] Implicitly, we are treating $\alpha(R_1)$ as a continuous variable in performing the differentiation. To do so, we implicitly incorporate the constraint $\alpha(R_1)(1 - \alpha(R_1)) = 0$, which ensures that implementable contracts must set $\alpha(R_1) \in \{0, 1\}$. The logic below is unaffected.

so that we have either $\chi(R_1) > 0$ or $\nu(R_1) > 0$. This implies that $x(R_1) = \min\{(1-b)R_1Y_0, x_u\}$ for all $R_\ell \leq R_1 \leq R_u$.

As a result, the optimal contract is a three-part structure. First, there is a threshold R_ℓ such that $\alpha(R_1) = 1$ and $x(R_1) = \gamma R_1 Y_0$ for $R_1 \leq R_\ell$, and $\alpha(R_1) = 0$ for $R_1 \geq R_\ell$. Second, there is a threshold $R_u \geq R_l$ such that $x(R_1) = \min\{(1-b)R_1Y_0, x_u\}$ for $R_1 \leq R_u$ and $x(R_1) = x_u$ for $R_1 \geq R_u$. Note finally that there cannot be a discontinuity in x at R_u. If there were a discontinuity, we would have

$$x_u > \lim_{R_1 \uparrow R_u} x(R_1) = (1-b)R_uY_0$$

and x would exceed pledgeable income at R_u. The capital structure is therefore continous at R_u.

Finally, the above capital structure can be implemented by a liabilities contract $L(R_1) = (1-b)R_\ell Y_0$ for $R_1 \leq R_\ell$ and $L(R_1) = x(R_1)$ for $R_1 > R_\ell$. This liability structure is monotone, and so we have implementable contracts.

In sum, the optimal contract lies within a class of contracts characterized by thresholds R_ℓ and R_u and corresponding liability structure above. This proves the first part of the proposition.

Now, we characterize the optimal thresholds R_ℓ and R_u. Considering the case where these thresholds are interior, $\underline{R} < R_\ell \leq R_u \leq \overline{R}$ we have the optimization problem

$$\max_{R_\ell, R_u, Y_0} \int_{R_\ell}^{R_u} bR_1Y_0 f_{1,H}(R_1)dR_1 + \int_{R_u}^{\overline{R}} [R_1 - (1-b)R_u]\, Y_0 f_{1,H}(R_1)dR_1$$

subject to

$$\int_{R_\ell}^{R_u} bR_1Y_0 \Big(1 - \Lambda_1(R_1)\Big) f_{1,H}(R_1)dR_1 + \int_{R_u}^{\overline{R}} [R_1 - (1-b)R_u]\, Y_0 \Big(1 - \Lambda_1(R_1)\Big) f_{1,H}(R_1)dR_1 \geq B_0Y_0$$

$$Y_0 - A_0 = \int_{\underline{R}}^{R_\ell} \gamma R_1 Y_0 f_{1,H}(R_1)dR_1 + \int_{R_\ell}^{R_u} (1-b)R_1Y_0 f_{1,H}(R_1)dR_1 + \int_{R_u}^{\overline{R}} (1-b)R_uY_0 f_{1,H}(R_1)dR_1$$

Under the same multiplier convention, the optimality condition for R_ℓ is

$$0 = -bR_\ell Y_0 - \mu b R_\ell Y_0 \Big(1 - \Lambda_1(R_\ell)\Big) + \lambda\,(\gamma - (1-b))\,R_\ell Y_0,$$

which reduces to

$$\mu b(\Lambda(R_\ell) - 1) = b + \lambda\,(1 - b - \gamma)\,.$$

Similarly, the optimality condition for R_u is

$$0 = \int_{R_u}^{\overline{R}} [-(1-b)Y_0 f_{1,H}(R_1) - \mu(1-b)Y_0(1-\Lambda_1(R_1))f_{1,H}(R_1) + \lambda(1-b)Y_0 f_{1,H}(R_1)]\,dR_1,$$

which reduces to

$$0 = \mathbb{E}\,[\lambda - 1 - \mu(1-\Lambda_1(R_1))|R_1 \geq R_u, e_0 = H]\,.$$

This completes the proof.

A.2.1 A Remark on Contract Uniqueness

The optimal contract is not generally unique in the following sense. In the region $R \leq R_\ell$, the bank only needs a liability face value that is sufficient to liquidate the bank, and so any contract with monotone face value $L(R_1) > (1-b)R_1Y_0$ in this region is optimal. We selected the contract with a flat face value below R_ℓ due to its correspondence to standard debt. The face value of liabilities above R_ℓ is uniquely determined. Moreover, in the presence of an $\varepsilon \to 0$ premium for standard debt (e.g. as in Appendix B.7), the implementation using standard debt becomes uniquely optimal.

A.3 Proof of Corollary 4

Consider the proposed liability structure. The amount $(1-b)R_\ell Y_0$ of standard debt liquidates the bank when $R_1 \leq R_\ell$, generating the lower region. $(1-b)(R_u - R_\ell)$ is written down in the region $R_\ell \leq R_1 \leq R_u$, so that the bank is always held to the agency rent over this region. The full debt level

$(1-b)R_u Y_0$ is repaid above R_u. Therefore, we replicate the contract in Proposition 3.

A.4 Proof of Proposition 5

We split the proof into the different cases.

Case 1: Suppose first that $B_0 = 0$, but $b > 0$ and $\gamma(s) < 1-b$. We impose $(1-b)\mathbb{E}[R] < 1$ to obtain a finite solution. Incentive compatibility at date 0 is now

$$E\left[c(R_1)\left(1-\Lambda(R_1)\right)|e_0 = H\right] \geq 0.$$

For any monotone c, we have

$$E\left[c(R_1)\left(1-\Lambda(R_1)\right)|e_0 = H\right] = \operatorname{cov}\left(c(R_1), 1-\Lambda(R_1)\right) \geq 0$$

where the inequality follows from MLRP. As a result, any monotone consumption rule c satisfies (IC-0). Thus the optimal contract structure is determined by date 2 payoffs of Lemma 2, with no liquidations at date 1. This can be interpreted either as bail-in debt or long-term debt.

Case 2: Consider next $b = 0$. The RHS of (12) then collapses to $\lambda(1-\gamma)$ while the LHS collapses to 0, and so banks never choose to liquidate. Optimal contracts use only bail-in debt.

Case 3: Consider finally $\gamma = 1$. Any face value $L(R_1) \leq R_1 Y_0$ can then be repaid by liquidating assets, so that bank consumption is $c(R_1) = R_1 Y_0 - L(R_1)$ for any $L(R_1) \leq R_1 Y_0$. Therefore for any liability structure $L(R_1)$, we can define

$$(c(R_1), x(R_1)) = \begin{cases} \left(R_1 Y_0 - L(R_1), L(R_1)\right), & L(R_1) \leq R_1 Y_0 \\ \left(0, R_1 Y_0\right), & L(R_1) \geq R_1 Y_0 \end{cases}$$

where the relevant liquidation function $\alpha(R_1)$ is defined from the liability structure. Minimum pledgeability never binds. One interpretation is that if $(1-b)R_1Y_0 < L(R_1) < R_1Y_0$, then we have liquidation with a liquidating dividend paid to equity.

Defining the problem in the repayment space, we then have

$$\max \int_{R_1} [R_1Y_0 - x(R_1)]\, f_{1,H}(R_1)dR_1,$$

subject to

$$\int_{R_1} [R_1Y_0 - x(R_1)]\,(1-\Lambda_1(R_1))\, f_{1,H}(R_1)dR_1 \geq B_0Y_0$$

$$Y_0 - A = \int_{R_1} x(R_1) f_{1,H}(R_1)dR_1$$

$$R_1 \geq R_1' \Rightarrow x(R_1) \geq x(R_1')$$

with $0 \leq x(R_1) \leq R_1Y_0$. Relaxing monotonicity, the FOC for $x(R_1)$ is given by

$$\frac{\partial \mathscr{L}}{\partial x(R_1))} = [-1 - \mu\,(1-\Lambda_1(R_1)) + \lambda]\, f_{1,H}(R_1)$$

yielding a non-monotone live-or-die contract (see the proof of Proposition 3). This results in an upper pooling region R_u with liabilities x_u. Because $R_u < R^*$ as in the proof of Proposition 3, we have $x(R_1) = R_1Y_0$ for all $R \leq R_u$. Continuity implies $L(R_1) = R_uY_0$ for all R, and so the contract is standard debt.

A.5 Proof of Proposition 6

Take the optimization problem of choosing the thresholds R_ℓ and R_u obtained from the proof of Proposition 3. In particular, we can rewrite the investor participation constraint as

$$Y_0 = \frac{1}{1 - \left[\int_{\underline{R}}^{R_\ell} \gamma R_1 f_H(R_1)dR_1 + \int_{R_\ell}^{R_u} (1-b)R_1Y_0 f_H(R_1)dR_1 + \int_{R_u}^{\overline{R}} (1-b)R_uY_0 f_H(R_1)dR_1\right]} A_0.$$

We substitute this into the objective function and denote the objective as the utility function

$$U(R_\ell, R_u) = \frac{\int_{R_\ell}^{R_u} bR_1 f_{1,H}(R_1)dR_1 + \int_{R_u}^{\overline{R}} [R_1 - (1-b)R_u] f_{1,H}(R_1)dR_1}{1 - \left[\int_{\underline{R}}^{R_\ell} \gamma R_1 f_{1,H}(R_1)dR_1 + \int_{R_\ell}^{R_u} (1-b)R_1 f_{1,H}(R_1)dR_1 + \int_{R_u}^{\overline{R}} (1-b)R_u f_{1,H}(R_1)dR_1\right]} A_0.$$

Finally, we can use incentive compatibility to represent $R_u(R_\ell)$, that is

$$\int_{R_\ell}^{R_u(R_\ell)} bR_1(1-\Lambda_1(R_1))f_{1,H}(R_1)dR_1 + \int_{R_u(R_\ell)}^{\overline{R}} [R_1 - (1-b)R_u(R_\ell)](1-\Lambda_1(R_1))f_{1,H}(R_1)dR_1 = B_0$$

which allows us to represent the utility function $U(R_\ell, R_u(R_\ell))$ as a utility function over a single variable R_ℓ, $\mathcal{U}(R_\ell)$. The first order condition for optimality is $\mathcal{U}'(R_\ell) = 0$. We are now ready to take a comparative static. In particular, in the usual manner we know that

$$\frac{\partial R_\ell}{\partial \gamma} = \frac{\frac{\partial^2 \mathcal{U}}{\partial R_\ell \partial \gamma}}{-\frac{\partial^2 \mathcal{U}}{\partial R_\ell^2}}$$

which has the same sign as $\frac{\partial^2 \mathcal{U}}{\partial R_\ell \partial \gamma}$. Thus, we look to evaluate the cross partial.

First taking the derivative in γ, note that incentive compatibility does not depend on R_ℓ, and therefore $R_u(R_\ell)$ is not a direct function of γ. Thus, we have

$$\frac{\partial \mathcal{U}}{\partial \gamma} = \mathcal{U}(R_\ell) \frac{\int_{\underline{R}}^{R_\ell} R_1 f_{1,H}(R_1)dR_1}{1 - \left[\int_{\underline{R}}^{R_\ell} \gamma R_1 f_{1,H}(R_1)dR_1 + \int_{R_\ell}^{R_u} (1-b)R_1 f_{1,H}(R_1)dR_1 + \int_{R_u}^{\overline{R}} (1-b)R_u f_{1,H}(R_1)dR_1\right]}.$$

Now, we can take the derivative in R_ℓ. Given that at an optimum $\mathcal{U}'(R_\ell) = 0$, then we have (since we have continuity of payoffs at the boundary R_u)

$$\frac{\partial^2 \mathcal{U}}{\partial\gamma\partial R_\ell} = \mathcal{U}(R_\ell)\frac{R_\ell f_{1,H}(R_\ell)\left[1-\left[\int_{\underline{R}}^{R_\ell}\gamma R_1 f_{1,H}(R_1)dR_1+\int_{R_\ell}^{R_u}(1-b)R_1 f_{1,H}(R_1)dR_1+\int_{R_u}^{\overline{R}}(1-b)R_u f_{1,H}(R_1)dR_1\right]\right]}{\left(1-\left[\int_{\underline{R}}^{R_\ell}\gamma R_1 f_{1,H}(R_1)dR_1+\int_{R_\ell}^{R_u}(1-b)R_1 f_{1,H}(R_1)dR_1+\int_{R_u}^{\overline{R}}(1-b)R_u f_{1,H}(R_1)dR_1\right]\right)^2}$$

$$+\mathcal{U}(R_\ell)\frac{\int_{\underline{R}}^{R_\ell}R_1 f_{1,H}(R_1)dR_1\cdot\left[\gamma R_\ell f_{1,H}(R_\ell)-(1-b)R_\ell f_H(R_\ell)+\int_{R_u}^{\overline{R}}(1-b)\frac{\partial R_u}{\partial R_\ell}f_{1,H}(R_1)dR_1\right.}{\left(1-\left[\int_{\underline{R}}^{R_\ell}\gamma R_1 f_{1,H}(R_1)dR_1+\int_{R_\ell}^{R_u}(1-b)R_1 f_{1,H}(R_1)dR_1+\int_{R_u}^{\overline{R}}(1-b)R_u f_{1,H}(R_1)dR_1\right]\right)^2}$$

which rearranges to

$$\frac{\partial^2 \mathcal{U}}{\partial\gamma\partial R_\ell} = \mathcal{U}(R_\ell)R_\ell f_{1,H}(R_\ell)\frac{\left[1-(1-b)\left[\int_{\underline{R}}^{R_u}R_1 f_{1,H}(R_1)dR_1+\int_{R_u}^{\overline{R}}R_u f_{1,H}(R_1)dR_1\right]\right]+\int_{R_u}^{\overline{R}}(1-b)\frac{\partial R_u}{\partial R_\ell}f_{1,H}(R_1)dR}{\left(1-\left[\int_{\underline{R}}^{R_\ell}\gamma R_1 f_{1,H}(R_1)dR_1+\int_{R_\ell}^{R_u}(1-b)R_1 f_{1,H}(R_1)dR_1+\int_{R_u}^{\overline{R}}(1-b)R_u f_{1,H}(R_1)dR_1\right]\right.}$$

The first expression in the numerator is positive since we assume pledgeable income is less than 1 to ensure finite scale ($(1-b)\mathbb{E}[R_1|e_0=H]<1$). The second term is positive as long as $\frac{\partial R_u}{\partial R_\ell}\geq 0$. To see why this is the case, differentiating incentive compatibility note that we have

$$bR_\ell\Big(1-\Lambda(R_\ell)\Big)f_{1,H}(R_\ell)=\frac{\partial R_u}{\partial R_\ell}\int_{R_u(R_\ell)}^{\overline{R}}(1-b)\left(1-\Lambda_1(R_1)\right)f_{1,H}(R_1)dR_1.$$

From Proposition 3, we know both terms are positive when $R_\ell \leq R_u$, and so we have $\frac{\partial R_u}{\partial R_\ell}\geq 0$. Therefore, we have $\frac{\partial^2\mathcal{U}}{\partial\gamma\partial R_\ell}\geq 0$ and hence $\frac{\partial R_\ell}{\partial\gamma}\geq 0$. Moreover from IC, we also have $\frac{\partial R_u}{\partial\gamma}\geq 0$.

Next, we want to ask how $R_u - R_\ell$ changes in γ. Suppose we are not at a corner solution, $R_\ell < R_u$. Then, using IC and Proposition 3 we have

$$\frac{1}{R_\ell f_{1,H}(R_\ell)}\frac{\partial R_u}{\partial\gamma}=\frac{b+\lambda(1-b)-\lambda\gamma}{b+\lambda(1-b)-1}\frac{\partial R_\ell}{\partial\gamma}$$

and therefore, we have

$$\frac{\partial[R_u - R_\ell]}{\partial\gamma} = \left[R_\ell f_{1,H}(R_\ell)\frac{b+\lambda(1-b)-\lambda\gamma}{b+\lambda(1-b)-1} - 1\right]\frac{\partial R_\ell}{\partial\gamma},$$

from which the required condition follows.

A.6 Proof of Proposition 7

First, the derivation of Lemma 2 follows exactly as before. Thus we once again work in the space (α, x, c) and then back out implementing L.

The program of the social planner is

$$\max \mathbb{E}\left[c(R_1)|e_0 = H\right]$$

subject to

$$\mathbb{E}\left[c(R_1)\left(1-\Lambda_1(R_1)\right)|\, e_0 = H\right] \geq B_0 Y_0$$

$$Y_0 - A = \mathbb{E}[x(R_1)|e_0 = H]$$

$$R_1 \geq R_1' \Rightarrow x(R_1) \geq x(R_1')$$

$$c(R_1) \geq (1-\alpha(R))bR_1Y_0$$

$$\gamma = \gamma(\Omega), \quad \Omega = \int \alpha(R_1)R_1 f_{1,H}(R_1)dR_1$$

where only the last equation is different from the private program in the proof of Proposition 3. The proof follows as in the proof of Proposition 3. First solve for the optimal contract without monotonicity. The first order condition for $c(R_1)$ is the same as in the proof of Proposition 3, since $c(R_1)$ does not directly affect Ω. This implies as before that we obtain a pooling region at the top, x_u and R_u.

As before, take R_u and x_u as given, and solve for the optimal contract for $R_1 \leq R_u$. The same steps imply that implementable contracts must satisfy $R_u < R^*$ for the same definition of R^*, else the contract would be non-monotone. The FOC for optimal liquidations $\alpha(R_1)$ is now

$$\frac{\partial \mathscr{L}}{\partial \alpha(R_1)} \propto \lambda\left(\gamma(\Omega)-1\right) R_1 Y_0 f_{1,H}(R) + \chi(R_1) b R_1 Y_0 f_{1,H}(R_1) + \nu(R_1)\left(1-\gamma\right) R_1 Y_0 f_{1,H}(R_1)$$
$$+ \frac{\partial \gamma(\Omega)}{\partial \Omega} \frac{\partial \Omega}{\partial \alpha(R_1)} \lambda \int_{R'} \alpha(R') R' Y_0 f_{1,H}(R') dR'$$

Substituting in the derivative $\frac{\partial \Omega}{\partial \alpha(R_1)} = R_1 f_{1,H}(R_1)$ and recalling the assumption of constant elasticity $\frac{\Omega}{\gamma}\frac{\partial \gamma}{\partial \Omega} = -\sigma$,[55] we obtain

$$\frac{\partial \mathscr{L}}{\partial \alpha(R_1)} \propto \lambda\left(\gamma - 1\right) + \chi(R_1) b + \nu(R_1)\left(1-\gamma\right) - \lambda \sigma \gamma$$

The additional wedge $-\lambda\sigma\gamma$ is negative and independent of R_1. Thus the same steps apply as in the proof of Proposition 3, yielding a liquidation threshold rule R_ℓ. Because as before $R_u < R^*$, we have $x(R_1) = \min\{(1-b)R_1 Y_0, x_u\}$ in the region $R_\ell \le R_1 \le R_u$, with continuity at R_u for the same reason. Thus, the set of candidate optimal contracts is the same as in the private equilibrium, and the implementation of Corollary 4 holds (optimal contracts combine standard and bail-in debt).

Lastly, we characterize the optimal choices of R_ℓ and R_u for interior solutions. The optimality condition for R_u is identical to the private optimality condition, since it does not affect the liquidation value. By contrast, the social optimality condition for R_ℓ satisfies

$$b + \lambda\left((1-b)-\gamma\right) = \mu b\left(\Lambda_1(R_\ell)-1\right) - \lambda\sigma\gamma.$$

which gives the result. Finally, the FOC for R_u is the same.

The only remaining part of the proposition is the comparative static (Proposition 6). Adopting notation of the proof of Proposition 6, we can write for the social planner

$$\frac{dU(R_\ell, R_u)}{dR_\ell} = \frac{\partial U}{\partial R_\ell} + \frac{\partial U}{\partial \gamma}\frac{\partial \gamma}{\partial R_\ell}$$

[55] Note the same steps can be used to prove the result even without constant elasticity.

The first term on the RHS is the private bank derivative. We know that $\frac{\partial U}{\partial \gamma} > 0$ and that $\frac{\partial \gamma}{\partial R_\ell} < 0$. Therefore, we have $\frac{dU}{dR_\ell} < \frac{\partial U}{\partial R_\ell}$. Thus at the private optimum, we have $\frac{dU}{dR_\ell} < 0$. Thus the planner lowers R_ℓ and there is less standard debt. Finally, the comparative static on total debt and bail-in debt is derived only from incentive compatibility, and hence the results on total and bail-in debt hold under an analogous condition as Proposition 6.

A.7 Proof of Corollary 8

We need only to verify that the ex post bail-in authority achieves the same outcome as the contractual liabilites of the social optimum. In the region $R_1 < R_\ell$, the non-bail-inable senior debt exceeds asset values, and the bail-in authority is unable to resolve the bank. The bank is liquidated.

In the region $R_\ell \leq R_1 < R_u$, if the bail-in authority does not intervene then the bank gets 0, senior non-bail-inable debt gets $x^S(R_1) = \min\{(1-b)R_\ell, \gamma R_1\}$, and junior bail-inable debt gets $\max\{\gamma R_1 - x^S(R_1), 0\}$. If by contrast the bail-in authority intervenes, it recapitalizes the bank with any haircut $R' - R_\ell \leq R_1 - R_\ell \leq R_u - R_\ell$ to bail-inable junior debt. Senior non-bail-inable debt gets fully repaid and is weakly better off. The bank gets payment bR' and is better off. Junior bail-inable debt gets $(1-b)(R' - R_\ell)$, and is better off local to $R' = R_1$ because $\gamma < 1-b$. Therefore, there is a Pareto efficient haircut. The haircut that maximizes total recovery value to creditors is $R' = R_1$, which is the same outcome as contractual bail-in debt.

In the region $R_u \geq R_1$, the bank is solvent, and all debt is repaid in full. A haircut on bail-inable debt is not Pareto efficient, and the bail-in authority does not act.

Hence, the bail-in authority implements the social optimum.

A.8 Proof of Proposition 9

Consider the program of the planner with uncertainty. Thus the program is identical to that in Proposition 7 except for the aggregate uncertainty, and we can write all expectations as being over σ and all policies as functions of (R_1, σ). Thus, all the same steps tell us we have a contract

$(R_\ell(\sigma), R_u(\sigma))$ that is σ-adapted. The first order condition for the $R_\ell(\sigma)$ is identical to the characterization in Proposition 7. Hence since the Lagrange multipliers (μ, λ) are invariant to σ, from MLRP we have that $R_\ell(\sigma)$ falls in $(1-\sigma)\gamma$. Suppose that $R_\ell(\sigma)$ were not decreasing in σ. Then there are states $\sigma_1 > \sigma_2$ such that $R_\ell(\sigma_1) > R_\ell(\sigma_2)$ and $(1-\sigma_1)\gamma_1 > (1-\sigma_2)\gamma_2$. But this means that $\gamma_1 > \gamma_2$, a contradiction. Hence, $R_\ell(\sigma)$ decreases in σ. Finally, the first order condition for R_u takes the same form as in the proof of Proposition 3 regardless of the σ realization, hence R_u does not depend on σ. This concludes the proof.

A.9 Proof of Proposition 10

First consider incentive compatibility. Investor utility after contracts have been signed is given by

$$\int c(R_1) f(R_1|e_0) dR_1 + \frac{1}{2} B_0 (1 - e_0^2) Y_0.$$

Thus differentiating in e_0, we obtain

$$\int c(R_1)(1 - \Lambda_1(R_1)) f_H(R_1) dR_1 = B_0 e_0 Y_0.$$

Therefore, the banks' program is identical to that in Proposition 3, except for the new definition of incentive compatibility and the fact that $e_0 \in [0,1]$ is also a choice variable. The Lagrangian is hence also identical. When $\mu > 0$ and hence there is positive value to higher effort, the proof that optimal contracts combine standard and bail-in debt proceeds using exactly the same steps as the proof of Proposition 3. For the socially optimal contract, note that total liquidations are now given by $\Omega = \int \alpha(R_1) R_1 f_{1,H}(R_1|e_0) dR_1$. Thus when $\mu > 0$ and there is positive value to higher effort, the proof that optimal contracts combine standard and bail-in debt also proceeds in the same manner as the proof of Proposition 7.

Note that the interpretation of $\mu > 0$ is the same as the baseline model. Define a wedge in incentive compatibility Δ by

$$\int c(R_1)(1-\Lambda_1(R_1))f_H(R_1)dR_1+\Delta=B_0e_0Y_0.$$

We therefore have

$$\frac{\partial\mathscr{L}}{\partial\Delta}=\mu,$$

that is μ is the marginal value of a marginal commitment to increase effort provision.

Finally, let us consider the privately and socially optimal choices of R_u. We know that both take the same form as Proposition 3 and 7, but a direct comparison can no longer be made due to differences in equilibrium values of e_0. From incentive compatibility, we know that local to the optimum

$$\frac{\partial e_0}{\partial R_u}=-\int_{R_u}^{\overline{R}}(1-b)(1-\Lambda_1(R_1))f_H(R_1)dR_1<0$$

which follows from the usual FOC for R_u.

Now, we can take the social first order condition over R_u. Representing the program in the form of Proposition 3 but internalizing the effort choice $e_0(R_\ell,R_u)$, we have the social first order condition for R_u given by

$$0=\frac{\partial\mathscr{L}}{\partial R_u}+\lambda\frac{\partial\mathscr{L}}{\partial\gamma}\frac{\partial\gamma}{\partial R_u}$$

where the second term is the part uninternalized by private agents. Previously, it had been equal to zero. However, now we have

$$\frac{\partial\gamma}{\partial R_u}=\frac{\partial\gamma}{\partial\Omega}\frac{\partial\Omega}{\partial e_0}\frac{\partial e_0}{\partial R_u}$$

We know that $\frac{\partial\gamma}{\partial\Omega}<0$ and, as just argued, $\frac{\partial e_0}{\partial R_u}<0$ local to the social optimum. Finally, we know that

$$\frac{\partial\Omega}{\partial e_0}=\int_{\underline{R}}^{R_\ell}R_1(1-\Lambda_1(R_1))f_{1,H}(R_1)dR_1<0$$

where the last line follows since $\Lambda_1(R_1)>\Lambda_1(R_\ell)>1$ for $R_1<R_\ell$.[56] Thus since $\frac{\partial\mathscr{L}}{\partial\gamma}>0$, we know

[56]For completeness, note that we must indeed have $\Lambda_1(R_\ell)>1$. If hypothetically we had $\Lambda_1(R_\ell)<1$, then a

there is a negative wedge on the margin in the social FOC for R_u relative to the private FOC for R_u, and hence there is a positive tax on total debt R_u in the decentralization. This concludes the proof.

A.10 Proof of Proposition 11

Recall from Section 2.6 that we have $\gamma(\sigma,R_\ell)=\overline{\gamma}(\sigma)\Omega(R_\ell)^{-\sigma}$, where $\overline{\gamma}(\sigma)=\overline{\gamma}\underline{\Omega}^{\sigma}$ and that we have assumed $\Omega\geq\underline{\Omega}$ in equilibrium. Note that we have

$$\frac{\partial^2 U}{\partial\sigma\partial R_\ell}=\frac{\partial}{\partial\sigma}\left[(1-\sigma)\gamma(\sigma,\Omega(R_\ell))\frac{\partial\Omega}{\partial R_\ell}\right]=\left[-\gamma+(1-\sigma)\frac{\partial\gamma}{\partial\sigma}\right]\frac{\partial\Omega}{\partial R_\ell}<0$$

where the last line follows since $\sigma<1$, $\frac{\partial\gamma}{\partial\sigma}<0$, and $\frac{\partial\Omega}{\partial R_\ell}>0$. Recall that the efficient rule $R_\ell(\sigma)$ of Proposition 9 is a decreasing function of σ. Thus we have a quasilinear mechanism design problem with a one-dimensional action R_ℓ and a one dimensional type σ that satisfies the usual Spence-Mirrlees condition, and so we have an incentive compatible mechanism for some T.[57] Note that because $R_\ell(\sigma)$ is an decreasing function and U is an decreasing function of R_ℓ, then $T(\sigma)$ is an increasing function of σ.

For completeness, we derive an incentive compatible mechanism when $\Sigma=\{\underline{\sigma},\overline{\sigma}\}$. The derivation follows in the usual manner and we only present the full argument for completeness. We will specify the smallest punishment $T(\overline{\sigma})>0$ possible when $T(\underline{\sigma})=0$. Incentive compatibility for type $\underline{\sigma}$ implies the smallest possible punishment is

$$T(\overline{\sigma})=U(\underline{\sigma},R_\ell(\overline{\sigma}))-U(\underline{\sigma},R_\ell(\underline{\sigma})).$$

Now, we verify incentive compatibility for type $\overline{\sigma}$. Monotonicity gives

$$U(\underline{\sigma},R_\ell(\underline{\sigma}))-U(\underline{\sigma},R_\ell(\overline{\sigma}))\geq U(\overline{\sigma},R_\ell(\underline{\sigma}))-U(\overline{\sigma},R_\ell(\overline{\sigma})).$$

reduction in R_ℓ improves private welfare, reduces fire sales directly, and also reduces fire sales indirectly by promoting effort. Thus, $\Lambda_1(R_\ell)<1$ is never efficient.

[57] To obtain the standard form of the condition as positive cross-partial and nondecreasing allocation, recall we can always redefine the type as $\theta=1-\sigma$.

Note the LHS is $-T(\overline{\sigma})$, so we get

$$U(\overline{\sigma},R_\ell(\overline{\sigma}))-T(\overline{\sigma})\geq U(\overline{\sigma},R_\ell(\underline{\sigma}))-T(\underline{\sigma})$$

giving incentive compatibility for type $\underline{\sigma}$. The final step comes from simply substituting in that $U(\underline{\sigma},R_\ell)=(\overline{\gamma}-1)\Omega(R_\ell)$ when $\underline{\sigma}=0$.

A.11 Proof of Proposition 12

We adopt the following proof strategy. We will consider a contract that results in bailouts, and show that it is equivalent to a contract that: (1) features bail-ins (rather than bailouts) ex post; and, (2) implements an ex ante lump sum transfer from taxpayers to the bank. Thus, all contracts with bailouts are equivalent to contracts without bailouts combined with lump sum transfers. We then construct the Pareto weight that equalizes marginal utility and hence rules out transfers.

Suppose that there is a liability structure with a bailout at return R_1, so that $L(R_1)>(1-b)R_1Y_0$ and $T_1(R_1)=L(R_1)-(1-b)R_1Y_0$. This generates consumption profile $c(R_1)=bR_1Y_0$ and a repayment to investors $x(R_1)=L(R_1)=(1-b)R_1Y_0+T_1(R_1)=\hat{x}(R_1)+T_1(R)$, where $\hat{x}(R_1)$ is repayment out of bank resources. Substituting into the participation constraint, we have

$$Y_0-A_0-\mathbb{E}[T_1(R_1)]\leq\mathbb{E}\Big[\hat{x}(R_1)\Big].$$

The problem is otherwise identical. Hence, from the bank perspective the bailout $T_1(R)$ is equivalent to a bail-in contract combined with a lump-sum transfer of equal expected value from taxpayers to the bank at date 0. Moreover, taxpayer optimization of B_0 implies the change in contract also has no impact on taxpayer welfare, since the taxpayer simply adjusts B_0 in response to maintain the same path of consumption.

Thus, we need merely to characterize the Pareto weight that equalizes marginal utility. Defining the social welfare weight on taxpayers to be $\omega^T=\frac{\lambda}{u_0'(c_0^T)}$, then the social planner is indifferent to transfers between banks and taxpayers at date 0. Thus we have equalized marginal utilities, meaning we have a Pareto efficient contract of Proposition 7 without bailouts. This concludes the proof.

B Extensions

In this Appendix, we provide a number of extensions. Appendix B.1 allows for positive arbitrageur welfare weights. Appendix B.2 allows for aggregate risk. Appendix B.3 characterizes optimal contracts without monotonicity. Appendix B.4 considers the interaction between macroprudential (asset-side) and liability-side regulation. Appendix B.5 studies the allocation of bail-in securities among heterogeneous investors with different risk tolerances and different exposures to the banking sector. Appendix B.6 incorporates a role for (outside) equity-like claims into the bank's capital structure by incorporating bank risk aversion and risk shifting. Appendix B.7 allows for standard debt to command a premium over other instruments, including bail-in debt. Appendix B.8 studies the trade-off between bailouts and bail-ins in protecting insured deposits when banks are allowed to issue insured deposits as part of their standard debt. Appendix B.9 studies bailouts without commitment.

B.1 Pareto Efficiency

We now study Pareto efficient social contracts, accounting for positive welfare weight ω^A on arbitrageurs. Recall that we have assumed that $u'(\overline{A}) > 1$. We obtain the following result.

Proposition 13. *Let $\sigma > 0$. Then, the socially optimal contract features R_ℓ given by*

$$\mu b(\Lambda_1(R_\ell) - 1) = b + \lambda\left((1-b) - \gamma\right) - \lambda\sigma\gamma\omega^*$$

where $\omega^* = 1 - \frac{1}{u'(\overline{A}-A_0)} > 0$. As a result, the privately optimal contract is not Pareto efficient given $u'(\overline{A}) > 1$.

Pareto efficient improvements arise because arbitrageurs are borrowing constrained, so that their marginal utility at date 0 exceeds that at date 1. Efficiency is achieved by transfering resources to arbitrageurs at date 0 in order to compensate them for resource losses from lower surplus from bank liquidations. This is a form of distributive externality.

When we take $\omega^A \to 0$, the associated point on the Pareto frontier is $A_0 \to \overline{A}$ and $u'(\overline{A} - A_0) \to +\infty$, and we obtain the first order condition of Proposition 7.

B.1.1 Proof of Proposition 13

We can characterize a Pareto efficient contract by adopting the welfare function

$$\mathbb{E}\left[c|e_0 = H\right] + \omega^A \left[u\left(\overline{A} - A_0\right) + \left(\mathscr{F}(\Omega) - \gamma(\Omega)\Omega\right)Y_0\right],$$

where ω^A is the welfare weight on arbitrageurs. The optimality of standard and bail-in debt follows the same steps as in the proofs of Propositions 3 and 7. However, in writing the optimal choice of threshhold R_ℓ, we now account for arbitrageur surplus and obtain

$$b + \mu b(\Lambda_1(R_\ell) - 1) = b + \lambda\left((1-b) - \gamma\right) - \left(\lambda - \omega^A\right)\sigma\gamma.$$

Finally, the optimality condition for A_0 is given by $\lambda = \omega^A u'(\overline{A} - A_0)$. Substituting in completes the proof.

B.2 Aggregate Risk

To incorporate aggregate risk into the model, we add an aggregate state $s \in S$ of the economy at date 1. For expositional simplicity, we assume that S is a finite set, with probability measure $\pi(s)$. Also for expositional simplicity, we assume s only affects the distribution over date 1 returns and not over date 2 returns. In this sense, we think of date 2 as being "the long run."

The aggregate state s affects the return distribution, so that we have $f_{1,e}(R_1|s)$. All contracts can be written on the aggregate state. MLRP now applies contingent on the aggregate state, and liability monotonicity is also contingent on the aggregate state.

From here, the characterization of privately optimal contracts follows almost identically to before.

Proposition 14. *A privately optimal bank contract has an aggregate-state-contingent form of that in Proposition 3, where both $R_\ell(s)$ and $R_u(s)$ now depend on s.*

Proof. The proof follows the same steps as the proof of Proposition 3, except that contracts are now adapted to (R_1, s). ■

In contrast to the baseline model, both instruments are contingent on the aggregate state, reflecting that the terms of bank contracts adjust to verifiable events that are beyond a bank's control. For example, if all else equal a state s has lower returns due to an aggregate (TFP) shock, equation (12) implies it should have a lower liquidation threshold.[58]

In the context of CoCos, conditioning the level of bail-in debt on both the idiosyncratic state (i.e. individual bank health) and aggregate state (i.e. banking sector health) can be thought of as a dual trigger (see also Proposition 9 in the main text). In this context, there is $R_\ell(s_{\min})$ of fully non-contingent debt, and $R_u(s_{\max})$ of bail-in debt with a dual-price trigger. The dual price trigger writes down bail-in debt automatically to $R_u(s)$ based on the aggregate state s, and allows for it to be additionally written down to $R_\ell(s) - R_\ell(s_{\min})$ to restore bank solvency. The dual price trigger thus conditions recapitalization of banks on the aggregate state as well as the idiosyncratic state.[59]

From here, the results on the socially optimal contract proceed identically, with the state contingency. Similarly, the bailout results can also be derived, where the result is that no bail-in debt is issued for state s whenever there are bailouts in state s. This helps to understand the limits of bank contingencies on verifiable aggregate risk. Although aggregate risk is verifiable and not

[58] See Dewatripont and Tirole (2012) for a related argument.

[59] Alternatively, we could consider it a combination of $R_\ell(s_{\min})$ of fully non-contingent debt, $R_u(s_{\max})$ of debt with a dual price trigger but no automatic write-down, and an aggregate risk hedge that mimicked the automatic write-down.

a result of bank shirking, banks neglect fire sales and expect to receive bailouts in bad aggregate states. This limits the extent to which they write contingencies on aggregate risk.

B.2.1 Bail-in Equivalence with Aggregate Risk

As highlighted above, the degree of bail-inability of debt depends on the aggregate state. Such rules either must be contractually pre-written into debt contracts, or must be written into the rules governing the operations of the bail-in authority. Provided that such rules are specified to govern ex-post resolution, the equivalence between ex-ante contractual provisions and ex-post resolution follows as in the baseline model.

In the US, such rules could be implemented using the organizational structure of the bank. Bank holding companies are required to maintain an amount of loss-absorbing debt at the level of the top-level holding company. The goal is to resolve the top-level holding company while allowing operating subsidiaries to continue operations without being affected by the resolution of the holding company. In principle, however, if a full write-down of the liabilities at the holding company level is not sufficient to recapitalize the bank, recapitalization would require bail-ins of debt at the operating subsidiaries. One could structure the governing rules of the bail-in authority to condition the ability of that authority to resolve operating subsidiaries based on the state of the economy. Operating subsidiaries could be resolved by the bail-in authority in crises, but not in normal times.

It is not clear whether aggregate state contingent rules governing the bail-in authority could credibly be implemented and followed. A bail-in authority is likely to be tempted to recapitalize a bank if there is enough long-term debt available to do so, suggesting the potential for time inconsistency in bail-ins.

B.3 Optimal Contract Without Monotonicity

In the main text, we impose that liability contracts must be monotone. In this appendix, we characterize the optimal contract without monotonicity. For simplicity, we focus solely on the date

0 effort choice.[60]

In order to do so, we will need to bound investor payoffs below. In particular, we impose the following limited liability constraint on investors,

$$x(R_1) \geq -\underline{x},$$

where $\underline{x} \geq 0$ is therefore the minimum payment that can be made to the bank by investors. If $\underline{x} = 0$, then this is a limited liability constraint in the standard sense that investors cannot be forced to pay money into the bank after date 0. We obtain the following characterization of a privately optimal contract.

Proposition 15. *A privately optimal bank contract has a liability structure*

$$x(R_1) = \begin{cases} (1-b)R_\ell Y_0, & R_1 \leq R_\ell \\ (1-b)R_1 Y_0, & R_\ell \leq R_1 \leq R_u \\ -\underline{x}, & R_u \leq R_1 \end{cases}$$

where R_l and R_u are given by

$$\underbrace{\mu b\left(\Lambda(R_\ell)-1\right)}_{\text{Incentive Provision}} = \underbrace{b+\lambda\left(1-b-\gamma\right)}_{\text{Liquidation Costs}} \tag{A.2}$$

$$\underbrace{\mu\left(1-\Lambda(R_u)\right)}_{\text{Incentive Provision}} = \underbrace{\lambda - 1}_{\text{Investor Repayment}} \tag{A.3}$$

Proof. The proof proceeds identically to the steps in the proof of Proposition 3, except that we do not need to impose monotonicity and hence get $x_u = -\underline{x}$. ∎

The optimal contract of Proposition 15 is a form of live or die contract (Innes 1990). As in

[60] We could always rederive Lemma 2 in a similar manner as a continuation live-or-die contract and an associated agency rent.

the model with monotonicity (Proposition 3), in the region $R_1 \leq R_\ell$ the bank is liquidated, while in the region $R_\ell \leq R_1 \leq R_\ell$ the bank is held to its agency rent via "bail ins." The key difference is the upper region. In the model with monotonicity, liabilities were $L(R_1) = (1-b)R_u Y_0$ in this region, corresponding to debt. Here instead, investors make the largest payment possible to banks in this region, $x(R_1) = -\underline{x} \leq 0$, in order to best incentivize effort. Note that taking $\underline{x} \to \infty$ results in $R_u \to \overline{R}$, that is the bank will receive an arbitrarily large repayment in only the highest return state.[61]

The following corollary provides a simple implementation of this contract, using one additional instrument relative to Corollary 4.

Corollary 16. *Absent monotonicity, the privately optimal contract can be implemented by a combination of three instruments:*

1. *Standard debt with face value* $(1-b)R_\ell Y_0$.
2. *Bail-in debt with face value* $(1-b)R_u Y_0$.
3. *An insurance contract (or option) that pays out* $(1-b)R_u Y_0 - \underline{x}$ *to the banker in the event that* $R_1 \geq R_u$*, and pays out* 0 *otherwise.*

Corollary 16 shows that removing monotonicity simply requires the addition of an insurance/option contract to the bank's capital structure. The contract pays off a fixed amount $(1-b)R_u Y_0 - \underline{x}$ in "success' states of high returns, $R_1 \geq R_u$. This allows the banker to repay all debtholders, as well as receive the highest payment $\underline{x}$ from investors possible.

From here, the core regulatory results of Proposition 7 proceeds as before. A social planner with a complete set of regulatory instruments implements the same contract structure as Proposition 15, but sets R_ℓ lower to account for the fire sale, thus increasing the use of bail-in debt. However, the planner agrees with the bank over choice of R_u and use of the insurance/option contract. Thus, the qualitative insights for bail-in policies remain in this case.

[61] This follows necessarily from the participation constraint of investors.

B.4 Macroprudential Regulation and Bail-ins

In the baseline model, the fact that banks have a single investment project means that liability-side regulation is sufficient. In practice, banks asset allocations also affect their risk profiles. We now show that macroprudential (asset-side) regulation is a necessary complement to bail-ins when banks can affect risks using both sides of their balance sheet.

We augment the model as follows. Banks choose a contractible vector $\theta = (\theta_1, ..., \theta_N)$ of asset allocations at date 0. The total return R_1 on bank scale Y_0 follows a density $f_{1,e}(R_1|\theta)$, which depends on the allocation θ. For simplicity, R_2 does not depend on θ. $f_{1,e}(R_1|\theta)$ satisfies MLRP (conditional on θ) over the relevant range of allocations θ. To simplify exposition, the support of R_1 is an interval $[\underline{R}, \overline{R}]$ that does not depend on θ. Otherwise, the setup is the same as before.[62]

As before, optimal liability contracts combine contingent and standard debt, and the trade-off between standard and bail-in debt reflects the same forces as before.[63] We now characterize the optimal asset allocation rule under the socially optimal contract.

Proposition 17. *The socially optimal contract has FOC for θ_n*

$$0 = \underbrace{\mathbb{E}\left[\left(\lambda x(R_1) + c(R_1)\left(1 + \mu\left(1 - \frac{\partial f_{1,L}(R_1|\theta)/\partial\theta_n}{\partial f_{1,H}(R_1|\theta)/\partial\theta_n}\right)\right)\right)\frac{\partial f_{1,H}(R|\theta)/\partial\theta_n}{f_{1,H}(R|\theta)}\right]}_{\text{Private Bank Benefit}}$$

$$+ \underbrace{\lambda E\left[\frac{\partial\gamma}{\partial\Omega}\Omega Y_0 \cdot \left[\int_{\underline{R}}^{R_\ell} R_1 \frac{\partial f_{1,H}(R_1|\theta)}{\partial\theta_n} dR_1\right]\right]}_{\text{Fire Sale Cost}}$$

The first line of Proposition 17 reflects the private trade-off to banks of a change in asset composition, corresponding to changes in the return distribution. These changes are weighted by the (weighted) sum of payoff to investors in those states, and to banks in those states, where the weighting reflects

[62] In Appendix B.4.1, we show how a standard asset allocation problem generates a density function of this form. If the shirking benefit $B(\theta)$ depended on the allocation, e.g. because riskier assets are more difficult to monitor, the planner and banker would agree on how θ affects B. Assets in our model all sell at the same discount and generate the same fire sale spillover. If they differed in terms of liquidation discounts and fire sale spillovers, there would be an additional regulatory incentive on this margin.

[63] Given that θ is contractible, the proof follows the same steps as Proposition 3.

both the direct value of payoffs, and the incentive value of payoffs. The second line of Proposition 17 reflects the *social* cost of changes in asset composition. The social cost arises when changes in the return distribution affect the magnitude of the fire sale spillover, by altering the measure Ω of bank liquidations. When an asset increases the probability that the banks' total return is lower than R_l, larger allocations to that asset result in more severe fire sale spillovers. The social cost term penalizes investment in such assets. The social cost term exists whenever $R_\ell > \underline{R}$, that is whenever liability-side regulation has not completely eliminated bank failures.

Proposition 17 illustrates that macroprudential (asset) regulation is a necessary complement to bail-ins (liability regulation). Macroprudential regulation and bail-ins co-exist in the regulatory regime because they control fire sales in different manners. For a given level of asset risk, bail-ins mitigate fire sales by reducing the liquidation threshold. For a given liquidation threshold, macroprudential regulation mitigates fire sales by reducing the probability that a bank will fall below that threshold.[64] These two aspects of regulation are not generally perfect substitutes, so they co-exist under the optimal regulatory regime.

Even though macroprudential regulation and bail-ins are not perfect substitutes, Proposition 17 suggests that bail-ins are a partial substitute for macroprudential regulation. Stronger liability regulation pushes the magnitude of the additional wedge in the asset allocation decision towards zero, by reducing the size of the liquidation region.

B.4.1 Multiple Assets Density Function

Suppose that there are $N+1$ assets between which the bank allocates its funds. Denote $\omega \in [\underline{\omega}, \overline{\omega}]$ to be the underlying idiosyncratic state of the bank, with associated density $f_e^\omega(\omega)$, where $e \in \{H, L\}$. Suppose that $f_e^\omega(\omega)$ satisfies MLRP, so that $\frac{\partial}{\partial \omega}\left(\frac{f_H^\omega(\omega)}{f_L^\omega(\omega)}\right) > 0$.

Asset $n \in \{1, ..., N+1\}$ generates a return $R_n(\omega)$ per unit. Let $\theta = (\theta_1, ..., \theta_{N+1})$ be a vector that determines the asset allocations $\theta_1 Y_0$,...,$\theta_{N+1} Y_0$. Allocations θ satisfy a technological restriction $\mathscr{F}(\theta) = 0$, for example there may be a concave technology. Note that to coincide with the previous

[64] Macroprudential regulation in our model closely risk weights on loss-absorbing capital.

parts, we assume the technology is linear in the scale Y_0, and only (potentially) concave in the asset weights. If $\mathscr{F}(\theta) = \sum_{n=1}^{N+1} \theta_n - 1$, we have a simple linear technology with equal cost of investment across assets.

We invert θ_{N+1} from $(\theta_1, ..., \theta_N)$ via $\mathscr{F}$, so that we can internalize the constraint. We denote the total return to the bank, given an asset allocation vector θ, by

$$R(\omega\theta) = \sum_{n=1}^{N+1} \theta_n R_n(\omega) Y_0$$

where θ_{N+1} is derived from the technology $\mathscr{F}(\theta) = 0$, given $\theta_1, ..., \theta_N$.

Suppose that conditional on θ, there is an injective mapping between ω and R_1. In this case, R_1 identifies ω, given θ, and we can write contracts on R_1. We assume that the mapping is injective over the relevant range of asset allocations θ. For example, this will be the case if asset allocations are non-negative ($\theta_n \geq 0$) and individual asset returns are monotone in ω. Without loss of generality, we assume the injective mapping is monotone increasing: high states ω identify high returns R_1, consistent with the interpretation of $e = H$ as "high effort."

Denote $R^{-1}(R_1|\theta)$ to be the inverse function mapping the total return R_1 into the idiosyncratic state ω. The inverse function does not depend directly on e, but rather the density will depend on e. We now derive the density of R_1, conditional on θ. We have

$$F_{1,e}(R_1|\theta) = \Pr(R(\omega|\theta) \leq R_1|e) = \Pr\left(\omega \leq R^{-1}(R_1|\theta)|e\right) = F_e^{\omega}\left(R^{-1}(R_1|\theta)\right).$$

Differentiating in R, we obtain the density function:

$$f_{1,e}(R_1|\theta) = f_e^{\omega}\left(R^{-1}(R_1|\theta)\right) \frac{\partial R^{-1}(R_1|\theta)}{\partial R_1}$$

We impose the simplifying assumption that the support $[\underline{R}, \overline{R}]$ of the density is invariant to the allocation θ. If the support depended on the portfolio allocation, we would have boundary terms in derivatives. The principal term of relevance would be how the lower boundary of the support moves in the asset allocation, which reflects changes in the measure of the liquidation region. These effects are qualitatively the same as the direct effects of changing the measure from changes in the density. For simplicity, we keep the support fixed.

Finally, we can show that this function satisfies monotone likelihood. Differentiating the likelihood ratio in R, we obtain

$$
\begin{aligned}
\frac{d}{dR_1}\left(\frac{f_{1,H}(R_1|\theta)}{f_{1,L}(R_1|\theta)}\right) &= \frac{d}{dR_1}\left(\frac{f_H^{\omega}\left(R^{-1}(R_1|\theta)\right)}{f_L^{\omega}\left(R^{-1}(R_1|\theta)\right)}\right) \\
&= \frac{\frac{\partial f_H^{\omega}}{\partial \omega}\frac{\partial R^{-1}(R_1|\theta)}{\partial R_1} f_L^{\omega} - \frac{\partial f_L^{\omega}}{\partial \omega}\frac{\partial R^{-1}(R_1|\theta)}{\partial R_1} f_H^{\omega}}{(f_L^{\omega})^2} \\
&= \frac{\partial}{\partial \omega}\left(\frac{f_H^{\omega}}{f_L^{\omega}}\right)\frac{\partial R^{-1}(R_1|\theta)}{\partial R_1} \\
&> 0
\end{aligned}
$$

where in the last line, we have used MLRP on f_e^{ω} combined with monotonicity of R^{-1}.

As a result, we obtain a representation of the problem as a density $f_{1,e}(R_1|\theta)$. Implicitly, we differentiate in $(\theta_1, ..., \theta_N)$, where we have internalized θ_{N+1} as arising from the technology.

B.4.2 Proof of Proposition 17

Consider the optimal contract of the social planner. Holding fixed the debt levels R_l and R_u, the derivative of the planner's Lagrangian in θ_n is given by

$$
\begin{aligned}
0 =& \mathbb{E}\left[c\frac{\partial f_{1,H}(R_1|\theta)/\partial \theta_n}{f_{1,H}(R_1|\theta)}\right] + \mu \mathbb{E}\left[c\left(\frac{\partial f_{1,H}(R_1|\theta)/\partial \theta_n}{f_{1,H}(R_1|\theta)} - \frac{\partial f_{1,L}(R_1|\theta)/\partial \theta_n}{f_{1,H}(R_1|\theta)}\right)\right] \\
&+ \lambda \mathbb{E}\left[x\frac{\partial f_{1,H}(R_1|\theta)/\partial \theta_n}{f_{1,H}(R_1|\theta)}\right] \\
&+ \lambda \mathbb{E}\left[\int_{\underline{R}}^{R_\ell} \frac{\partial \gamma(\Omega)}{\partial \Omega}\frac{\partial \Omega}{\partial \theta_n} R_1 Y_0 f_{1,H}(R_1|\theta) dR_1\right]
\end{aligned}
$$

where the first two lines reflect the private bank trade-off, and the last line reflects the social trade-off. Liquidations are given by

$$\Omega = \int_{\underline{R}}^{R_\ell} R_1 f_{1,H}(R_1|\theta) dR_1$$

so that we have

$$\frac{\partial \Omega}{\partial \theta_n} = \int_{\underline{R}}^{R_\ell} R_1 \frac{f_{1,H}(R_1|\theta)}{\partial \theta_n} dR_1.$$

Substituting in above, we obtain

$$0 = \underbrace{\mathbb{E}\left[\left(\lambda x(R_1) + c(R_1)\left(1 + \mu\left(1 - \frac{\partial f_{1,L}(R_1|\theta)/\partial \theta_n}{\partial f_{1,H}(R_1|\theta)/\partial \theta_n}\right)\right)\right) \frac{\partial f_{1,H}(R_1|\theta)/\partial \theta_n}{f_{1,H}(R_1|\theta)}\right]}_{\text{Private Bank Benefit}}$$
$$+ \underbrace{\lambda \mathbb{E}\left[\frac{\partial \gamma}{\partial \Omega} \Omega Y_0 \cdot \left[\int_{\underline{R}}^{R_\ell} R_1 \frac{\partial f_{1,H}(R_1|\theta)}{\partial \theta_n} dR_1\right]\right]}_{\text{Fire Sale Cost}}$$

giving the result.

B.5 Heterogeneous Investors and the Allocation of Securities

In the baseline model, investors are homogeneous and risk neutral, so that the distribution of standard and bail-in debt among investors is irrelevant. A key practical concern is what investors should hold what form of debt, since bail-in debt holders will experience losses when it is written down. Particular concern has been expressed about protecting retail investors from losses that are large relative to their wealth[65], and to preventing institutional investors who are potentially exposed to fire sales from bearing losses from bail-ins.[66]

To capture these elements, we extend the model to include two classes of bank investors, "institutional" and "retail." To make the problem interesting, we include aggregate risk. Institutional investors are able to invest across all banks, but still retain exposure to the aggregate state and

[65] The resolution of four Italian banks in 2015 sparked a political backlash due to losses to retail investors. Financial Times, "Italy bank rescues spark bail-in debate as anger at Renzi grows," December 22, 2015.

[66] Article 44 of BRRD states that "[m]ember states shall ensure that in order to provide for the resolvability of institutions and groups, resolution authorities limit...the extent to which other institutions hold liabilities eligible for a bail-in tool."

have preferences that may depend on bank liquidation discounts. Retail investors are only able to invest in a single bank and retain exposure to the idiosyncratic return of that bank. For simplicity, we abstract away from other potential components of these investors' portfolio choice problems, instead allowing for state dependent preferences. All investors are price takers, and purchase state-contingent payoffs from the banks they invest in. Nevertheless, we show that in equilibrium all investors purchase a combination of the standard and bail-in debt contracts issued by banks.

To streamline notation, we relabel R_1 as R and $f_{1,e}$ as f_e throughout this appendix. Denote $q(R,s)$ the (endogenous) probability-normalized price of a unit of payoff from a bank that realizes state (R,s).[67] Institutional investors are indexed by $i \in I$, have initial wealth w_0^i, and preferences $u_0^i(c_0^i) + E\left[u_1^i\left(c_1^i|s,\gamma(s)\right)\right]$. Retail investors are indexed by $j \in J$, have initial wealth w_0^j, and preferences $u_0^j(c_1^j) + E\left[u_1^j(c_1^j|s)\right]$. Both I and J are finite sets, and we interpret each investor type as corresponding to a continuum of (atomistic) agents of that type. Both types of agents have period-0 budget constraints given by

$$c_0^k + \sum_s \pi(s) \int_R q(R,s) x^k(R,s) f_H(R|s) dR = w_0^k, \quad k \in I \cup J.$$

However, they differ in their choice of c_1. Institutional investors are able to diversify across banks, so that $c_1^i(s) = \int_R x^i(R,s) f_H(R|s) dR$. Retail investors are not able to diversify across banks, and so have $c_1^j(R,s) = x^j(R,s)$. Given the contract payoff $x(R,s)$ from the bank, market clearing for liabilities is given by

$$\sum_{k \in I \cup J} \mu^k x_1^k(R,s) = x(R,s)$$

where μ^k is the mass of investors of type $k \in I \cup J$.

We focus on the case where the mass of retail investors is sufficiently small that it does not exhaust the returns of the bank in any state (R,s). That is, $\sum_j \mu^j x_1^j(R,s) < x(R,s)$. As a result, both retail and institutional investors price bank liabilities on the margin. We now characterize the

[67]Note that the bank will go bankrupt in some states, implying not all liabilities are repaid at full face value. For simplicity, we price units of payout directly, rather than face value.

equilibrium of the private economy without government intervention.

Proposition 18. *Suppose that in equilibrium $\sum_j \mu^j x_1^j(R,s) < x(R,s)$. In the private equilibrium:*

1. *The price $q(R,s) = q(s)$ depends only on the aggregate state s.*

2. *Optimal bank contracts combine standard and bail-in debt.*

3. *Retail investors only purchase standard debt, and their consumption profile $c_1^j(R,s) = c_1^j(s)$ only depends on the aggregate state s. Consumption profiles of retail investors are given by*

$$\frac{\partial u_1^j\left(c_1^j(s)|s\right)}{\partial c_1^j(s)} = q(s)\frac{\partial u_0^j\left(c_0^j\right)}{\partial c_0^j}$$

4. *Institutional investors purchase both standard and bail-in debt. Consumption profiles of institutional investors are given by*

$$\frac{\partial u_1^i\left(c_1^i(s)|s,\gamma(s)\right)}{\partial c_1^i(s)} = q(s)\frac{\partial u_0^i\left(c_0^i\right)}{\partial c_0^i}$$

Even though retail investors are tied to a specific bank, their equilibrium consumption profile does not depend on the idiosyncratic state. This implies not only that retail investors exclusively purchase standard debt, but also that retail investors are first in line for repayment in the event of bank liquidation. In other words, in equilibrium they purchase claims that have the highest priority for repayment. Since retail investors are often depositors, one natural interpretation of this result is that of deposit priority.[68] However, it extends beyond deposits, and furthermore suggests that retail bondholders may also benefit from priority. This suggests a role for non-bail-inable long-term debt, as a way to codify protection for retail investors.

Institutional investors are not exposed to the idiosyncratic state due to their ability to diversify, but are exposed to the aggregate state. Institutional investors face greater losses on the aggregate

[68]These deposits are not insured in this section, but are repaid due to their priority. In Appendix B.8, we consider deposit insurance.

state when either they are more risk tolerant, or less exposed to bank fire sales. This suggests that the ideal holders of bail-in debt will be institutional investors with limited risk aversion (or ability to diversify using other securities) and limited commonality with the banking sector, so that they are not affected by fire sales.

Finally, consider what would happen if we relaxed the assumption $\sum_j \mu^j x_1^j(R,s) < x(R,s)$. Consider an aggregate state s where $\sum_j \mu^j x_1^j(R,s) = x(R,s)$ for a range of returns $R \leq R^*$. For $R > R^*$, institutional investors are the marginal pricing agent, and $q(R,s) = q(s)$ is a constant. For $R < R^*$, retail investors are the marginal pricing agents, and $q(R,s) \geq q(s)$. Given monotone liabilities contracts, $q(R,s)$ will be falling in R. Contracts will still be debt, but the optimal thresholds are affected by the fact that retail investors suffer larger losses in liquidation, pushing $q(R,s)$ higher above $q(s)$. This generates an additional trade-off for the bank in deciding the optimal composition of standard and bail-in debt.

B.5.1 Proof of Proposition 18

Suppose that there is a state-contingent Arrow price $q(R,s) = q(s)$ that depends only on the aggregate state. Contracts still take the form of standard and bail-in debt, following the same steps as in the proof of Proposition 3.

Now, consider the investor side. Begin first with instituional investors, whose Lagrangian is given by

$$\begin{aligned}\mathscr{L}^i =& u_0^i\left(c_0^i\right) + \sum_s \pi(s) u_1^i\left(c_1^i|s,\gamma(s)\right) + \lambda^i\left[w_0^i - c_0^i - \sum_s \pi(s)\int_R q(R,s)x^i(R,s)f_H(R|s)dR\right] \\ &+ \sum_s \pi(s)\mu^i(s)\left[\int_R x^i(R,s)f_H(R|s)dR - c_1^i(s)\right].\end{aligned}$$

Given the non-negativity constraint $x^i(R,s) \geq 0$, we have

$$\frac{\partial \mathscr{L}^i}{\partial x^i(R,s)} = -\left[\lambda^i q(R,s) - \mu^i(s)\right]\pi(s)f_H(R|s) \leq 0.$$

This equation holds with equality only at the lowest value of $q(R,s)$ in state s. In other words, investors only purchase $x^i(R,s) > 0$ if $q(R,s) = q(s)$, where $q(s)$ is defined to be the lowest price of a state-contingent security for some return state R in state s.

Suppose then that in equilibrium $\sum_j \mu^j x_1^j(R,s) < x(R,s)$. Then, at least one institutional investor i is purchasing $x^i(R,s) > 0$. As a result, we have $q(R,s) = q(s)$ for all R in state s, that is the price is constant in aggregate state s. Moreover, $q(s)\lambda^i = \mu^i(s)$.

From here, we can obtain λ^i from the FOC for c_0^i and $\mu^i(s)$ from the FOC for c_1^i. Substituting in, we obtain

$$\frac{\partial u_1^i\left(c_1^i(s)|s,\gamma(s)\right)}{\partial c_1^i(s)} = \frac{\partial u_0^i\left(c_0^i\right)}{\partial c_0^i} q(s).$$

giving us the characterization of the consumption rules of institutional investors.

Finally, consider type-j (retail) investors. Given the constant price $q(s)$, their Lagrangian is

$$\mathscr{L}^j = u_0^j\left(c_0^j\right) + E\left[u_1^j\left(c_1^j(R,s)|s\right)\right] + \lambda^j\left(w_0^j - c_0^j - \sum_s \pi(s)\int_R q(s) c_1^j(R,s) f_H(R|s) dR\right),$$

so that we have optimality condition for $c_1^i(R,s)$

$$\frac{\partial u_1^j\left(c_1^j(R,s)|s\right)}{\partial c_1^j(R,s)} = \lambda^j q(s).$$

As a result, $c_1^j(R,s) = c_1^j(s)$ is constant within state s. The indifference condition follows immediately by combining with the FOC for c_0^j. This concludes the proof.

B.6 Outside Equity

The baseline model featured no role for (outside) equity-like instruments in the bank's capital structure. We extend the model to incorporate risk aversion and risk shifting, ingredients known to generate a role for equity-like claims. Optimal contracts still feature a region of liquidations and a region of "bail-ins," where the bank is held to its continuation agency rent. Above the bail-in region, the contract involves equity-like claims.[69]

To streamline notation, we relabel R_1 as R and $f_{1,e}$ as f_e throughout this appendix. Banks are risk averse and have utility $u(c_1+c_2)$ from consumption, while investors are risk averse and have utility $v(x_1+x_2)$. Bank utility and marginal utility are finite at 0, and we normalize $u(0)=0$. We incorporate risk shifting by extending the bank's monitoring decision to $e \in \{L, H, RS\}$, where $e = RS$ is "risk shifting" and $e \in \{L, H\}$ are the high and low monitoring choices from before. Risk shifting does not generate a private benefit but affects the return density, $f_{RS}(R)$.[70] Define the likelihood ratios $\lambda_{L,H}(R) = \frac{f_L(R)}{f_H(R)}$ and $\lambda_{RS,H}(R) = \frac{f_{RS}(R)}{f_H(R)}$. Risk shifting inefficiently pushes mass towards the extremes of the distribution, which we formalize by defining a point $R_{RS} \in [\underline{R}, \overline{R}]$ such that $\frac{\partial \lambda_{RS,H}(R)}{\partial R} < 0$ for $R < R_{RS}$ and $\frac{\partial \lambda_{RS,H}(R)}{\partial R} \geq 0$ for $R \geq R_{RS}$.

As before, we assume optimal contracts enforce $e = H$. The no-risk-shifting constraint is

$$\int_R u(c(R))\,(f_H(R) - f_{RS}(R))\,dR \geq 0 \tag{A.4}$$

while the incentive constraint is the same as before, except with $u(c(R))$. Investor participation is given by

$$Y_0 - A = \int_R v(x(R)) f_H(R) dR.$$

Define $\overline{\lambda}_H(R) = \frac{\mu_L}{\mu}\lambda_{L,H}(R) + \frac{\mu_{RS}}{\mu}\lambda_{RS,H}(R)$ and $\mu = \mu_L + \mu_{RS}$.

To simplify exposition, we will assume that the characterization that follows satisfies both consumption monotonicity for the bank and liability monotonicity for investors.[71] Characterization of contracts in settings that do not satisfy monotonicity is beyond the scope of this paper. Moreover, we assume that the region $1 + \mu(1 - \overline{\lambda}_H(R)) < 0$ is a connected set. This simplifies exposition.

Proposition 19. *Let $|S| = 1$. Suppose that the region $1 + \mu(1 - \overline{\lambda}_H(R)) < 0$ is a connected set. The prviately optimal contract is as follows.*

1. *In the region where $1 + \mu(1 - \overline{\lambda}_H(R)) < 0$, there are liquidations and bail-ins.*

[69] See e.g. Hilscher and Raviv (2014) for analysis of CoCo design on risk shifting.
[70] We could incorporate a private benefit or cost of risk shifting without qualitatively changing results.
[71] Note that because both agents are risk averse, there is less scope for live-or-die contracts.

2. *In the region where* $1+\mu(1-\overline{\lambda}_H(R)) \geq 0$*, there are bail-ins and "equity." The equity sharing rule is*

$$u'(c(R))\left(1+\mu(1-\overline{\lambda}_H(R))\right) = \lambda v'\left(RY_0 - c(R)\right)$$

The motivations behind the liquidation region and the bail-in region are as in the baseline model (we interpret bail-in region as automatically wiping out outside equity, but again there is an equivalence between principal write down and debt-equity conversion for bail-in debt). In the liquidation region, all other liabilities are wiped out. In the bail-in region, only "debt" holders are repaid. Consider next the "outside equity" region. First, bank risk aversion moderates payouts to the bank, smoothing the bank consumption profile on the upside and so giving away some of the equity value to investors. Second, bank consumption decreases with the average likelihood $\overline{\lambda}_H(R)$. In the region $R \leq R_{RS}$, $\overline{\lambda}_H(R)$ is decreasing in R and so banker consumption is increasing. However, when $R \geq R_{RS}$, $\lambda_{L,H}$ is falling while $\lambda_{RS,H}$ is rising. This second effect, which comes from the risk shifting motivation, moderates payoffs to banks in high return states, which signal a higher likelihood that the bank engaged in risk shifting.

We could also derive the socially optimal contract, which would internalize the fire sale spillover cost of liquidations. However, conditional on not liquidating, bank and planner incentives are aligned, suggesting that the planner needs only to control the trade-off between liquidations and non-liquidations, and not the trade-off between bail-ins and "equity."[72]

B.6.1 Proof of Proposition 19

Given the assumption of consumption monotonicity, if there is a liquidation region, it satisfies a threshold rule $R \leq R_l$. We define the optimal contract in terms of this threshold rule and in terms of

[72] If effort were a continuous choice variable that affected bank returns, there would be an incentive to govern this margin. See Mendicino et al. (2018) for a numerical study of this problem.

liaiblities $x(R)$ above this threshold. The bank's Lagrangian is given by

$$\begin{aligned}\mathscr{L} = &\int_{R\geq R_l} u(c(R)) f_H(R) dR \\ &+ \mu_L \left[\int_{R\geq R_l} u(c(R))\,(f_H(R)-f_L(R))\,dR - BY_0\right] + \mu_{RS}\left[\int_R u(c(R))\,(f_H(R)-f_{RS}(R))\,dR\right] \\ &+ \lambda\left[A + \int_{R\leq R_l} v(\gamma R Y_0) f_H(R) dR + \int_{R\geq R_l} v\,(RY_0 - c(R))\, f_H(R) dR - Y_0\right] \\ &+ \int_{R\geq R_l} \chi(R)\,[c(R) - bRY_0]\, f_H(R) dR\end{aligned}$$

Define $\overline{\lambda}_H(R) = \frac{\mu_L}{\mu}\lambda_{L,H}(R) + \frac{\mu_{RS}}{\mu}\lambda_{RS,H}(R)$ and $\mu = \mu_L + \mu_{RS}$. We can combine the second line and obtain

$$\begin{aligned}\mathscr{L} = &\int_{R\geq R_l} u(c(R)) f_H(R) dR \\ &+ \mu\left[\int_{R\geq R_l} u(c(R))\left[1-\overline{\lambda}_H(R)\right] f_H(R) dR - \frac{\mu_L}{\mu} BY_0\right] \\ &+ \lambda\left[A + \int_{R\leq R_l} v(\gamma R Y_0) f_H(R) dR + \int_{R\geq R_l} v\,(RY_0 - c(R))\, f_H(R) dR - Y_0\right] \\ &+ \int_{R\geq R_l} \chi(R)\,[c(R) - bRY_0]\, f_H(R) dR\end{aligned}$$

The derivative in R_l is given by

$$\frac{1}{f_H(R_l)}\frac{\partial \mathscr{L}}{\partial R_l} = -u(c(R_l))\left[1+\mu\left(1-\overline{\lambda}_H(R_l)\right)\right] + \lambda\left[v(\gamma R Y_0) - v(RY_0 - c(R_l))\right]$$

so that liquidations may be optimal when $1+\mu\left(1-\overline{\lambda}_H(R_l)\right) < 0$, that is when the average likelihood ratio is high. At low values of R_l, both the risk shifting and shirking problems have high likelihoods, so that $\overline{\lambda}_H$ is large. As a result, bank consumption contributes negatively to welfare. Provided that this negative contribution outweighs the resource cost to investors, we have $R_l > \underline{R}$.

Next, consider the region above R_l. The FOC for consumption $c(R)$ is

$$0 = u'(c(R))\left(1+\mu(1-\overline{\lambda}_H(R))\right) - \lambda v'(RY_0 - c(R)) + \chi(R)$$

so that we have $\chi(R) > 0$ when $1+\mu(1-\overline{\lambda}_H(R)) < 0$. As a result, for all values $1+\mu(1-\overline{\lambda}_H(R)) < 0$, we either have liquidation or bail-in.

Finally, for $1+\mu(1-\overline{\lambda}_H(R)) > 0$, we either have bail-in or an interior consumption value. When consumption is interior, it satisfies a risk sharing rule

$$u'(c(R))\left(1+\mu(1-\overline{\lambda}_H(R))\right) = \lambda v'(RY_0 - c(R))$$

giving us an "equity" sharing rule.

Finally, the only role of assuming $1+\mu(1-\overline{\lambda}_H(R)) < 0$ is a connected set in the proof is to ensure that it there are no points with $1+\mu(1-\overline{\lambda}_H(R)) \geq 0$ below R_l.

B.7 Premium for standard debt

In the baseline model, the incentive problem is the only motivation for issuance of standard debt. In practice, standard debt can enjoy a premium relative to all other instruments, meaning it can pay a lower rate of return to investors. There are two natural stories for such a premium. The first is that standard debt takes the form of demand deposits, which enjoy a liquidity premium and require a lower rate of return. The second is that standard debt enjoys preferential tax treatment. We show that contracts still feature standard and bail-in debt, and that the trade-off is largely the same up to the consideration of the return premium. We then discuss potential issues with a pure premium story for standard debt.

To streamline notation, we relabel R_1 as R and $f_{1,e}$ as f_e throughout this appendix.

Suppose that standard debt has required return $\frac{1}{1+r}$, where $r > 0$. We obtain the following result.

Proposition 20. *Suppose the model is extended to include a premium for standard debt. Optimal contracts combine standard and bail-in debt. The private optimality condition for standard debt is*

$$\mu b\left(\frac{f_L(R_l))}{f_H(R_l)}-1\right)=b+\lambda\left[(1-b)-\gamma\right]+r\left[\lambda\left[(1-b)-\gamma\right]-\lambda\frac{1-F_H(R_l)}{R_l f_H(R_l)}\right].$$

while the optimality condition for bail-in debt is the same as in Proposition 3. The tax on R_l that decentralizes the socially optimal contract is

$$\tau_l=-(1+r)R_l f_H(R_l)\frac{\partial\gamma(\Omega)}{\partial\Omega}\int_{\underline{R}}^{R_l} RY_0 f_H(R)dR$$

while the tax on bail-in debt is $\tau_u=0$.

Relative to the baseline case where $r=0$, when $r>0$ we have the term

$$r\left[\lambda\left[(1-b)-\gamma\right]-\lambda\frac{1-F_H(R_l)}{R_l f_H(R_l)}\right]$$

in the private optimality condition, reflecting an additional cost/benefit trade-off of increasing use of standard debt. This term contains two additional effects of the presence of the liquidity premium. On the one hand, the higher liquidity premium implies that the costs of liquidation go up, because the resources lost would have been repaid to investors who have a high willingness to pay. On the other hand, replacing bail-in debt with standard debt increases payoff to investors with high willingness to pay in non-liquidation states. The bank privately trades off these two forces in choosing the optimal standard debt level, in addition to the incentive forces.

Premium versus Incentive Problems. If $r>0$, then the bank is willing to issue standard debt even in the absence of an incentive problem, that is if $B=b=0$ and hence $\mu=0$. The premium story alone can generate use of standard debt in the bank's capital structure. However, in the absence of the incentive problem the logic of Corollary 5 applies. The bank (without loss of generality) uses equity as its other instrument.[73] The planner can implement optimal regulation with an equity requirement. By including the incentive problem, our model provides a role for bail-in debt in optimal contracts.

What if instead $B > 0$, $b = 0$, and $r > 0$, so that standard debt has value from a premium perspective, but not from an incentive perspective (relative to bail-in debt). In this case, the optimal contract would combine standard and bail-in debt. However, this story on its own is problematic for two reasons.

The first is that because bail-ins typically apply to long-term debt, which were also non-contingent prior to the crisis, the premium story revolves around premiums on long-term debt, which is likely due to tax incentives. But if the government is subsidizing (non-contingent) long-term debt, this suggests it must provide some fundamental economic benefit. Our model provides a fundamental economic benefit of non-contingent long-term debt.

A second and closely related way to understand this issue is that in the event that $b = 0$, banks have strong incentives to protect themselves against liquidations by backing their non-contingent claims with liquid assets such as treasuries. This relates to a fundamental question in the banking literature: why are illiquid assets paired with fragile (often deposit) financing? Our model endogenously pairs illiquid assets with fragile (non-contingent) financing, rather than exogenously imposing it. Optimal regulation in our model respects the fundamental activity of banks: backing illiquid assets with fragile funding. A model that relies exclusively on a standard debt premium naturally lends itself to a "narrow banking" result, where not only the planner but also banks prefer to use safe treasuries to keep the bank from ever failing.

We could nevertheless adopt this view. The main result that would change is the non-optimality of bailouts (Proposition 12), which would no longer generically hold. We would be back into an incomplete markets world, in which bailouts may be desirable to mitigate fire sales, in a standard way. Moreover in the case of deposit insurance, the planner would always prefer to bail out the

[73] As a technical aside, of course a bank with no incentive problem and an expected return greater than 1 would, given linear technology, scale up to infinity. This issue is fixed simply by assuming that banks operate a concave technology $Y_0 = f(I_0)$ to produce projects.

bank, rather than liquidating and repaying depositors. Bailing out the bank would save resources without distorting bank incentives, and so would be strictly preferred to liquidation.

B.7.1 Proof of Proposition 20

Relative to the baseline model, the only change is that the participation constraint becomes

$$Y_0 - A = \int_{\underline{R}}^{R_l} (1+r)\gamma R Y_0 f_H(R) dR + \int_{R \geq R_l}^{R_u} \left[(1+r)(1-b)R_l Y_0 + x_1(R)\right] Y_0 f_H(R) dR$$

where $x_1(R)$ is repayment pledged to other investors. Note that it is immediate that standard debt enjoys priority over other liabilities, since it has the lower required rate of return. The proof that optimal contracts combine standard and bail-in debt follows as in the proof of Proposition 3. As a result, the optimization problem that determines R_l and R_u is the same as before, except that the participation constraint is now

$$\begin{aligned} Y_0 - A = &\int_{\underline{R}}^{R_l} (1+r)\gamma R Y_0 f_H(R) dR + \int_{R_l}^{R_u} \left[(1+r)(1-b)R_l + (1-b)(R-R_l)\right] Y_0 f_H(R) dR \\ &+ \int_{R_u}^{\overline{R}} \left[(1+r)(1-b)R_l + (1-b)(R_u - R_l)\right] Y_0 f_H(R) dR \end{aligned}$$

This yields the private optimality condition for R_l

$$\begin{aligned} 0 = &- bR_l Y_0 f_H(R_l) - \mu b R_l Y_0 \left(1 - \frac{f_L(R_l))}{f_H(R_l)}\right) f_H(R_l) \\ &+ \lambda \left[(1+r)\gamma R_l Y_0 f_H(R_l) - (1+r)(1-b) R_l Y_0 f_H(R_l)\right] + \lambda \int_{R_l}^{\overline{R}} r(1-b) Y_0 f_H(R) dR \end{aligned}$$

which rearranges to

$$\mu b \left(\frac{f_L(R_l))}{f_H(R_l)} - 1\right) = b + \lambda\left[(1-b) - \gamma\right] + r\left[\lambda\left[(1-b)-\gamma\right] - \lambda \frac{1 - F_H(R_l)}{R_l f_H(R_l)}\right]$$

Because R_u is not directly impacted by the liquidity premium, the optimality condition for R_u is as before, assuming that $R_u > R_l$.

The planning problem features a wedge of the same form as before. The only difference is that the wedge is now weighted by $1+r$, reflecting the higher liquidation losses. In other words, the planning problem is decentralized by the tax

$$\tau_l = -(1+r)R_l f_H(R_l)\frac{\partial\gamma(\Omega)}{\partial\Omega}\int_{\underline{R}}^{R_l} RY_0 f_H(R)dR.$$

As before, R_u does not contribute to liquidations, and therefore $\tau_u = 0$.

B.8 Safety Premia and Insured Deposits

In this appendix, we study the interaction between our unobservable effort model and a safety premium story. This synergizes well with the observation that protecting insured deposits is another goal of bail-in regimes. It also allows us to shed further light on bail-ins versus bailouts as means of protecting insured deposits.

To streamline the model, we assume there are no fire sales (fixed γ).

There are special depositors who place an excess value $\beta > 0$ on a completely safe bank deposit at date 0, that is they are willing to pay $1+\beta$ for a safe deposit. At date 1, special depositors will withdraw their funds and be replaced by regular investors if rollover occurs. The number of special depositors that show up to a given bank is $(1-b)R_dY_0$ for a fixed $R_d > \underline{R}$. We thus abstract away from the optimal level R_d and focus on the residual capital structure and how the planner protects special depositors. We assume that the planner extends deposit insurance to special depositors, so that banks can treat special deposits as completely safe. The bank is always insolvent if $R < R_d$, absent intervention, regardless of its other liabilities. Because deposits are insured, the planner is liable for any shortfall relative to the face value $(1-b)R_dY_0$. Insured deposits are always at the top of the creditor hierarchy in liquidation.[74]

The planner chooses bailouts with commitment. Bailouts have a constant variable cost $tau > 0$.

[74] In practice, banks may issue wholesale funding which is not insured but runs prior to resolution.

Thus a bailout that recapitalizes an insolvent bank costs

$$\text{Cost}_{\text{No Liquidation}} = \tau\left(L(R_1) - (1-b)R_1Y_0\right)$$

where $L(R_1)$ is total liabilities including insured deposits.When the planner instead allows the bank to fail, insured deposits receive the entire liquidation value and are covered by deposit insurance, so that the cost in taxpayer funds is

$$\text{Cost}_{\text{Liquidation}} = \min\{\tau\left((1-b)R_dY_0 - \gamma R_1Y_0\right), 0\}$$

Note that even when $L(R_1) = (1-b)R_dY_0$ and there are only insured deposits remaining, the cost of rescuing the bank with a bailout is lower than the cost of rescuing the bank under liquidation, due to the loss of pledgeable income in liquidation.

The planner solves for the optimal contract, which includes the rescue decision (either via bailout or via liquidation and repayment by insurance).[75] We constrain bank consumption to be *monotone*, that is $c(R_1)$ must be nondecreasing in R_1,[76] which was satisfied by optimal contracts in the baseline model. This implies that bailouts must be monotone: if a an insolvent bank R_1 is bailed out, then all insolvent banks $R_1' \geq R_1$ must also be bailed out. This rules out the possibility that the planner bails out a bank with $R_1 < R_d$ to protect depositors but liquidates a bank with $R_1 > \frac{1-b}{\gamma}R_d$ for incentive reasons.

Proposition 21. *The socially optimal contract consists of insured deposits R_d, standard (uninsured) debt $R_\ell \geq R_d$, and bail-in debt $R_u \geq R_\ell$. The following are true:*

1. *If $R_\ell > R_d$, there is deposit insurance but no bailouts. The bank is liquidated when $R_1 \leq R_\ell$.*
2. *If $R_\ell = R_d$, there is a threshold $R_L \leq R_d$ such that the bank is liquidated when $R_1 \leq R_L$ and*

[75] A technical aside is that it is possible that the planner does not find it optimal to allow the bank to scale up as much as possible due to the cost of insuring deposits. We assume this is not the case, for example if R_d is close to $\underline{R}$.

[76] If $c(R_1) > c(R_1')$ but $R_1 < R_1'$, the bank could increase its payoff ex post by destroying assets to bring its return down to R_1. We look for contracts where value destruction is not ex post optimal.

bailed out when $R_L \leq R_1 \leq R_d$. The indifference condition is for bailouts (when interior) is

$$\mu b\left(\Lambda_1(R_L)-1\right)=b+\tau\left(1-b-\gamma\right).$$

Proposition 21 illustrates the capital structure decision and method of protecting insured deposits. If $R_\ell > R_d$, the optimal contract combines insured deposits and uninsured deposits. Intuitively, this will tend to occur when liquidation costs γ and bailout costs τ are not too high. If these costs are high, then $R_\ell = R_d$ and there are only insured deposits. This arises due to the trade-off between deposit insurance and bailouts for protecting special depositors. Bailing out the bank reduces the taxpayer cost of deposit insurance, but provides worse incentives for the bank. Whenever the planner allows use of standard debt in excess of insured deposits, that is $R_\ell > R_d$, then necessarily the planner will commit to rescue depositors but not the bank. In this case, there is deposit insurance but no bailouts. If $R_\ell = R_d$ and $R_L < R_d$, the planner uses bailouts ex post in order to reduce the cost of protecting depositors. Interestingly, this is a case where special depositors play both roles.

B.8.1 Proof of Proposition 21

Due to consumption monotonicity, there is a threshold $R_L \geq \underline{R}$ for bank liquidation, with $R_L = \underline{R}$ corresponding to no liquidations. As in the proof of Proposition 12, there are no bailouts above R_d, due to the taxpayer burden. We can thus split the problem into two parts.

First, suppose that the liquidation threshold satisfies $R_L > R_d$, and suppose that the planner finds it optimal to engage in bailouts in a states $R_1 < R_d$. By consumption monotonicity, there are also bailouts for $R_d \leq R_1 \leq R_L$. But then because transfers to regular investors are inefficient, it is optimal to set $R_L = R_d$, as in the proof of Proposition 12. The optimal contract does not feature both $R_L > R_d$ and bailouts. Consider then the form of the optimal contract when $R_L > R_d$. Because there are no bailouts, the social objective function is

$$\int c(R_1)f_{1,H}(R_1)dR_1 - \int_{R_1\leq R_L} \tau \max\{(1-b)R_d - \gamma R_1, 0\}Y_0 f_{1,H}(R_1)dR_1$$

while the corresponding investor participation constraint is

$$Y_0 - A_0 = \beta(1-b)R_d Y_0 + \int_{\underline{R}}^{R_L} \max\{(1-b)R_d, \gamma R_1\} Y_0 f_{1,H}(R_1) dR_1 + \int_{R_1 \geq R_L} ((1-b)R_d + x(R_1)) f_{1,H}(R_1) dR_1$$

where x is repayment to regular investors, and where incentive compatibility is the same a in the baseline model. From here, note that the trade-off above R_L is the same as in the baseline model. The model again combines standard and bail-in debt, as in the baseline model.

Consider next the optimal contract when $R_L < R_d$. R_L then also corresponds to the bailout threshold, such that there are bailouts when $R_L \leq R_1 \leq R_d$, and where $R_L = R_d$ corresponds to no bailouts. The resulting social objective function is

$$\int c(R_1) f_{1,H}(R_1) dR_1 - \int_{\underline{R}}^{R_L} \tau \left[(1-b)R_d - \gamma R_1\right] Y_0 f_{1,H}(R_1) dR_1 - \int_{R_L}^{R_d} \tau(1-b)\left(R_d - R_1\right) Y_0 f_{1,H}(R_1) dR_1$$

while investor repayment is given by

$$Y_0 - A = (1+\beta)(1-b)R_d Y_0 + \int_{R_d}^{\overline{R}} x(R_1) f_{1,H}(R_1) dR_1$$

reflecting that depositors are always repaid. Finally, incentive compatibility is as in the baseline model. Optimal contracts again combine standard and bail-in debt.

Consider the choice of the liquidation threshold R_L. The trade-off is the same as in the baseline model, expect that an increase in the liquidation threshold leads to a tax burden on taxpayers rather than a cost to investors. That is, the FOC for the liquidation threshold is

$$0 = -bR_L - \mu b R_L \left(1 - \Lambda_1(R_L)\right) - \tau\left[(1-b)R_d - \gamma R_L - (1-b)(R_d - R_L)\right]$$

which simplifies to

$$b + \tau\left(1 - b - \gamma\right) = \mu b \left(\Lambda_1(R_L) - 1\right).$$

The only change is that the effective costs of liquidations has risen, due to the greater burden on taxpayers ($\tau > \lambda$). If the solution to this equation features $R_L < R_d$, then there are bailouts in states $R_L \leq R_1 \leq R_d$.

B.9 Bailouts and Time Consistency

It is useful to consider how lack of commitment can change our result.

We begin by studying *collective moral hazard* (Farhi and Tirole 2012), where bailout temptation results from many small banks failing simultaneously. Formally, we suppose that there is a fixed cost of engaging in any bailouts, FY_0, which scales with the size of the overall banking system. After paying the fixed cost, the cost of bailouts is simply the utilitarian transfer cost of 1. For simplicity, suppose there is no fire sale and γ is fixed (although we can obtain similar results with a fire sale).

Consider the problem of a utilitarian bailout authority ex post. The total loss from liquidations is given by

$$\int_{\underline{R}}^{R_\ell} (1-\gamma) R_1 Y_0 f_{1,H}(R_1) dR_1.$$

Since $\gamma < 1$, if the planner engages in any bailouts, then the planner bails out all insolvent banks. Therefore, the planner engages in bailouts when

$$\int_{\underline{R}}^{R_\ell} R_1 f_{1,H}(R_1) dR_1 > \frac{F}{1-\gamma}$$

The left hand side is an increasing function of R_ℓ, whereas the RHS is constant in R_ℓ. Therefore, there is a threshold R_ℓ^* such that all banks are bailed out if $R_\ell > R_\ell^*$, and no banks are bailed out if $R_\ell < R_\ell^*$. R_ℓ^* increases in the fixed cost and in the liquidation value.

From here, we obtain the following result.

Proposition 22. *In the model with collective moral hazard, the social planner imposes the requirement $R_\ell \leq R_\ell^*$. No bailouts occur.*

Proposition 22 illustrates that when time consistency results from collective moral hazard, it can still be efficient to rule out bailouts. Here, ruling out bailouts is distortionary when R_ℓ^* is lower than the optimal level without bailouts. This means welfare is reduced by the need to rule out bailouts.

Next, we suppose that rather than collective moral hazard, the social planner has limited fiscal capacity. Formally, the planner has total resources XY_0 available from taxpayers to engage in bailouts at date 1. These resources have no cost beyond utilitarian cost, but there is no fixed cost of accessing them.

When $X > 0$, it is easy to see that whenever $R_\ell > \underline{R}$, some bailouts occur. The reason is that the marginal insolvent bank, $R_1 \uparrow R_\ell$, requires a bailout of size $T \downarrow 0$ to recapitalize, but results in a resource saving of $(1-\gamma)R_\ell$. More generally, the utilitarian cost means the planner ex post uses its entire fiscal capacity to bail banks out. The cost of bailing out a bank with $R_1 < R_\ell$ is $(1-b)(R_\ell - R_1)Y_0$, so the planner bailouts out banks starting from the highest return in order to capitalize on fiscal slack. In other words, total bailouts result in a new threshold R_ℓ^x given by

$$\int_{R_\ell^x}^{R_\ell} (1-b)(R_\ell - R_1) f_{1,H}(R_1) dR_1 = X.$$

From here, we can differentiate the above equation to obtain

$$-\frac{\partial R_\ell^x}{\partial R_\ell}(1-b)(R_\ell - R_\ell^x) + \int_{R_\ell^x}^{R_\ell} (1-b) f_{1,H}(R_1) dR_1 = 0$$

meaning that R_ℓ^x is an increasing function of R_ℓ, that is more banks are liquidated as R_ℓ rises. Intuitively, higher R_ℓ forces the planner to exhaust its fiscal slack more quickly, allowing more liquidations.

If fiscal slack is not too large, it turns out the planner can implement the private optimum in a manner that results in ex post bailouts (recall that we have abstracted away from fire sales).

Proposition 23. *There is a threshold fiscal capacity $\overline{X}$ such that if $X < \overline{X}$, then the planner can* *implement the private optimum in a Pareto efficient manner. In particular, the planner sets $R_\ell \leq R_u$ such that R_ℓ^x is equal to the privately optimal liquidation rule of Proposition 3. Bailouts occur for banks $R_\ell^x < R_1 \leq R_\ell$. The planner requires a lump sum transfer of X from banks to taxpayers at date* 0 *to compensate taxpayers for bailouts.*

The intuition of Proposition 23 is that when bailouts are costless (aside from the transfer), then the planner cannot rule bailouts out ex post without requiring banks to set $R_\ell = \underline{R}$. However, the planner can exhaust the ex post fiscal capacity by pledging R_ℓ *higher* than the private optimum. In particular if fiscal capacity is not too large, the planner can set $R_\ell \leq R_u$ to be high enough that, after fiscal capacity is exhausted, the marginal liquidated bank R_ℓ^x is exactly the one from the privately optimal contract. The requirement $X < \overline{X}$ is needed to ensure that the level of standard debt needed to achieve R_ℓ^x while exhausting fiscal capacity is below the total debt level R_u associated with the private optimum. Finally, to achieve a Pareto efficient allocation the planner needs to engage in a lump sum transfer of X from banks to taxpayers, so that taxpayers are no worse off than if there were no bailouts.

If instead $X > \overline{X}$, the planner would have to set $R_\ell > R_u$ to get X to its privately optimal level, but this no longer implements the private optimum. Thus, the planner has to move to a different (privately suboptimal) contract. In this case, bailouts are costly in the sense that they distort the private optimum, similarly to collective moral hazard.

In both cases, contract distortions arise when it is easier for the planner to engage in ex post bailouts. In the first case, distortions are larger as F falls and hence R_ℓ^* falls. In the second case, distortions occur when fiscal capacity becomes sufficiently large, $X > \overline{X}$. In this sense, both cases are consistent with the notion that a planner would want to try to tie their hands ex ante to make ex post bailouts harder.

B.9.1 Proof of Proposition 22

Suppose that the planner chose $R_\ell > R_\ell^*$. In this case, all banks are bailed out ex post. Hence, this contract is equivalent to a contract featuring only bail-in debt, $R_\ell = \underline{R}$, combined with an ex ante lump sum transfer from taxpayers to banks. Selecting an ex ante welfare weight on taxpayers of $\omega^T = \lambda$, bailout transfers reduce welfare due to the fixed cost. Hence, mandating $R_\ell = \underline{R}$ is superior to choosing $R_\ell > \overline{R}$ and generating bailouts. Hence, the planner restricts banks to choose $R_\ell \leq R_\ell^*$, and no bailouts occur.

B.9.2 Proof of Proposition 23

Let R_u be the total debt level under the privately optimal contract. Note that R_ℓ^x decreases in fiscal capacity X and increases in R_ℓ. Set R_ℓ^x to the threshold under the private optimum. Then, an increase in fiscal capacity X decreases R_ℓ^x, and so requires an increase in R_ℓ. Therefore, there is a maximum fiscal capacity $\overline{X}$ so that $R_\ell \leq R_u$. From here, suppose that $X < \overline{X}$ and the planner chooses (R_ℓ^x, R_u) according to the private optimum. The required R_ℓ then lies below R_u. Bailouts of banks $R_\ell^x < R_1 \leq R_\ell$ occur, meaning that banks with $R_1 \leq R_\ell^x$ are liquidated and banks with $R_\ell^x \leq R_1 \leq R_u$ continue in such a manner that banks are just paid their minimum agency rent. Thus we have the same feasible contract as the private optimum, except that banks' participation constraint is slackened by the bailout transfer X, that is we effectively have inside equity $A_0 + X$. Imposing a lump sum transfer X from banks to taxpayers reduces banks' inside equity back to A_0. This implements the privately optimal contract in a manner that makes taxpayers no worse off, and hence is efficient.